New Phenomena in Josephson *SINIS* Junctions

A. F. Volkov
Institute of Radioengineering and Electronics of the Russian Academy of Sciences, Mokhovaja 11, Moscow 103907, Russia
(Received 5 December 1994)

We analyze the dc and ac Josephson effects in S_aINIS_b junctions in which an additional bias current flows in the N layer. The case of low temperatures and voltages (eV, $T \ll \Delta$) is considered in the dirty limit. We show that the critical Josephson current may change sign, and the considered *SINIS* junction may become a π junction if the voltage drop across the N/S_a interface exceeds a certain value ($eV_N > \Delta/\sqrt{2}$). The ac Josephson effect may arise even if the current flows only through the N/S_a interface, whereas the current through the S_b/N interface is absent.

PACS numbers: 74.50.+r

In recent years, considerable attention has been attracted to studying *SINIS* junctions (S, I, and N denote a superconducting, insulating, and normal metal layer, respectively). In particular, measurements have been carried out on *S-Sm-S* junctions which behave like *SINIS* junctions (*Sm* is a heavily doped semiconductor with a three- or two-dimensional electron gas) [1–6]. The role of insulating barrier is played by the Schottky barrier which arises at the interface between the superconductor and the semiconductor. A mismatch between electron parameters such as the effective masses and Fermi momenta in *Sm* and *S* leads also to an additional scattering at the interface. The *SINIS* junctions in which a normal metal or a semiconductor are used as the middle layer were studied to clarify, for example, the mechanism of the charge transfer through the S/N interface at low voltages ($eV < \Delta$) and temperatures ($T \ll \Delta$) [1–8]. Phenomena connected with Coulomb blockade and parity effects may appear in *SINIS* junctions with a very small N region [9,10]. But even if the size of the N region is not small enough to observe these phenomena, some new peculiarities arise in these systems at low temperatures and voltages. It has been established first in Ref. [1(b)] that the subgap conductance of the *SIN* contact (Nb/InGaAs) increases with decreasing V and reaches a value comparable with the conductance of the contact in the normal state. The authors of Refs. [4(b),7,8] observed an oscillatory dependence of the subgap conductance in *SINIS* junctions, shown schematically in Fig. 1(a), as a function of an applied magnetic field H. An explanation for the phenomenon of the subgap conductance enhancement and its dependence on H has been suggested in Refs. [4(b),11–16], and this enhancement was interpreted in Refs. [1(b),11–13] as a result of an anomalous proximity effect enhancing at low voltages V and temperatures T. The condensate current induced by an external magnetic field suppresses the proximity effect. Because the condensate momentum is an oscillatory function of the magnetic field H threading the superconducting loop, the subgap conductance oscillates with increasing H also.

One can formulate an inverse problem: how the current through the N layer will influence the Josephson effects in the system shown in Fig. 1(b) [our approach is applicable also the system in Fig. 1(a) if the distance between superconductors S_a and S_b is less than the coherence length ξ_N]. Note that the voltage V_b between S_a and S_b (the potential V_a is taken to be zero) may be zero, whereas the potential of the N layer, V_N, is not zero. In this Letter we analyze the influence of V_N on the Josephson effects in a *SINIS* junction [17]. In particular, we will show that the ac Josephson effect appears even in the case when there is no current through the superconductor S_b ($I = 0$).

We consider the system shown in Fig. 1 and assume that the thickness of the N layer is less than ξ_N. We also assume the dirty limit. Averaging equations for the Green's functions $\hat{G}^{R(A)}$ over the thickness of the N layer, d_N, we obtain [13(b)]

