Scaling Behavior in the Current-Voltage Characteristic of One- and Two-Dimensional Arrays of Small Metallic Islands

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We have measured the current-voltage (I-V) characteristics of one- and two-dimensional arrays of normal metal islands linked by small tunnel junctions. The tunneling resistance is large compared to the resistance quantum, and a ground plane reduces the screening length to much less than the interisland spacing. At temperatures well below the island charging energy, we find a threshold voltage V_T below which little current flows. For $V > V_T$, I scales as $(V/V_T - 1)^\zeta$ where $\zeta = 1.36 \pm 0.1$ (1D) and 1.80 ± 0.16 (2D). We interpret this behavior as a dynamic critical phenomenon.

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The transport of interacting objects through quenched disorder is an ubiquitous phenomenon. Several well-known examples, including sliding charge-density waves [1], flux-line lattices in type-II superconductors [2], and fluids in disordered media [3], have been the subject of extensive investigation. Such systems typically display a threshold behavior: Below some critical force F_c the system is static and the velocity of objects in it is zero, while above F_c the system enters a dynamic conducting state in which the objects move, producing transport through the system. Fisher suggested that this behavior can be analyzed as a dynamic critical phenomenon [4], and critical exponents associated with the conduction transition have been calculated in a variety of models. Recently, Middleton and Wingreen (MW) [5] proposed that one- and two-dimensional (1D and 2D) arrays of small normal metal islands linked by tunnel junctions, in which transport occurs through the stochastic flow of discrete charges, should provide a novel model system for the study of such dynamic critical phenomena. In these arrays the microscopic degrees of freedom and their range of interaction are well understood and under good experimental control. Furthermore, the possible sources of microscopic disorder are clear. Thus, such arrays offer a unique opportunity to investigate the relationship between microscopic parameters and universality classes for dynamic critical phenomena.

In the model, the arrays consist of small normal metal islands linked by tunnel junctions of resistance R and capacitance C and located close to a ground plane to which each island has a capacitance C_g. We assume $R \gg R_Q = h/e^2$ and $(e^2/2)\max[C, C_g] \gg k_B T$. The proximity of each island to the ground plane allows us to neglect capacitive coupling between non-neighboring islands [6]. An excess charge placed on an island will polarize surrounding islands; the polarization drops away from the charge exponentially with a screening length λ which increases with C/C_g. The excess charge and its associated polarization constitute a soliton. In the absence of disorder, soliton dynamics in long-screening-length ($C \gg C_g$) arrays have been studied extensively, both theoretically [6] and experimentally [7]. However, little theoretical work and to our knowledge no experimental work has been reported in the limit of short screening length ($C \ll C_g$), with or without the effects of disorder. In this limit MW consider 1D arrays of N islands ($50 < N < 2000$) and square 2D arrays of $N \times N$ islands ($40 < N < 400$), as shown in the insets to Fig. 1; electrical contact is made via leads to opposite sides of the array. MW include disorder in the form of offset charges q_i associated with each island, representing the charge induced by charged impurities scattered randomly throughout the array. Large offset charges will be partially neutralized by an integral number of mobile charges, so that $0 < q_i < e$. Furthermore, MW assume that the disorder is maximal, that is, q_i is independently and randomly distributed between 0 and e. When a voltage V is applied between the leads, no current flows below a threshold voltage V_T, while for $V > V_T$ the current-voltage (I-V) characteristic obeys a scaling law $I \propto (V/V_T - 1)^\zeta$. For infinite arrays in the limit of short screening length ($C \ll C_g$), they argue analytically that the exponent $\zeta = 1$ and 5/3 for 1D and 2D; their computer simulations for arrays of finite size give $\zeta = 1.0$ and 2.0 ± 0.2 for 1D and 2D. The threshold voltage (averaged over disorder) increases linearly in the array size as $V_T = \alpha(C/C_g)Ne/C_g$, where $\alpha(C/C_g) \to 1/2$ and 0.338 for 1D and 2D as $C/C_g \to 0$; α decreases rapidly as C/C_g increases [5].

