Can the Nambu-Goldstone Boson Live on the Light Front?

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We show that the Nambu-Goldstone (NG) boson restricted on the light front (LF) can only exist if we regularize the theory by introducing the explicit symmetry breaking NG-boson mass m_π. The NG-boson zero mode, when integrated over the LF, must have a singular behavior $\sim 1/m_\pi^2$ in the symmetric limit of $m_\pi^2 \rightarrow 0$. In the discretized LF quantization this peculiarity is clarified in terms of the zero-mode constraints in the linear σ model. The LF charge annihilates the vacuum, while it is not conserved in the symmetric limit in the NG phase.

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Recently there has been renewed interest in light-front (LF) quantization [1] as a promising approach to solve nonperturbative dynamics [2,3]. Based on the trivial vacuum structure, the LF quantization with a Tamm-Dancoff truncation has successfully described bound state spectra and their wave functions in several field theoretical models in (1+1) dimensions, particularly within the framework of the discretized LF quantization (DLFQ) [4,5]. However, realistic theories such as QCD in (3+1) dimensions include rich structures such as confinement, spontaneous symmetry breaking (SSB), etc., which are basically on account of the nontrivial vacuum in the conventional equal-time quantization. How can one reconcile such a nontrivial structure of the theory with the trivial vacuum of the LF quantization? It seems to be now a general consensus that the zero mode [4] plays an essential role to realize the spontaneous symmetry breaking on the LF [3,6,7]. The problem of the zero mode in the LF vacuum was first addressed back in 1976 by Maskawa and Yamawaki [4], who discovered, within the canonical theory of DLFQ, the second class constraint, the so-called zero mode constraint, through which the zero mode is not an independent degree of freedom but a complicated operator-valued function of all other modes. One may thus expect that solving the vacuum state in the ordinary equal-time quantization is traded for solving the operator zero mode in the LF quantization. Actually, several authors have recently argued in (1+1)-dimensional models that the zero-mode solution might induce the spontaneous breaking of discrete symmetries [7]. However, the most outstanding feature of spontaneous symmetry breaking is the existence of the Nambu-Goldstone (NG) boson for continuous symmetry breaking. Thus the real question to be addressed is whether or not the zero-mode solution automatically produces the NG phase, particularly in (3+1) dimensions.

In this paper we shall show, in the context of DLFQ, how the NG phenomenon is realized due to the zero modes in (3+1) dimensions, while the vacuum remains in the trivial LF vacuum. We encounter a striking feature of the zero mode of the NG boson: Naive use of zero-mode constraints does not lead to the NG phase at all ("no-go theorem"), in contrast to the current expectation mentioned above. Within the DLFQ, it is inevitable to introduce an infrared regularization by the explicit symmetry breaking mass of the NG boson m_π. The NG phase can only be realized via peculiar behavior of the zero mode of the NG-boson fields: The NG-boson zero mode, when integrated over the LF, must have a singular behavior $\sim 1/m_\pi^2$ in the symmetric limit $m_\pi^2 \rightarrow 0$. This we demonstrate both in a general framework of the LSZ reduction formula and in a concrete field theoretical model, the linear σ model, within a framework of DLFQ. The NG phase is, in fact, realized in such a way that the vacuum is trivial while the LF charge is *not conserved* in the symmetric limit $m_\pi^2 \rightarrow 0$.

Let us first prove a no-go theorem that the naive LF restriction of the NG-boson field leads to vanishing of both the NG-boson emission vertex and the corresponding current vertex; namely, the NG phase is not realized in the LF quantization.

Based on the LSZ reduction formula, the NG-boson emission vertex $A \rightarrow B + \pi$ may be written as

$$\begin{aligned}\langle B\pi(q)|A\rangle &= i\int d^4x\, e^{iqx}\langle B|\Box\pi(x)|A\rangle \\ &= i(2\pi)^4\delta(p_A^- - p_B^- - q^-)\delta^{(3)}(\vec{p}_A - \vec{p}_B - \vec{q}\,) \\ &\quad\times \langle B|j_\pi(0)|A\rangle, \end{aligned} \tag{1}$$

where $\pi(x)$ and $j_\pi(x) = \Box\pi(x) = (2\partial_+\partial_- - \partial_\perp^2)\pi(x)$ are the interpolating field and the source function of the NG boson, respectively, and $q^\mu = p_A^\mu - p_B^\mu$ are the NG-boson four momenta and $\vec{q} \equiv (q^+, q^\perp)$ [8]. It is customary [9] to take the collinear momentum, $\vec{q} = 0$ and $q^- \neq 0$ (not a soft momentum), for the emission vertex of the exactly massless NG boson with $q^2 = 0$. Here we adopt the DLFQ, $x^- \in [-L, L]$, with a periodic boundary condition [10] in the x^- direction and take the continuum limit $L \rightarrow \infty$ in the end of the whole calculation [4]. Without specifying the boundary condition, we would not be able to formulate consistently the LF quantization anyway, even in the continuum theory [11]. Then the