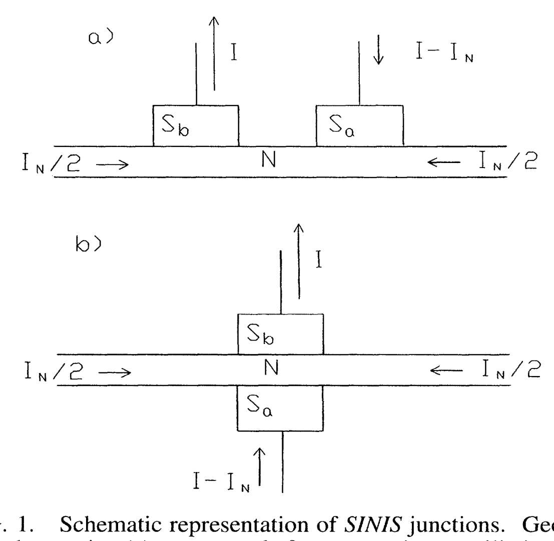

FIG. 1. Schematic representation of *SINIS* junctions. Geometry shown in (a) was used for measuring oscillations of the subgap conductance of the *SIN* interfaces in an applied magnetic field. In this case the superconductors S_a and S_b were connected by a superconducting loop.

 0031-9007/95/74(23)/4730(4)$06.00

$$\varepsilon_a[\hat{G},\hat{G}_a]_- + \varepsilon_b[\hat{G},\hat{G}_b]_- + i\varepsilon[\hat{\sigma}_z,\hat{G}]_- - (\gamma/2)[\hat{\sigma}_z\hat{G}\hat{\sigma}_z,\hat{G}]_- = 0\,. \quad (1)$$

The indices $R(A)$ are omitted for brevity. Here $\varepsilon_{a,b} = \rho D/(2R_{a,b}d_N)$ are characteristic energies related to the barrier transmittance, γ is the depairing rate, $R_{a,b}$ are the resistances of the $S_{a,b}/N$ interfaces per unit area, and D and ρ are the diffusion coefficient and specific resistance of the N layer. We suppose that the barrier transmittances are not high and the condition $\varepsilon_{a,b} \ll \Delta d_S/d_N$ is fulfilled. Then the Green's functions $\hat{G}_{a,b}$ in the S electrodes can be considered undisturbed by the proximity effect. The electron-phonon coupling constant in the N layer is assumed to be vanishingly small, so that the order parameter Δ_N in the N layer is equal to zero. In order to calculate the supercurrent, we must determine the Green's function $\hat{G} = G\hat{\sigma}_z + \hat{F} = G\hat{\sigma}_z + i(F_x\hat{\sigma}_x + F_y\hat{\sigma}_y)$ in the N layer. Solutions for $\hat{G}$ can be found from Eq. (1) in two limiting cases: (a) $\gamma \gg \varepsilon_{a,b}$ (low barrier transmittances) and (b) $\gamma \ll \varepsilon_{a,b} \ll \Delta(d_S/d_N)$ (moderate barrier transmittances) [13(b)].

In the first case we obtain $G^{R(A)} = \pm[1 + (F^{R(A)})^2/2]$ and

$$\hat{F}^{R(A)} = \pm i(\varepsilon_a\hat{F}_a + \varepsilon_b\hat{F}_b)^{R(A)}/(\varepsilon + i\gamma)\,. \quad (2)$$

Here $\hat{F}_a^{R(A)} = i\hat{\sigma}_y\Delta/\xi_\varepsilon^{R(A)}$, $\hat{F}_b^{R(A)} = i(\hat{\sigma}_x\sin\phi + \hat{\sigma}_y\cos\phi)\Delta/\xi_\varepsilon^{R(A)}$, $\xi_\varepsilon^{R(A)} = [(\varepsilon \pm i0)^2 - \Delta^2]^{1/2}$, and ϕ is the phase difference of the order parameter between S_a and S_b. The excitation spectrum in the N layer remains gapless, although it has peculiarities at $\varepsilon \cong \gamma$ and $\varepsilon \cong \Delta$. In the second case (b) we obtain for $\varepsilon \ll \Delta$

$$G = \varepsilon/\tilde{\xi}_\varepsilon,\; F_x = (\varepsilon_b/\tilde{\xi}_\varepsilon)\sin\phi,$$
$$F_y = (\varepsilon_a + \varepsilon_b\cos\phi)/\tilde{\xi}_\varepsilon\,. \quad (3)$$

Here $\tilde{\xi}_\varepsilon = (\varepsilon^2 - \varepsilon_g^2)^{1/2}$, $\varepsilon_g = (\varepsilon_a^2 + \varepsilon_b^2 + 2\varepsilon_a\varepsilon_b \times \cos\phi)^{1/2}$ is an energy gap in the excitation spectrum in the N layer induced by the proximity effect (recall that $\Delta_N = 0$). The energy gap ε_g oscillates as a function of ϕ.