In this Letter we report measurements of the I-V characteristics of a single 1D array and a single 2D array designed to lie in the short-screening-length limit. The arrays consist of Al islands linked by $Al/Al_xO_y/Al$ tunnel junctions and are fabricated with electron-beam lithography and a shadow evaporation technique [8]. The electrical leads are separated by $N = 440$ (1D) and $N = 38$ (2D) islands; the 2D array is 40 islands wide. The substrates, which act as the ground plane, are degenerately doped Si thermally oxidized to a thickness of

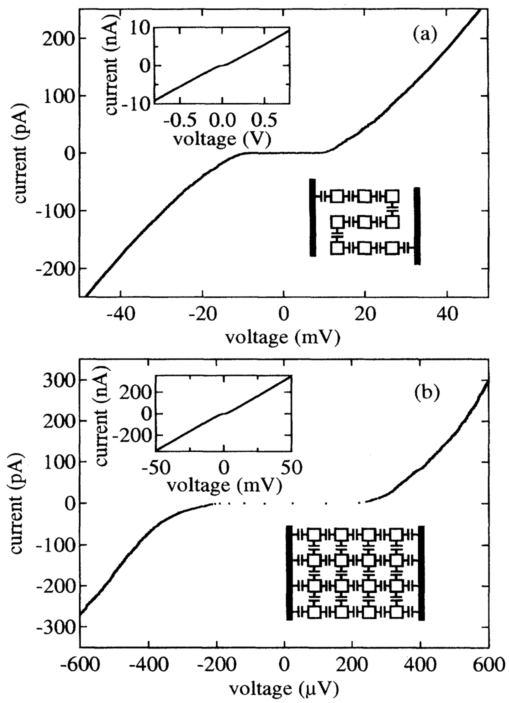

FIG. 1. I-V characteristics for (a) 1D and (b) 2D arrays in the normal state. The presence of a threshold voltage above which nonlinear conduction occurs is clearly present in both characteristics. The left insets show asymptotic linear behavior at high current; the right insets are schematic diagrams of the arrays.

102 ± 5 nm to provide electrical isolation. Each substrate has four Ohmic contacts, allowing electrical contact to the ground plane. The islands were designed to give $C_g \approx 1.5$ fF. Typical measured junction areas were approximately 70 by 80 nm^2 (1D) and 70 by 70 nm^2 (2D). Other work [9] implies a specific capacitance of ≈ 70 fF/μm^2 for junctions with size and resistance similar to ours, leading us to expect $C \approx 0.35$ fF. Disorder in our arrays arises from variations in the junction resistance and capacitance as well as from offset charges q_i; measured junction areas vary by roughly 20%. Since we have no control over the naturally occurring q_i, it is not clear which form of disorder dominates.

We made electrical measurements in a dilution refrigerator at temperatures of 30 to 40 mK, using a four-probe technique. The sample leads were carefully filtered by microwave [10] and radio-frequency filters at 4.2 K, and a second set of microwave filters at the mixing chamber temperature. The measurement electronics were battery powered, except for a plotter used to digitize the data. Radio-frequency π filters at room temperature were used to reject any noise from the plotter. We performed all measurements in a screened room, and occasionally recorded data with an analog XY recorder to verify that the digital electronics did not affect the results. The array was current biased for $V \gg V_T$, and approximately voltage biased for $V \lesssim V_T$ where the array resistance generally exceeded the bias resistor, $10^9\ \Omega$. We measured the current with a current-sensitive amplifier with a low-current-noise input, and the voltage across the array with a high-input-impedance ($\sim 10^{15}\ \Omega$) amplifier. To bias the array symmetrically with respect to the ground plane, we sampled the voltages V_L and V_R on the two sides of the array with high-impedance ($\sim 10^{15}\ \Omega$) buffers and applied the average voltage $(V_L + V_R)/2$ to the substrate.

To check the quality of the junctions, we measured the I-V characteristics in the superconducting state, and found a gap voltage of 370 mV (1D) and 32.0 mV (2D). This voltage is expected to be $4(N + 1)\Delta/e$ where Δ is the superconducting energy gap. We obtain average values for 2Δ of 0.42 and 0.41 meV for the 1D and 2D arrays, indicating the superconducting transition temperature may be slightly above the bulk value. This measurement has especially important implications for the 2D sample; the average conducting path through the array must pass through 39 junctions, indicating that at most a few were open or shorted. After making all measurements, we inspected the samples in a scanning electron microscope for imperfections such as broken lines; we found none.

To make measurements in the normal state we applied a 0.5 T magnetic field perpendicular to the plane of arrays. In Fig. 1 we show typical I-V characteristics of the 1D and 2D arrays. As shown in the insets, the I-V characteristics at high current are linear, yielding an average asymptotic junction resistance $R = 188$ kΩ (1D) and 138 kΩ (2D). Both values of R are significantly larger than the resistance quantum $R_Q = 25.8$ kΩ, so that effects of cotunneling [11] should be minimal. In previous studies [7] of long-screening-length arrays, the offset voltage $V_{\rm off}$ in the asymptotic regime was used as a measure of the junction capacitance; however, in our arrays the value of $V_{\rm off}$ is dominated by C_g [6] and is not useful as a measure of the junction capacitance.

