Quantum Interference of a Single Vortex in a Mesoscopic Superconductor

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(Received 9 January 1995)

We propose an Aharonov-Bohm-type interference experiment for a vortex in a superconductor to *directly* test the vortex velocity part of the Magnus force. Based on the recent advances in the nanofabrication technology as well as the progresses in both theoretical and experimental studies of vortex dynamics we show that this conceptually simple experiment can be easily realized. If established, the proposed vortex interference experiment not only demonstrates the quantum coherence phenomenon at a macroscopic level, but also has a potential for technical applications.

PACS numbers: 74.20.De, 03.65.Bz, 74.50.+r, 74.60.Ge

It is commonly accepted that vortices in a 2D (or straight vortex lines in a 3D) superconductor behave as charged particles in a magnetic field [1]. In the equation of motion governing the vortex dynamics, the Magnus force has recently been shown, first at zero temperature [2] then at finite temperatures [3], to be an intrinsic property of type II superconductors, insensitive to details such as disorder and pinnings. Although there is an active theoretical exploration of its consequences [4,5], experimentally the Magnus force has not been well established yet. Many of the experimental attempts of demonstrating its existence come from the Hall angle measurements in the mixed state of superconductors. In those measurements the Hall angle is, however, usually small, and sometimes changes its sign as a function of the temperature and the magnitude of an applied magnetic field [6]. The latter is in an apparent disagreement with a simple minded application of the Magnus force, and could be interpreted as evidence against its existence. It has been shown elsewhere that the Magnus force is actually consistent with the Hall angle measurements after a consideration of the many-body correlation and pinning [7]. However, this can only be viewed as an indirect confirmation, because the Hall angle depends strongly on the details of a sample, which are unfortunately not well controlled. The purpose of the present paper is to propose a vortex interference experiment which can *directly* test the Magnus force. The proposed interference experiment is the usual two-slit type [8], with the interference pattern controlled by tuning the Magnus force, similar to the case in the Aharonov-Bohm-type experiment [9]. We will show that our design is simple considering the modern nanofabrication technology, and it is also feasible with the present understanding of vortex dynamics in superconductors. In the following we will first describe the proposed experimental setup, then discuss important relevant issues such as the vortex damping and a possible quantum electrodynamic interference effect due to the magnetic flux carried by a moving vortex. It should be emphasized that if the proposed interference experiment is established, it demonstrates the quantum coherence effect at a macroscopic scale. Therefore it has a profound fundamental implication. Furthermore, in views of the great technical applications of all the known quantum interference phenomena, the vortex interference effect should have its own preferable ones.

The equation of motion for a vortex in a 2D superconductor with mass m at position $\mathbf{r}$ is

$$m\ddot{\mathbf{r}} = \tfrac{1}{2}q_v h \rho_s(\mathbf{r},T)[\mathbf{v}_s(\mathbf{r}) - \dot{\mathbf{r}}] \times \hat{\mathbf{z}} + \mathbf{F}_p - \eta\dot{\mathbf{r}} + \mathbf{f}(t) . \quad (1)$$

Here h is the Planck constant, ρ_s the 2D superfluid electron density at the position of the vortex, T the temperature of the superconductor, $\mathbf{v}_s$ the superelectron velocity, η the vortex viscosity, $\mathbf{F}_p$ the pinning force, $\mathbf{f}(t)$ the fluctuating force related to the friction by the fluctuation-dissipation theorem, and $q_v = \pm 1$ the vorticity. The vortex mass has many contributions, and its precise value is not known yet. Because a steady state will be considered, the vortex mass is not relevant as long as it is not infinite, which has been shown to be the case [10]. The first term on the right-hand side of Eq. (1) is the Magnus force [1–3]. The pinning force in principle can be calculated microscopically [11], but in general it is a quite complicated procedure. What is needed here is of a simple type: a guided motion. The damping is important in our proposed interference experiment, and we will show that it can be very small with a proper choice of material parameters. Its discussion is deferred to after the description of the proposed experimental setup.