The condensate current through the S_a/N interface is determined by the formula

$$I_{S,a} = -(w/16R_a)\,\mathrm{Tr}\,\hat{\sigma}_z\int d\varepsilon[(\hat{F}^R\hat{F}_a^R - \hat{F}^A\hat{F}_a^A)(f_0 + f_{0a}) + (\hat{F}^R\hat{F}_a^A - \hat{F}^A\hat{F}_a^R)(f_0 - f_{0a})], \quad (4)$$

where w is the width of the S layers, and f_0, f_{0a} are the distribution functions in the N and S_a layers (they are odd functions of ε). We assume that they have the equilibrium form, e.g., $f_0 = [\tanh(\varepsilon + V_N)\beta + \tanh(\varepsilon - V_N)\beta]/2$ and $f_{0a} = \tanh(\varepsilon\beta)$ (the electrical charge is included into V). This assumption is correct if $\varepsilon_{a,b} \ll \max\{\tau_\varepsilon^{-1}, D/w^2\}$, where τ_ε is the energy relaxation time [13(a)]. On the other hand, solutions (2) and (3) are correct provided that the width w is large as compared to ξ_S: $w^2 \gg D/\Delta$. Calculating the integral in Eq. (4), we obtain

$$I_{S,a}(V_N)R = \varepsilon_0\ln(\Delta^2 - V_N^2)/(V_N^2 + \gamma^2)\sin\phi, \quad \text{(case a)}, \quad (5)$$

$$I_{S,a}(V_N) = [I_c(0) + \delta I_c(V_N)]\sin\phi\,, \quad \text{(case b)}. \quad (6)$$

We assumed for simplicity that the temperature is low enough: $T \ll \gamma$ (a) and $T \ll \varepsilon_{a,b}$ (b). Here $I_c(0)R = 2\varepsilon_0\ln(2\Delta/\varepsilon_g)$, $\delta I_c(V_N)R = -2\varepsilon_0\vartheta(V_N - \varepsilon_g)\ln[(V_N + (V_N^2 - \varepsilon_g^2)^{1/2})/\varepsilon_g]$ at $V_N \ll \Delta$, and $\delta I_c(V_N)\,R = -2\varepsilon_0\ln[2\Delta V_N/\varepsilon_g(\Delta^2 - V_N^2)^{1/2}]$ at $V_N \gg \varepsilon_g$; $\varepsilon_0 = (\varepsilon_a + \varepsilon_b)/2$, $R = (R_a + R_b)/w$ is the total resistance of the interfaces. In Fig. 2 we show the dependence of the critical current on V_N. One can see that $I_c(V_N)$ changes sign at $V \cong \Delta/\sqrt{2}$. The condensate current through the S_b/N interface is determined by the same formulas (5) and (6) if V_N is replaced by $V_b - V_N$.

Now we calculate the dissipative component of the current I_{diss}. It consists of two parts (see, for example, [20,21]):

$$I_{\mathrm{diss}} = I_{\mathrm{qp}} + I_{\mathrm{int}}\,. \quad (7)$$

Here $I_{\mathrm{qp}} = (w/2R_a)\int d\varepsilon\,\nu\nu_a(\varepsilon)[f_1 - f_{1a}]$ is the quasiparticle current through the S_a/N interface; ν and $\nu_a(\varepsilon)$