To measure C_g we bias the array just above threshold and measure changes in the voltage across the array as we ramp the substrate voltage [12]. We expect the array voltage to be periodic in the gate voltage with period e/C_g, and for the 2D array measure a period of 130 μV, giving $C_g = 1.2$ fF. For the 1D array the large array resistance and threshold voltage made it difficult to determine the period precisely. However, the island design and substrate are identical for both arrays so that we expect $C_g = 1.2$ fF for the 1D array as well. The values of C and C_g yield a screening length $\lambda \approx 0.6$ for both 1D and 2D [6].

Figure 1 also shows that the conduction is very small below a threshold voltage V_T, less than $2.9 \times 10^{-12}\ \Omega^{-1}$ (1D) and $1.2 \times 10^{-10}\ \Omega^{-1}$ (2D). Above threshold we see nonlinear conduction in both arrays. We interpret the data in Fig. 1 as dynamic critical phenomena associated with the transition to a dynamic conducting state at V_T when the applied potential is large enough to populate the entire array with electrons, causing current to flow.

In measuring V_T we encounter two sources of uncertainty. First, for a given set of data there is an uncertainty in V_T of roughly $\pm 100\ \mu$V (1D) and $\pm 10\ \mu$V (2D) due to rounding of the transition [13]. Second, the measured value of V_T was found to vary on a time scale of hours. This variation could be due to the motion of impurities in the substrate, leading to a different disorder realization and hence a different threshold after sufficient time has elapsed. Alternatively, because the period of oscillations associated with changes in the gate voltage is small (only 130 μV), it is possible that changes in thermal voltages in the sample leads and offset drift in the buffer amplifiers contributed to changes in the measured V_T. However, in general, the changes occurred slowly enough for us to make 4 (1D) or 6 (2D) consecutive sweeps on different current ranges and to piece them together to obtain a single characteristic with no detectable discrepancies. In this way we measured 4 (1D) and 5 (2D) sets of I-V characteristics for the arrays, and found an average $V_T = 9.4 \pm 0.3$ mV (1D) and $230 \pm 20\ \mu$V (2D). Calculations in the model of MW using our values of C and C_g and assuming maximal disorder yield $V_T \approx 13$ mV (1D) and 580 μV (2D) [14], larger than the measured values of V_T by a factor of about 1.5 and 2.5, respectively

Since we do not know the exact amount of disorder present in our arrays, we also compared our results to theoretical predictions in the absence of disorder. In a disorder-free array there is a threshold voltage V_s associated with soliton injection [6]. The voltage V_s is an edge effect more or less independent of both array dimension and size for $C \gg C_g$. We estimate $V_s \approx$ 110 μV (1D) and 90 μV (2D) for the parameters of our arrays; for the 1D array, this prediction is two orders of magnitude below our measured value. While in our 2D array V_T is only a factor of 2.5 larger than V_s, this result nonetheless is in marked contrast to results on long-screening-length 2D arrays for which measured values of V_T are typically a factor of 2 to 10 smaller than V_s [15]. Finally, in the absence of disorder V_s is reduced by a factor of 2 if the array is asymmetrically biased with respect to ground [16]; we made measurements of V_T for the 2D array with one side fixed at the substrate voltage, but found no significant decrease in V_T. This result indicates that it is the voltage applied between the ends of the array, rather than the voltage between the edges and ground plane, that determines primarily when conduction will occur, implying disorder plays an important role in transport through our arrays.

To examine the scaling behavior, we plot current vs reduced voltage $\nu = (V/V_T - 1)$ on a log-log plot, as shown in Fig. 2 for two typical sets of data. The threshold voltages were chosen to give a straight line over the widest range of reduced voltage, but in all cases the corresponding current was between one and two times