The discussion of the quantum interference requires us to extend the classical equation, Eq. (1), to the corresponding one in quantum mechanics. This can be made, for example, by following the formulation of Caldeira and Leggett [12] for the dissipative quantum dynamics as done in Ref. [4]. This formulation goes beyond the linear velocity damping in Eq. (1). Therefore a more general discussion of dissipation and dephasing can be conducted within it, which is the case for the damping contribution outside of the vortex core as discussed below. However, in order to illustrate the essential physics and keep the mathematics minimum, we first drop the friction in Eq. (1), and we will show later that it can be

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small. Without friction the Hamiltonian corresponding to Eq. (1) is

$$H = \frac{1}{2m}(\mathbf{P} - q_v \mathbf{A})^2 + U(\mathbf{r}), \tag{2}$$

with the "vector" potential $\mathbf{A}$ determined by

$$\nabla \times \mathbf{A} = \frac{h}{2} \rho_s \hat{\mathbf{z}}, \tag{3}$$

and the scalar potential U determined by

$$-\nabla U = \frac{h}{2} q_v \rho_s \mathbf{v}_s \times \hat{\mathbf{z}} + \mathbf{F}_p. \tag{4}$$

As usual, there is an arbitrary constant in the scalar potential which will not influence any physical quantity. The arbitrariness in the vector potential also has no observable consequence. With the Hamiltonian for a vortex, Eq. (2), its quantum dynamics is governed by the familiar Schrödinger equation

$$i\hbar|\dot{\Psi}\rangle = H|\Psi\rangle, \tag{5}$$

with Ψ the vortex wave function. It has been shown [2,3] that for a vortex moving along a closed loop such as the one in Fig. 1, a geometric phase Φ in addition to the usual dynamical phase will be accumulated according to

$$\begin{aligned}\Phi &= -\frac{q_v}{\hbar}\oint \mathbf{A}\cdot d\mathbf{r} = -q_v 2\pi \int ds\, \frac{1}{2}\rho_s \\ &= -2\pi q_v N_{\mathrm{pair}}, \end{aligned} \tag{6}$$

where N_{pair} is the total number of Cooper pairs enclosed by the loop. Since the superfluid density ρ_s varies continuously, so does the enclosed Cooper pairs number N_{pair}. Equation (6) is similar to the case of a charged particle in the presence of a magnetic field, with the number of magnetic flux quanta replaced by the number of Cooper pairs, and is an alternative and equivalent expression for the vortex velocity part of the Magnus

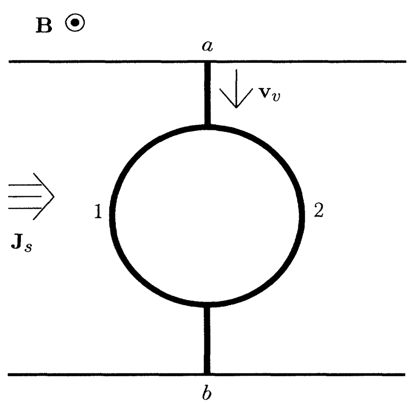

FIG. 1. A schematic diagram for the proposed vortex interference experiment. $\mathbf{J}_s = e\rho_s \mathbf{v}_s$ is the applied supercurrent. $\mathbf{v}_v$ is the vortex velocity. $\mathbf{B}$ is the applied magnetic field, pointing towards the reader. A vortex moves from the point a to the point b, through either the guided route 1 or the guided route 2. A voltage drop along the supercurrent measures the interference effect of the guided motion.

force in Eq. (1). The above equation is the base for our proposed vortex interference experiment.

Consider a vortex moving across the superconducting film from the point a at one edge to the point b at the other edge in Fig. 1. It can pass through either the guided route 1 or the guided route 2. If Ψ_1 and Ψ_2 are the transition amplitudes from a to b via routes 1 and 2, respectively, the total transition probability is

$$\begin{aligned}|\Psi|^2 &= |\Psi_1 + \Psi_2|^2 \\ &= |\Psi_1|^2 + |\Psi_2|^2 + 2|\Psi_1||\Psi_2|\cos(\Phi), \end{aligned} \tag{7}$$

with Φ given by Eq. (6). Here the unimportant dynamical phase factors of Ψ_1 and Ψ_2 are omitted, because during the variation of the geometric phase Φ they can be kept unchanged. It is evident from Eq. (7) that the geometric phase Φ controls the vortex interference. To observe the interference as manifested through Eqs. (6) and (7), we first discuss the generating of vortices, the motion from a to b, and the measurement of the interference. Then we discuss the tuning of the geometric phase Φ.