NG-boson emission vertex should vanish on the LF due to the periodic boundary condition:

$$(2\pi)^3 \delta^{(3)}(\vec{p}_A - \vec{p}_B) \langle B | j_\pi(0) | A \rangle = \int d^2 x^\perp \lim_{L\to\infty} \langle B | \Big(\int_{-L}^{L} dx^- \, 2\partial_- \partial_+ \pi \Big) | A \rangle = 0 \,. \quad (2)$$

Another symptom of this disease is the vanishing of the current vertex (analog of g_A in the nucleon matrix element). When the continuous symmetry is spontaneously broken, the NG theorem requires that the corresponding current J_μ contains an interpolating field of the NG boson $\pi(x)$, that is, $J_\mu = -f_\pi \partial_\mu \pi + \widehat{J}_\mu$, where f_π is the "decay constant" of the NG boson and $\widehat{J}_\mu$ denotes the nonpole term. Then the current conservation $\partial_\mu J^\mu = 0$ leads to

$$0 = \langle B | \int d^3\vec{x} \; \partial_\mu \widehat{J}^\mu(x) | A \rangle_{x^+=0} = -i(2\pi)^3 \delta^{(3)}(\vec{q}\,) \, \frac{m_A^2 - m_B^2}{2p_A^+} \langle B | \widehat{J}^+(0) | A \rangle \,, \quad (3)$$

where $\int d^3\vec{x} \equiv \lim_{L\to\infty} \int_{-L}^{L} dx^- \, d^2x^\perp$ and the integral of the NG-boson sector $\Box\pi$ has no contribution on the LF because of the periodic boundary condition as we mentioned before. Thus the current vertex $\langle B | \widehat{J}^+(0) | A \rangle$ should vanish at $q^2 = 0$ as far as $m_A^2 \neq m_B^2$.

This is actually a manifestation of the conservation of a charge $\widehat{Q} \equiv \int d^3\vec{x}\, \widehat{J}^+$, which is constructed only from the nonpole term. Note that $\widehat{Q}$ is equivalent to the full LF charge $Q \equiv \int d^3\vec{x}\, J^+$, since the pole part always drops out of Q due to the integration on the LF, i.e., $Q = \widehat{Q}$. Therefore the conservation of $\widehat{Q}$ inevitably follows from the conservation of Q: $[\widehat{Q}, P^-] = [Q, P^-] = 0$, which, in fact, implies the vanishing current vertex mentioned above. This is in sharp contrast to the charge integrated over usual space $\boldsymbol{x} = (x^1, x^2, x^3)$ in the equal-time quantization: $Q^{\mathrm{et}} = \int d^3\boldsymbol{x}\, J^0$ is conserved while $\widehat{Q}^{\mathrm{et}} = \int d^3\boldsymbol{x}\, \widehat{J}^0$ is not.

Thus the NG bosons are totally decoupled, i.e., the NG phase is not realized on the LF. Note that this is a direct consequence of the periodic boundary condition and the first-order form of $\Box = 2\partial_+\partial_- - \partial_\perp^2$ in $\partial_\pm$ in contrast to the second-order form in ∂_0 in the equal-time quantization.