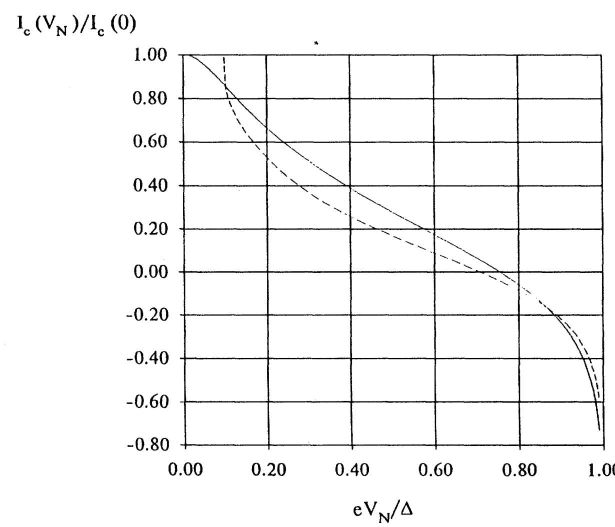

FIG. 2. Normalized critical current $I_c(V_N)/I_c(0)$ vs normalized voltage drop across the N/S_a interface, eV_N/Δ, for a weak [case (a), solid line] and moderate [case (b), dashed line] barrier transmittances; voltage drop across S_a and S_b is taken to be zero. The following parameters were used: $\gamma/\Delta = 0.1$ (a) and $\varepsilon_g/\Delta = 0.1$ (b).

are the density of states in N and S_a. The functions f_{1a} and f_1 are the distribution functions in S_a and N; they are even functions of ε and equal in equilibrium to $f_1 = [\tanh(\varepsilon + V)\beta - \tanh(\varepsilon - V)\beta]/2$ and $f_{1a} = 0$ because we put $V_a = 0$. At low temperatures and low voltages ($V < \Delta$), the current $I_{\rm qp}$ is exponentially small. The current $I_{\rm int}$ is known in the theory of the Josephson effect as the current of the interference between pairs and quasiparticles. It leads to the subgap conductance anomaly and can be expressed through $\hat{F}^{R(A)}$ in the following way:

$$I_{{\rm int},a} = (w/16R_a)\,{\rm Tr}\int d\varepsilon\,(\hat{F}^R + \hat{F}^A) \times (\hat{F}^R_a + \hat{F}^A_a)(f_1 - f_{1a}). \tag{8}$$

Using Eqs. (2) and (3), one can easily find $I_{\rm int}$. Assuming again the low temperature case, we obtain for $I_{\rm int}$

$$I_{{\rm int},a}R = 2\varepsilon_0(p + \cos\phi) \times \begin{cases} \arctan(V_N/\gamma) & \text{(case a)} \\ \arcsin(V_N/\varepsilon_g)\vartheta(\varepsilon_g - |V_N|) + (\pi/2)\vartheta(|V_N| - \varepsilon_g) & \text{(case b)}. \end{cases} \tag{9}$$

Here $p = \varepsilon_a/\varepsilon_b$. Formulas for $I_{{\rm int},b}$ have the same form with p and V_N replaced by $\bar{p} \equiv p^{-1}$ and $V_b - V_N$, respectively.

The conservation law for the currents through the S_a/N and S_b/N interfaces reads

$$\begin{aligned} I &= I_{c,b}\sin\phi + (\bar{p} + \cos\phi)A(V_b - V_N) \\ &\quad + (V_b - V_N)/r_b + C_b\partial_t^2(V_b - V_N), \\ I - I_N &= I_{c,a}\sin\phi + (p + \cos\phi)A(V_N) \\ &\quad + V_N/r_a + C_a\partial_t^2 V_N. \end{aligned} \tag{10}$$

Here the function $A(V_N)$ is defined in Eq. (9); the third term describes either a small quasiparticle current or a current through possible shunt and microshorts. The last terms are the displacement currents. Because the distribution functions are assumed to have the equilibrium form, the voltage V_b is connected with the phase difference by the Josephson relation: $2eV_b = \partial_t\phi(\hbar = 1)$. Equations (10) describe the system under consideration.

Consider first the dc Josephson effect ($\partial_t\phi = 0$). Excluding V_N from Eqs. (10), we obtain for the case of identical interfaces $I = I_N/2 + I_c(V_N)\sin\phi$. This relation connects the currents I and I_N with the phase difference. As noted above, the critical current changes sign at $V_N > \Delta/\sqrt{2}$, and an instability of a state with $|\phi| < \pi/2$ must arise in the systems under consideration.

