Observation of Nonlinear Neoclassical Pressure-Gradient–Driven Tearing Modes in TFTR

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A detailed comparison is made between the tearing-type modes observed in TFTR supershot plasmas and the nonlinear, neoclassical pressure-gradient–driven tearing mode theory. Good agreement is found on the nonlinear evolution of single helicity magnetic islands ($m/n = 3/2$, 4/3, or 5/4, where m and n are the poloidal and toroidal mode numbers, respectively). The saturation of these neoclassical tearing-type modes requires $\Delta' < 0$ (where Δ' is the well-known parameter for classical current-driven tearing instability), which is also consistent with the numerical calculation using the experimental data.

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Understanding the tearing-type MHD (magnetohydrodynamic) instabilities observed in TFTR neutral-beam (NB) heated supershot [1] plasmas has long been a challenge for plasma theory. These modes typically have low frequency ($f < 50$ kHz) and low mode numbers ($m/n =$ 3/2, 4/3, and 5/4). The $m/n = 2/1$ modes are not usually seen in the high-performance supershot plasmas. The important effects of these MHD modes on plasma performance have been discussed in Ref. [2]. It is found that when these modes are large they can cause a strong deterioration in plasma performance [2] as measured by the DD or DT neutron rate, plasma-stored energy, energy confinement time, etc. Considerable effort has been expended on the theoretical interpretation and numerical simulation of these modes. These works have been mostly based upon the classical current-driven tearing mode theory [3,4]. However, the results have been unsatisfactory [5,6]. In this Letter, we compare the experimental results with a relatively new theory, the neoclassical *pressure-gradient–*(∇p) driven tearing mode theory [7,8]. The results are found to be very encouraging.

The evolution of two typical tearing-type modes in the high power NB heated, high β (plasma pressure/magnetic field pressure) supershot plasmas is shown in Fig. 1. Discharge A developed an $m/n = 3/2$ mode. Discharge B developed an $m/n = 4/3$ mode. Detailed analyses of the MHD modes and their deterioration effect on plasma transport [see Fig. 1(a)] have been reported in Ref. [2]. Here, we will concentrate only on the nonlinear evolution of the mode. Generally speaking, as shown in Fig. 1(b), these modes start before or around the peak performance of the plasma, which is typically ~300–450 ms after the beam injection. They have a long (~100–300 ms) and mostly linear growth phase before the saturation. The negative spikes of the mode amplitude are a common feature of these MHD modes, which may be related to some weak interaction between the MHD and fast beam ions. The mode amplitude gradually decays to the noise level after the beam turnoff. The mode frequency during the NB phase is nearly constant and decays exponentially after the NB phase. It is found that within the measurement uncertainty the mode frequency can be attributed to the plasma toroidal rotation.

High-temperature supershot plasmas (typically, $T_i \sim$ 20–35 keV, $T_e \sim 10$ keV) are in the low collisionality "banana" regime. The existence of trapped particles in these plasmas changes the plasma physics in many different ways. One of the important effects is the neoclassical bootstrap current which is induced by the damping of the poloidal flow due to the trapped-untrapped