Now, we propose to regularize the theory by introducing explicit symmetry breaking mass of the NG boson m_π. The essence of the NG phase with a small explicit symmetry breaking can well be described by the old notion of the PCAC (partial conservation of axial vector current) hypothesis: $\partial_\mu J^\mu(x) = f_\pi m_\pi^2 \pi(x)$, with $\pi(x)$ being the interpolating field of the (pseudo) NG boson π. From the PCAC relation the current divergence of the nonpole term $\widehat{J}^\mu(x)$ reads $\partial_\mu \widehat{J}^\mu(x) = f_\pi(\Box + m_\pi^2)\pi(x) = f_\pi j_\pi(x)$. Then we obtain

$$\langle B | \int d^3\vec{x} \; \partial_\mu \widehat{J}^\mu(x) | A \rangle = f_\pi m_\pi^2 \langle B | \int d^3\vec{x} \; \pi(x) | A \rangle = \langle B | \int d^3\vec{x} \; f_\pi j_\pi(x) | A \rangle \,, \quad (4)$$

where the integration of the pole term $\Box\pi(x)$ is dropped out as before. The second expression of (4) is nothing but the matrix element of the LF integration of the π zero mode (with $P^+ = 0$) $\omega_\pi \equiv (1/2L) \int_{-L}^{L} dx^- \, \pi(x)$. Suppose that $\int d^3\vec{x}\, \omega_\pi(x) = \int d^3\vec{x}\, \pi(x)$ is regular when $m_\pi^2 \to 0$. Then this leads to the no-go theorem again. Thus in order to have the nonzero NG-boson emission vertex [right hand side (rhs) of (4)] as well as the nonzero current vertex [left hand side (lhs)] at $q^2 = 0$, the π zero mode $\omega_\pi(x)$ must behave as

$$\int d^3\vec{x} \; \omega_\pi \sim \frac{1}{m_\pi^2} \quad (m_\pi^2 \to 0) \,. \quad (5)$$

This situation may be clarified when the PCAC relation is written in the momentum space:

$$\frac{m_\pi^2 f_\pi j_\pi(q^2)}{m_\pi^2 - q^2} = \partial^\mu J_\mu(q) = \frac{q^2 f_\pi j_\pi(q^2)}{m_\pi^2 - q^2} + \partial^\mu \widehat{J}_\mu(q) \,. \quad (6)$$

What we did when we reached the no-go theorem can be summarized as follows. We first set the lhs of (6) to zero [or equivalently, assumed implicitly the regular behavior of $\int d^3\vec{x}\, \omega_\pi(x)$] in the symmetric limit in accord with the current conservation $\partial^\mu J_\mu = 0$. Then in the LF formalism with $\vec{q} = 0$ ($q^2 = 0$), the first term (NG-boson pole term) of the rhs was also zero due to the periodic boundary condition or the zero-mode constraint. Thus we arrived at $\partial^\mu \widehat{J}_\mu(q) = 0$. However, this procedure is equivalent to playing a nonsense game:

$$\lim_{m_\pi^2, q^2 \to 0} \left(\frac{m_\pi^2 - q^2}{m_\pi^2 - q^2} \right) = 0$$

as far as $f_\pi j_\pi \neq 0$ (NG phase). Therefore the "$m_\pi^2 = 0$" theory with a vanishing lhs is ill defined on the LF, namely, the no-go theorem is false. The correct procedure should be to take the symmetric limit $m_\pi^2 \to 0$ *after* the LF restriction $\vec{q} = 0$ ($q^2 = 0$) [12], although (6) itself yields the same result $f_\pi j_\pi = \partial^\mu \widehat{J}_\mu$, irrespective of the order of the two limits $q^2 \to 0$ and $m_\pi^2 \to 0$. Then (5) does follow. This implies that at quantum level the LF charge $Q = \widehat{Q}$ is not conserved, or the current conservation does not hold for a particular Fourier component with $\vec{q} = 0$ even in the symmetric limit:

$$\frac{1}{i} [Q, P^-] = \partial^\mu J_\mu |_{\vec{q}=0} = f_\pi \lim_{m_\pi^2 \to 0} m_\pi^2 \int d^3\vec{x} \; \omega_\pi \neq 0 \,. \quad (7)$$

Let us now demonstrate that (5) and (7) indeed take place *as the solution of the constrained zero modes* in the NG phase of the O(2) linear σ model:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{2}\mu^2(\sigma^2 + \pi^2) - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2 + c\sigma \,, \quad (8)$$

where the last term is the explicit symmetry breaking, which regularizes the NG-boson zero mode.