The generating of vortices can be done by applying a small magnetic field perpendicular to the film. The small magnetic field generates a dilute gas of vortices. To avoid a possible complication the applied field should be small enough so that the vortex interaction is negligible in the proposed interference experiment. By applying a supercurrent a vortex will move across the film under the influence of the superfluid velocity part of the Magnus, the so-called Lorentz force. The guided motion can be created by making the thick line part of the film in Fig. 1 much thinner than the rest, a small fraction of the average thickness. Then a vortex will prefer to move along the guide. Modern nanofabrication can make the film very homogeneous. Therefore the pinning in the film and along the guide can be controlled. The largest effect that inhibits the vortex motion may come from the edge pinning, which can be reduced, too, by making tampered edges around points a and b. Hence a vortex may move rather freely from a to b along the guided routes 1 and 2, not in another part of the film. If the average number of vortices moving from a to b per unit time is n_v, according to the Josephson relation the voltage drop along the supercurrent due to this vortex motion is

$$V = \frac{h}{2e} n_v |\Psi|^2, \tag{8}$$

with Ψ given by Eq. (7). Here e is the charge of an electron. Therefore the periodic dependence of $|\Psi|^2$ on Φ is manifested by the same dependence in the voltage drop V, which gives the direct measurement of the interference effect in our proposed interference experiment.

The tuning of the Cooper pair number in the enclosed loop in Fig. 1, therefore the change of the phase Φ in Eq. (8) according to Eq. (6), can be achieved in several ways. One method is to put a voltage gate beneath the loop in Fig. 1. If the temperature is low enough, we can ignore the normal electron fraction in the voltage gate control and take the superfluid electron equal to the total electron density. Then if the charging energy is not big enough to break a Cooper pair, the change of the total electron number inside the loop caused by the gate voltage is equal to twice the total Cooper number change. In this case the variation δN_{pair} of the number of Cooper pairs enclosed by the loop is directly controlled by the gate voltage V_G:

$$\delta N_{\mathrm{pair}} = \frac{C_G}{2e} V_G , \tag{9}$$

where C_G is the capacitance between the loop and the voltage gate. Another method to change the N_{pair} is to change the temperature, such as using laser heating, because the superfluid electron density is a function of temperature. In this case there is no net charge change inside the loop. The low temperature dependence of the superfluid density for a conventional superconductor is exponentially weak because of the energy gap, while for a high T_c superconductor it seems to be a power law. Because essentially one Cooper pair change inside the loop will be observed, this difference requires a different combination of the temperature control and the geometric size of a sample for those two types of superconductors: better temperature control and/or smaller loop are desired for high T_c superconductors. This completes our discussion of the experimental setup.

We now discuss the vortex damping and the associated random fluctuation, the important quality which is decisive in our proposed vortex interference experiment. We will demonstrate that it can be negligibly small with a proper choice of materials and parameters. The vortex damping may be divided into two separated parts: the contribution from the vortex cores studied by Bardeen and Stephen [13] and one from outside the core such as due to vortex-density fluctuation interaction [10]. We first discuss the core contribution because it is usually the dominant one. Although for a large vortex core the damping may be strong, for a small vortex core, which is typical for high T_c superconductors, the core damping contribution can be very small at low temperatures due to the discrete nature of core states. Because the size of a vortex is an order of the superconducting coherence length ξ, the core level spacing is therefore an order of $\hbar^2/\xi^2 m_e$ with m_e the electron mass. If the temperature is smaller than the core level spacing, the Bardeen-Stephen approach [13] should be extended and a low damping limit will be expected. The core damping in this situation comes from interlevel transitions, an example of the Landau-Zener transitions [14]. Although a complete and consistent study of the small core damping is lacking, a straightforward calculation has indeed shown an exponentially small damping for low vortex velocities [15]. For high T_c superconductors this crossover temperature to low damping is found to be $\sim$20 K, and has been confirmed experimentally [16]. The same argument applying to dirty (and some clean) conventional superconductors leads to a crossover temperature as high as an order of 0.1 K. In an interesting paper the ballistic motion of vortices with a mean free path much larger than 1 μm in a dirty conventional superconductor was reported even at a temperature of a few K [17]. We should also point out that the guided motion can be realized by a thin layer insulator separating the loop area and the rest of the superconducting film as in a superconductor-insulator-superconductor Josephson junction. In this case there is no vortex core damping contribution. The conclusion of this discussion is that the core damping is small for some materials at low enough temperatures, and has been observed experimentally. The damping contribution from outside the core is generally believed to be small, and cannot be simply represented by a linear vortex velocity damping [10]. In a superconductor, the coupling of the density fluctuation to the electromagnetic field generates a plasma gap. Those density fluctuations can always follow a slow vortex motion. The corresponding damping is then small in this case, as one may infer from a general discussion in Ref. [10]. For a 2D superconductor there is no gap in the plasma mode, and the resulting vortex damping contribution might be large. However, one may coat the film with a layer of gold to make a finite gap in the plasma mode, then again eliminate the damping contribution outside the core. In view of the above discussion of the vortex damping, in the proposed vortex interference experiment the size of the loop, such as in Fig. 1, should be smaller than the phase breaking length for a vortex, which can be an order of 1 μm with the materials and parameters discussed above. This length scale is nevertheless much larger than the vortex core size, the "microscopic" length scale. In this sense our proposed vortex interference experiment is a mesoscopic one.