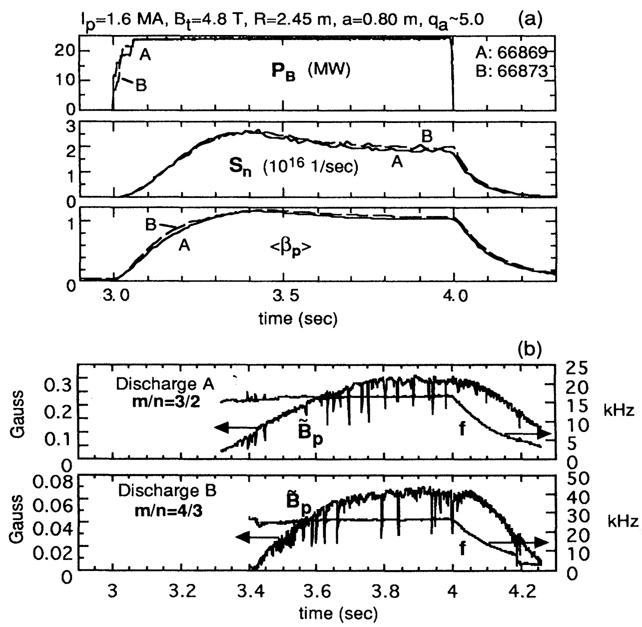

FIG. 1. (a) Two typical supershot discharges with different MHD modes. Here, P_B is the neutral beam power, S_n is the total DD neutron rate, and $\langle\beta_p\rangle$ is the volume-averaged poloidal beta. (b) Discharge A has a dominant $m/n = 3/2$ mode. Discharge B has a 4/3 mode. The mode amplitude ($\tilde{B}_p$) and frequency (f) are measured by a Mirnov coil.

particle viscous stresses in these plasmas [9]. Inclusion of the perturbed bootstrap current induced by magnetic islands leads to a nonlinear island evolution equation [7,8,10,11]

$$\frac{dw}{dt} = k_1 \frac{\eta_{\rm nc}}{\mu_0}\left(\Delta'(w) + k_2 \frac{w_c}{w}\right), \qquad (1)$$

with

$$w_c \equiv \sqrt{\epsilon}\,\beta_p \frac{L_q}{L_p}, \qquad (2)$$

where w (>0) is the magnetic island width, $k_1 = 1.22$ [11,12], k_2 is a constant of order unity [10,11], $\eta_{\rm nc}$ is the neoclassical plasma resistivity, $\mu_0 \equiv 4\pi \times 10^{-7}$ is the permeability of free space, $\Delta'(w)$ is the usual resistive MHD stability parameter [3], $\epsilon \equiv r/R_0 \ll 1$ is the tokamak inverse aspect ratio where r and R_0 are the plasma minor and major radii, $\beta_p \equiv p/(B_p^2/2\mu_0)$ is the local ratio of plasma pressure to the *poloidal* magnetic field pressure, $L_q \equiv q(dq/dr)^{-1}$ is the magnetic shear length (q is the plasma safety factor), and $L_p \equiv -p(dp/dr)^{-1}$ is the unperturbed, plasma pressure-gradient scale length. All the quantities are evaluated at the mode rational surface [$r = r_s$, $q(r_s) = m/n$]. In the limit of small toroidicity ($\epsilon = 0$), Eq. (1) reduces to the well-known Rutherford equation [4]. The second term in Eq. (1) is a new instability driving term (for normal tokamak plasma profiles with $dp/dr < 0$ and $dq/dr > 0$), and we will refer to it as a ∇p term. According to this theory, the ∇p term will drastically affect the island evolution. First of all, when the island is small, i.e., $w \ll |k_2 w_c/(-\Delta')|$, its evolution will be dominated by the ∇p term:

$$w^2 \simeq 2k_1 k_2 \frac{\eta_{\rm nc}}{\mu_0}\sqrt{\epsilon}\,\beta_p \frac{L_q}{L_p}\,t. \qquad (3)$$

Namely, the island grows with $w \sim \sqrt{t}$ in contrast to the Rutherford linear growth ($w \sim \Delta' t$). The second and more important feature of Eq. (1) is that when a ∇p-driven saturated island is formed, the plasma has to be stable to the classical Δ'-driven tearing mode, i.e., $\Delta'(w) < 0$. The saturated island width is given by

$$w_{\rm sat} = k_2 \frac{w_c}{-\Delta'(w)} = \frac{1}{-\Delta'(w)} k_2 \sqrt{\epsilon}\,\beta_p \frac{L_q}{L_p}. \qquad (4)$$

The comparison of the island evolution (discharges A and B in Fig. 1) between the measurement and Eq. (1) is shown in Fig. 2. Here, the "measured" island width (curves labeled a) is from a calculation using the standard cylindrical island width formula [13]:

$$w = 4r_s\sqrt{\frac{1}{m}\frac{\tilde{B}_r}{B_p}\frac{L_q}{r}} \quad \text{at } r = r_s. \qquad (5)$$