In the DLFQ we can clearly separate the zero modes (with $P^+ = 0$), $\pi_0 \equiv (1/2L) \int_{-L}^{L} dx^- \, \pi(x)$ (similarly for σ_0), from other oscillating modes (with $P^+ \neq 0$), $\varphi_\pi \equiv \pi - \pi_0$ (similarly for φ_σ). Through the Dirac quantization of the constrained system the canonical commutation relation for the oscillating modes reads [4]

$$[\varphi_i(x), \varphi_j(y)] = -\frac{i}{4}\left\{\epsilon(x^- y^-) - \frac{x^- y^-}{L}\right\} \times \delta_{ij}\delta^{(2)}(x^\perp - y^\perp), \qquad (9)$$

where each index stands for π or σ, and the $\epsilon(x)$ is the sign function. By use of (9) we can introduce creation and annihilation operators simply defined by the Fourier expansion of φ_i with respect to x^- even when the interaction is included. Thus the physical Fock space is constructed upon the LF vacuum ("trivial vacuum") which is defined to be annihilated by the annihilation operators without recourse to the dynamics.

On the other hand, the zero modes are not independent degrees of freedom but are implicitly determined by φ_σ and φ_π through the second class constraints (so-called zero-mode constraints) [4]:

$$\chi_\pi \equiv \frac{1}{2L}\int_{-L}^{L} dx^- [(\mu^2 - \partial_\perp^2)\pi + \lambda\pi(\pi^2 + \sigma^2)] = 0, \qquad (10)$$

and similarly, $\chi_\sigma \equiv (1/2L) \int_{-L}^{L} dx^- \{[\pi \to \sigma] - c\} = 0$. Note that through the equation of motion these constraints are equivalent to the characteristic of the DLFQ with periodic boundary condition, $\chi_\pi = -(1/2L) \int_{-L}^{L} dx^- \, 2\partial_+ \partial_- \pi = 0$ (similarly for σ), which we have used to prove the no-go theorem for the case of $m_\pi^2 \equiv 0$.

Actually, in the NG phase ($\mu^2 < 0$) the equation of motion of π reads $(\Box + m_\pi^2)\pi(x) = -\lambda(\pi^3 + \pi\sigma'^2 + 2v\pi\sigma') \equiv j_\pi(x)$, with $\sigma' = \sigma - v$ and $m_\pi^2 = \mu^2 + \lambda v^2 = c/v$, where $v \equiv \langle\sigma\rangle$ is the classical vacuum solution determined by $\mu^2 v + \lambda v^3 = c$. Integrating the above equation of motion over $\vec{x}$, we have

$$\int d^3\vec{x}\, j_\pi(x) - m_\pi^2 \int d^3\vec{x}\, \omega_\pi(x) = \int d^3\vec{x}\, \Box\pi(x) = -\int d^3\vec{x}\, \chi_\pi = 0, \qquad (11)$$

where $\int d^3\vec{x}\, \omega_\pi(x) = \int d^3\vec{x}\, \pi(x)$. Were it not for the singular behavior (5) for the π zero mode ω_π, we would have concluded $(2\pi)^3\delta^{(3)}(\vec{q})\langle\pi|j_\pi(0)|\sigma\rangle = -\langle\pi| \int d^3\vec{x}\, \chi_\pi|\sigma\rangle = 0$ in the symmetric limit $m_\pi^2 \to 0$. Namely, the NG-boson vertex at $q^2 = 0$ would have vanished, which is exactly what we called the no-go theorem now related to the zero-mode constraint χ_π. On the contrary, direct evaluation of the matrix element of $j_\pi = -\lambda(\pi^3 + \pi\sigma'^2 + 2v\pi\sigma')$ in the lowest order perturbation yields a nonzero result even in the symmetric limit $m_\pi^2 \to 0$: $\langle\pi|j_\pi(0)|\sigma\rangle = -2\lambda v\langle\pi|\varphi_\sigma\varphi_\pi|\sigma\rangle = -2\lambda v \neq 0$ ($\vec{q} = 0$), which is in agreement with the usual equal-time formulation. Thus we have seen that naive use of the zero-mode constraints by setting $m_\pi^2 \equiv 0$ leads to the internal inconsistency in the NG phase. The no-go theorem is again false.

We now study the solution of the zero-mode constraints in the perturbation around the classical (tree level) SSB vacuum, since we need to formulate the NG phase on the LF at least for the theory whose SSB is already described at the tree level in the equal-time quantization. It is convenient to divide the zero modes π_0 (or σ_0) into a classical constant piece v_π (or v_σ) and an operator part ω_π (or ω_σ), as do the zero-mode constraints. The classical part of the zero-mode constraints is nothing but the condition that determines the minimum of the classical potential, and we have chosen a solution that $v_\pi = 0$ and $v_\sigma \equiv v$; i.e., $\pi_0 = \omega_\pi$, $\sigma_0 = \omega_\sigma + v$. The operator zero modes are solved perturbatively by substituting the expansion $\omega_i = \sum_{k=1} \lambda^k \omega_i^{(k)}$ under the Weyl ordering.