We should emphasize that the ballistic motion of vortices is crucial in the above proposed vortex interference experiment. This requires that both the pinning along the guided routes and the vortex damping strength should be sufficiently small, which are, fortunately, the common properties of both some conventional superconductors and all high T_c superconductors. It is interesting to point out that a weak pinning, a nuisance in certain applications as indicated by the recent enormous effort in trying to increase the pinning in high T_c superconductors, will find its warm welcome in the possible applications associated with the vortex interference.

There is an interesting quantum electrodynamic interference effect associated with the magnetic flux carried by a vortex, the so-called Aharonov-Casher effect [18]. The geometric phase associated with this effect has a different sign compared to the one due to the Magnus force as given by Eq. (6). In principle this sign difference is experimentally observable. For example, if the charge imbalance felt by the moving magnetic flux is equal to the number change of Cooper pairs, the Aharonov-Casher phase will cancel the phase due to the Magnus force, and there will be no net phase change. However, here we wish to demonstrate that the Aharonov-Casher effect itself may be difficult to observe for a vortex moving in a superconductor. We note that the observation of this effect necessarily requires a charge imbalance in the superconductor and a well defined magnetic flux during a vortex motion. In order to have a well defined magnetic flux, the size of the loop such as in Fig. 1 must be larger than the size of the magnetic flux, which is the effective magnetic screening length and is usually much larger than the London penetration depth. For highly granular superconductors, such as Josephson junction arrays, the effective magnetic screening length can be as large as the size of the whole sample itself. To make the magnetic flux feel the charge imbalance in the superconductor, the voltage gate should be placed far away from the loop such that the magnetic flux should not see the Aharonov-Casher effect due to the voltage gate, because the whole superconductor-voltage gate system is charge neutral. The reason is that the electromagnetic field is in 3D, in contrast to the 2D hydrodynamical nature of the Magnus force. This distance is again presumably an order of the effective magnetic screening length. Note that the discussion of the vortex damping has shown that a very large loop is unsuitable for the interference experiment, because the damping and dephasing effect increases with the size of the loop. In view of the large effective magnetic screening length in dirty superconductors and high T_c superconductors, the conditions for the observation of the Aharonov-Casher effect are very restrictive there.

We note that in the proposed vortex interference experiment the loop area is large and well connected to the rest of the film. The usual quantum size effects such as the Coulomb blockade [19] and the even-odd number parity [20] are therefore not present. However, those effects do not suppress the vortex interference. It is likely that indications for the vortex interference effect due to the Magnus force have been observed experimentally [21], but have been confused with other processes such as the Aharonov-Casher effect. In the present proposed experiment there will be no such confusion.

In conclusion, we have proposed a vortex interference experiment in a superconductor to *directly* test the vortex velocity part of the Magnus force. We have shown that this experiment is a feasible one. This vortex interference experiment, if established, demonstrates the quantum coherence effect at a macroscopic level, and is important from a fundamental physics point of view. For a loop of size order of 1000 Å, and the film thickness order of 100 Å, the total number of Cooper pairs enclosed by the loop is of the order of 10^8. The vortex interference effect will allow us to measure the effect of a single Cooper pair. Thus a relative accuracy of an order of 10^{-8} may be readily achieved. This indicates that the vortex interference effect also has a good technical application potential.

We thank Per Delsing, David Haviland, Petter Minnhagen, Jørgen Rammer, and Andrei Shelankov for stimulating discussions.

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