The radial component of the magnetic fluctuation $\tilde{B}_r$ on the mode rational surface in Eq. (5) is obtained by integrating the well-known MHD equation [3,14]:

$$r\frac{d}{dr} r \frac{d\tilde{\psi}}{dr} - m^2\tilde{\psi} - \frac{q}{1 - nq/m}\, r\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\frac{r^2}{q}\right)\tilde{\psi} = 0, \qquad (6)$$

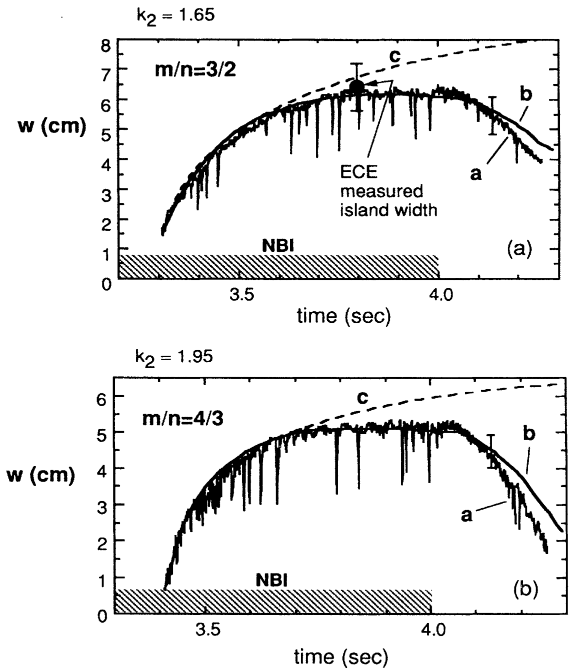

FIG. 2. Comparison of the "measured" magnetic island (curves labeled a) with the neoclassical ∇p-driven tearing mode theory. (a) $m/n = 3/2$ island. Also shown is the island width measured by the multichannel ECE diagnostics. (b) $m/n = 4/3$ island. Curves labeled b use the time-dependent parameters (see Fig. 3). Curves labeled c use fixed parameters [for 3/2 (4/3) mode: $\eta_{\rm nc}/\mu_0 = 120$ (110) cm^2/s, $w_c = 0.40$ (0.38), and $\Delta' = -0.07$ (-0.11) cm^{-1}].

and using the Mirnov coil measured edge $\tilde{B}_p$. In Eq. (6) $\tilde{\psi}$ is the perturbed poloidal magnetic flux, and $\tilde{B}_p = \partial\tilde{\psi}/\partial r$, $\tilde{B}_r = -im\tilde{\psi}/r$. The $q(r,t)$ profile is taken from a transport analysis code (TRANSP [15]), which can be partly justified by comparing the location of the mode rational surfaces with a local temperature fluctuation measurement using ECE (electron-cyclotron-emission) diagnostics. Also, it is found that the island width obtained from Eq. (5) agrees reasonably well with the multichannel ECE (~5–6 cm separation) measurement when the island is large; see Fig. 2(a). Curves labeled b in Fig. 2 are obtained by integrating Eq. (1) as an initial value problem beginning at the mode starting time. All the parameters, except the constant k_2, are taken from the TRANSP simulation using the measured $T_e, T_i, n_e, Z_{\rm eff}$, etc. For simplicity we use $\eta_{\rm nc} = \eta_s/(1 - \sqrt{\epsilon})^2$, where η_s is the Spitzer resistivity. Only the thermal plasma pressure is used in the calculation of w_c (i.e., we exclude the beam ion component in the pressure). The $\Delta'(w)$ in Eq. (1) is obtained from a numerical integration of Eq. (6) using the $q(r)$ from the TRANSP code. The finite island effect is modeled using

$$\Delta'(w) = (d\tilde{B}_r/dr|_{r_s+w/2} - d\tilde{B}_r/dr|_{r_s-w/2})/\tilde{B}_r(r_s). \qquad (7)$$

For $m \gtrsim 3$ and small w it is found that the calculated Δ' agrees with the analytic formula [16] very well. The constant k_2 is adjusted to match the observed saturated island width. The time evolution of the three parameters [η_{nc}/μ_0, w_c, and $\Delta'(w)$] used to obtain curves labeled b in Fig. 2 is shown in Fig. 3.