The analysis of the ac Josephson effect is more complicated because, when deriving expressions for I_S and $I_{\rm int}$, we assumed that voltage temporal variations are slow. Therefore, we will restrict ourselves to the analysis of the case (a) and assume mainly that the voltages are small enough: $\{V_b, V_N\} \ll \gamma$. Let the current I_N be absent. Then for identical S/N contacts, we find from Eq. (10)

$$\begin{aligned} I &= I_c\sin\phi + 2(\varepsilon_0/R)(1 + \cos\phi)\arctan(\partial_t\phi/4e\gamma) \\ &\quad + \partial_t\phi/4er + C\partial_t^2\phi/4e. \end{aligned} \tag{11}$$

If $\partial_t\phi \ll \gamma$, Eq. (11) is reduced to an ordinary equation for a tunnel Josephson junction taking into account the displacement and interference currents [20,21]. If $\partial_t\phi \gg \{\gamma, I_c/r\}$, the voltage $V_b = \partial_t\phi/2$ does not depend on time in the main approximation, and the $I(V_b)$ characteristic has the form $I = \pi\varepsilon_0/R + V_b/r$, that is, an excess current appears at $\gamma \ll V_b \ll \Delta$.

Let the current I equal zero. Then, for small voltages ($|\partial_t\phi| < \gamma$), we obtain again Eq. (11) in which the current I and the term $\arctan(\partial_t\phi/4e\gamma)$ should be replaced by $-I_N/2$ and $\partial_t\phi/4e\gamma$, respectively. Therefore, even if the current flows only through one superconducting electrode (S_a), the ac Josephson effects arise in the system at $I_N > 2I_c$. If both currents (I and I_N) flow in the system, one can control the critical current and other characteristics of the system varying the current I_N in the middle electrode. Note that the Josephson effect in the absence of the current may arise in an usual tunnel Josephson *SIS* junction when the superconducting electrodes are maintained at different temperatures (the Josephson current is compensated for by the thermoelectric current) [22].

In summary, we have analyzed the behavior of a S_aINIS_b Josephson junction in which an additional bias current can flow in the N layer. It is shown that the critical current depends on the voltage V_N between N and S_a and can change sign at a certain value of V_N. This effect is observable under some restrictions on parameters of the system. In the case (a) the interface transmittance is small ($\varepsilon_0 \ll \gamma \ll \Delta$) and the width of the S layers must satisfy the condition $w^2 \gg D/\Delta$. In the case (b) restrictions on w are more severe: $D/\Delta \ll w^2 \ll D/\varepsilon_0$. We note that the suggested method for obtaining π contacts differs from those proposed earlier (see, for instance, [23] and references cited therein) because it allows one to transfer a conventional Josephson junction into a π contact by varying the electric potential at the middle electrode. The ac Josephson effects may arise even if the current flows only through the N/S_a interface and there is no current through the S_b electrode.

The author is indebted to P. H. C. Magnee for reading the manuscript and for comments and to V. Pavlovsky for technical assistance. Financial support from the International Science Foundation (Grant No. MRC000) and the Russian Scientific Council on HTSC problem (Project No. 92061) is gratefully acknowledged.

[1] (a) A.W. Kleinsasser *et al.,* Appl. Phys. Lett. **57**, 1811 (1990); (b) A. Kastalsky *et al.,* Phys. Rev. Lett. **67**, 1326 (1991); A.W. Kleinsasser and A. Kastalsky, Phys. Rev. B **47**, 8361 (1993).

[2] J. Nitta *et al.,* Phys. Rev. B **46**, 14 286 (1992); N. van der Post, J. Nitta, and H. Takayanagi, Appl. Phys. Lett. **63**, 2555 (1993).

[3] C. Nguen *et al.,* Appl. Phys. Lett. **57**, 87 (1990); C. Nguen, H. Kroemer, and E.L. Hu, Phys. Rev. Lett. **69**, 2847 (1992).