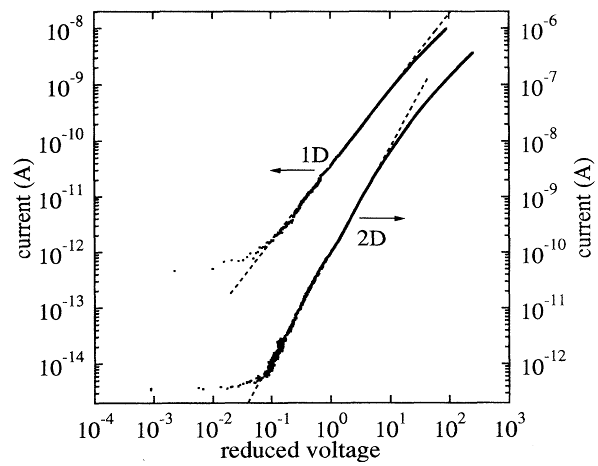

FIG. 2. Typical composite curves of current vs reduced voltage $(V/V_T - 1)$ plotted on logarithmic axes; 1D data are plotted on the left axis, 2D data on the right. Dashed lines show scaling relation (1) for the values of V_T and ζ fitted to these two data sets.

the current noise (in the measurement bandwidth below 0.3 Hz) at threshold. As can be seen, the scaling law (1) is obeyed by both arrays for $0.1 \lesssim \nu \lesssim 8$, corresponding to over $2\frac{1}{2}$ (1D) and nearly 4 (2D) orders of magnitude in current. We extract the value of the exponent ζ from the measured slope of the data in the region where (1) is obeyed and find $\zeta = 1.36 \pm 0.1$ (1D) and 1.8 ± 0.16 (2D). Above $\nu \approx 8$ we see a knee in the I-V characteristic and a transition to the linear asymptotic regime.

Our data are qualitatively in good agreement with the simulations of MW. We find threshold voltages significantly larger than those predicted in the absence of disorder, and a larger threshold for the 1D array, as expected. We also observe scaling behavior above threshold and find a measured value of the exponent ζ for the 2D array quite close to the calculated value of 2.0 ± 0.2 [5]. While the measured value $\zeta = 1.36 \pm 0.1$ for the 1D array is larger than the calculated value of 1.0, it is definitely smaller than the measured ζ for the 2D array, as predicted.

With regard to the threshold voltages, we note that the agreement between the measured and calculated values of V_T is much better for the 1D array than the 2D array. In fact, the measured V_T in the 2D array lies closer to that predicted in the absence of disorder. However, disorder in the actual samples may vary more slowly than in the model of MW. If the disorder consists of slowly varying hills and valleys rather than of white noise, we would expect a reduced threshold voltage in the 2D array since the electrons could flow around significant obstructions. On the other hand, V_T for the 1D array should remain relatively unaffected by slowly varying disorder. It is also possible that variations in the junction capacitance provide the electrons with built-in favorable paths through the 2D array. Furthermore, despite the fact that $R \gg R_Q$, cotunneling processes may still play a non-negligible role. In future experiments we shall address these issues by fabricating arrays with different numbers of junctions, intentionally introduced disorder and larger junction resistance.

We have also neglected the effects of temperature in our data analysis, but we expect that thermal fluctuations will tend to round the transition, and may decrease the size of the threshold voltage. We found, however, that a moderate increase in temperature (to roughly 60 mK) did not cause a significant change in the value of V_T or the shape of the I-V characteristic. In fact, quantum fluctuations resulting from the finite size of R/R_Q may be more important than the thermal fluctuations in this regime [16]. Further measurements are needed to shed light on the temperature dependence of both V_T and conduction below V_T.

In conclusion, we have presented measurements of the I-V characteristics of 1D and 2D arrays of normal metal islands in the short-screening-length limit. Each array exhibited a threshold voltage below which there is almost no conduction and above which the current scales as a power ζ of the reduced voltage ν. The values of the threshold voltages $V_T = 9.4$ mV and 230 μV, and of the exponent $\zeta = 1.36$ and 1.80, in 1D and 2D are in qualitative agreement with simulations interpreting the threshold behavior as a dynamic critical phenomenon. Our results are not consistent with a theory based on soliton injection.

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[12] This is a more direct method of determining C_g than using the measured value of V_{off} with the results of Ref. [6]; furthermore, our arrays do not lie in an asymptotic limit, so that no analytic expression for V_{off} in terms of C_g and C is available.
[13] Both thermal and quantum fluctuations of charge can round the transition. However, limitations of our measurement technique probably dominate the uncertainty for the 1D array. Because V_T is much larger, its relative uncertainty for the 1D array is nonetheless much smaller than for the 2D array.
[14] A.A. Middleton (private communication).
[15] See Tighe *et al.* and Delsing *et al.* (1994) in Ref. [7]. The small measured values of V_T are attributed to local lowering of threshold due to disorder or to quantum fluctuations of charge.
[16] See Delsing *et al.* (1994) in Ref. [7].