The lowest-order solution of the zero-mode constraints χ_π and χ_σ for ω_π takes the form

$$(-m_\pi^2 + \partial_\perp^2)\,\omega_\pi = \frac{\lambda}{2L}\int_{-L}^{L} dx^-(\varphi_\pi^3 + \varphi_\pi\varphi_\sigma^2 + 2v\varphi_\pi\varphi_\sigma). \qquad (12)$$

Then (5) immediately follows [13]:

$$\lim_{m_\pi^2 \to 0} m_\pi^2 \int d^3\vec{x}\, \omega_\pi = -\lambda \int d^3\vec{x}\,(\varphi_\pi^3 + \varphi_\pi\varphi_\sigma^2 + 2v\varphi_\pi\varphi_\sigma) \neq 0. \qquad (13)$$

This is our main result. This actually ensures a nonzero $\sigma \to \pi\pi$ vertex through (11): $\langle\pi|j_\pi(0)|\sigma\rangle = -2\lambda v$, which agrees with the previous direct evaluation as it should.

Let us next discuss the LF charge operator. The O(2) current in this model is given by $J_\mu = \partial_\mu\sigma\pi - \partial_\mu\pi\sigma$. As was noted in Ref. [4], the corresponding LF charge $Q = \hat{Q} = \int d^3\vec{x}\,(\partial_-\varphi_\sigma\varphi_\pi - \partial_-\varphi_\pi\varphi_\sigma)$ *contains no zero modes* including the π pole term which was dropped by the integration due to the periodic boundary condition and the ∂_-, so that Q is well defined even in the NG phase and hence annihilates the vacuum simply by the P^+ conservation:

$$Q|0\rangle = 0. \qquad (14)$$

This is also consistent with explicit computation of the commutators: $\langle[Q, \varphi_\sigma]\rangle = -i\langle\varphi_\pi\rangle = 0$ and $\langle[Q, \varphi_\pi]\rangle =$

$i\langle\varphi_\sigma\rangle = 0$ [14], which are contrasted to those in the usual equal-time case, where the spontaneously broken charge does not annihilate the vacuum: $\langle[Q^{\rm et}, \sigma]\rangle = -i\langle\pi\rangle = 0, \langle[Q^{\rm et}, \pi]\rangle = i\langle\sigma\rangle \neq 0$.

Since the PCAC relation is now an operator relation for the canonical field $\pi(x)$ with $f_\pi = v$ in this model, (13) ensures $[\hat{Q}, P^-] \neq 0$ or a nonzero current vertex $\langle\pi|\hat{J}^+|\sigma\rangle \neq 0$ ($q^2 = 0$) in the symmetric limit. Noting that $Q = \hat{Q}$, we conclude that the regularized zero-mode constraints indeed lead to nonconservation of the LF charge in the symmetric limit $m_\pi^2 \to 0$:

$$[Q, P^-] = iv \lim_{m_\pi^2 \to 0} m_\pi^2 \int d^3\vec{x}\, \omega_\pi \neq 0\,. \qquad (15)$$

This can also be confirmed by direct computation of $[Q, P^-]$ through the canonical commutator and explicit use of the regularized zero-mode constraints. At first sight there seems to be no distinction between the spontaneous and the explicit symmetry breakings on the LF. However, the singular behavior of the NG-boson zero mode (5) or (13) may be understood as a characterization of the spontaneous symmetry breaking.

Our result implies that solving the zero-mode constraints *without regularization* would *not* lead to the NG phase at all in contradiction to the naive expectation [7]. Our treatment of the zero modes in the canonical DLFQ is quite different from that proposed recently by Wilson *et al.* [3], who eliminate the zero modes by hand in the continuum theory instead of solving the zero-mode constraints. They also arrived at the nonconservation of the LF charge *without zero mode*. The relationship between these two approaches is not clear at the moment. Finally, it should be noted that there exists another no-go theorem that forbids any LF field theory (even the free theory) satisfying the Wightman axioms [15]. This no-go theorem is also related to the zero modes but has not yet been overcome by the DLFQ or any other existing approach and is beyond the scope of this paper.

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