[4] (a) W.M. van Huffelen, T.M. Klapwijk, and L. de Lange, Phys. Rev. B **45**, 535 (1992); (b) B. van Wees *et al.,* Phys. Rev. Lett. **69**, 510 (1992); (c) H. Dimoulas *et al.,* Phys. Rev. Lett. **74**, 602 (1995); (d) P.H.C. Magnee *et al.,* Phys. Rev. B **50**, 4594 (1994).

[5] Z. Ivanov, T. Claeson, and T. Andersson, Jpn. J. Appl. Phys. **26**, 1617 (1987); Appl. Phys. Lett. **63**, 2079 (1993)Appl. Phys. Lett. **63**, 2079 (1993).

[6] M. Kuhlmann *et al.,* Z. Phys. B (to be published).

[7] H. Pothier, S. Gueron, D. Esteve, and M.H. Devoret, Phys. Rev. Lett. **73**, 2488 (1994).

[8] V.T. Petrashov, V.N. Antonov, P. Desling, and T. Claeson, Phys. Rev. Lett. **70**, 347 (1993).

[9] D. Averin and Yu.V. Nazarov, in *Proceedings of the NATO Workshop on Mesoscopic Superconductivity, Karlsruhe, Germany, 1994* [Physica (Amsterdam) **203B**, 310 (1994)].

[10] R. Bauernschmitt, J. Siewert, Yu.V. Nazarov, and A.A. Odintsov, Phys. Rev. B **49**, 4076 (1994).

[11] A.V. Zaitsev, JETP Lett. **51**, 41 (1990); Physica (Amsterdam) **185–189C**, 2539 (1991); in *Proceedings of the NATO Workshop on Mesoscopic Superconductivity* [Physica (Amsterdam) **203B**, 274 (1994)].

[12] A.F. Volkov, A.V. Zaitsev, and T.M. Klapwijk, Physica (Amsterdam) **210C**, 21 (1993).

[13] (a) A.F. Volkov, Phys. Lett. A **174**, 144 (1993); (b) **187**, 404 (1994); in *Proceedings of the NATO Workshop on Mesoscopic Superconductivity* [Physica (Amsterdam) **203B**, 267 (1994)].

[14] C.W.J. Beenakker, Phys. Rev. B **46**, 12 841 (1992); I.K. Marmorkos, C.W.J. Beenakker, and R.A. Jalabert, *ibid.* **48**, 2811 (1993).

[15] F.W. Hekking and Yu.V. Nazarov, Phys. Rev. Lett. **71**, 1525 (1993); Phys. Rev. B **49**, 6847 (1994).

[16] C. Lambert, in *Proceedings of the NATO Workshop on Mesoscopic Superconductivity* [Physica (Amsterdam) **203B**, 201 (1994)].

[17] The Josephson effect in *SINIS* junctions was theoretically studied first in Ref. [18] on the basis of a tunnel Hamiltonian method and a perturbation theory in the barrier transmittances. The results obtained are correct for not too low temperatures ($T \gg \varepsilon_0, \gamma$) and voltages ($V > \Delta$). Some results concerning the dc Josephson effect were presented in Ref. [19]. The influence of the voltage V_N on the Josephson effects was not analyzed in these works, however.

[18] L.G. Aslamasov, A.I. Larkin, and Yu.N. Ovchinnikov, Sov. Phys. JETP **28**, 171 (1969).

[19] M.Yu. Kupriyanov and V.F. Lukichev, Sov. Phys. JETP **67**, 1163 (1988); A.A. Golubov, M.A. Verhoeven, I.A. Devyatov, M.Yu. Kupriyanov, G.J. Geritsma, and H. Rogalla, in *Proceedings of the International Conference on Materials and Mechanisms of Superconductivity High Temperature Superconductors IV, Grenoble, France, July 5–9, 1994* [Physica (Amsterdam) **235–240C**, 3261 (1994)].

[20] K.K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).

[21] A. Barone and G. Patterno, *Physics and Application of the Josephson Effect* (Wiley, New York, 1981).

[22] M.V. Karzovnik, V.V. Ryasanov, and V.V. Schmid, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 373 (1981) [JETP Lett. **33**, 356 (1981)].

[23] D. Khomskii, J. Low Temp. Phys. **95**, 205 (1994).