As Fig. 2 shows, the agreement between the theory (using time-dependent parameters) and experiment is very good within the analysis uncertainty for both $m/n = 3/2$ and $m/n = 4/3$ modes. The model not only closely mimics the island evolution during the beam phase, but also follows the data in the postbeam phase. The fast decay of the island after the beam turnoff is mainly due to the fast decrease of plasma β and quick increase in η. If the parameters are fixed in time (see curves labeled c in Fig. 2), the model will still agree with the data in the rising phase, but then it fails to match the saturated island width and the postbeam decay. The difference between curves b and c indicates the importance of the nonlinear effects due to the self-consistent changes of the plasma parameters during the island evolution. Similar results are obtained for the $m/n = 5/4$ tearing mode cases. More than 20 discharges have been analyzed. Similar agreement is found for most cases. Figure 4 shows a comparison of the saturated island widths between the theory and experiment. Since a fixed k_2 (=1.7) is used, the scatter in the data indicates the statistical uncertainty.

Although our analysis shows that the neoclassical ∇p-driven tearing mode theory provides a very promising explanation for the ($m \gtrsim 3$) tearing-type modes observed in TFTR, there are still many questions that remain to be resolved. The present theory [Eq. (1)] does not provide

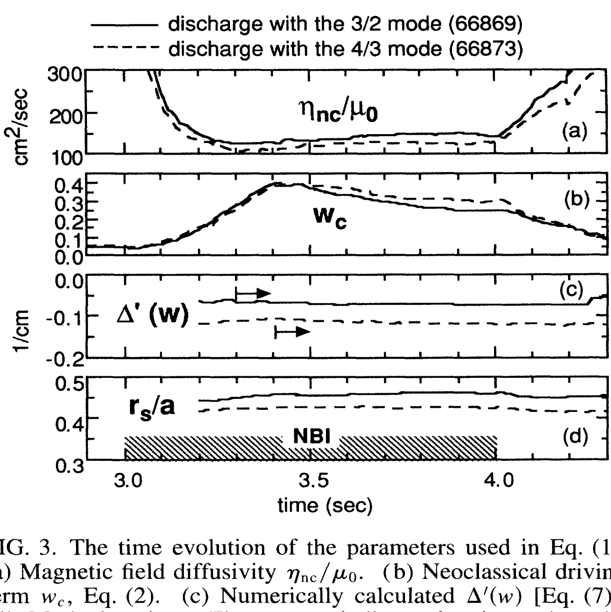

FIG. 3. The time evolution of the parameters used in Eq. (1). (a) Magnetic field diffusivity η_{nc}/μ_0. (b) Neoclassical driving term w_c, Eq. (2). (c) Numerically calculated $\Delta'(w)$ [Eq. (7)]. (d) Mode location. The arrows indicate the time when the MHD mode begins.

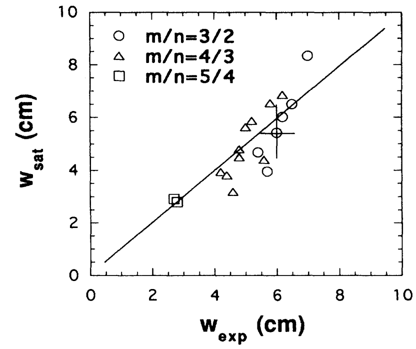

FIG. 4. Theory-experiment comparison of saturated magnetic island width. The w_{exp} is from Eq. (5). The w_{sat} is from Eq. (4). A constant $k_2 = 1.7$ has been used for all discharges.

a threshold island width. Therefore, the theory cannot predict when and what mode should start to grow. Some theoretical work has recently focused on the effects of the finite parallel thermal conductivity [11] and diamagnetic drift [17]. It is shown that both effects can lead to a threshold island width. In addition, physically the fluid theory needs to be modified when the island width is small, so the kinetic effects become important, such as Landau damping and finite particle orbit effects. Also, it is not clear how the fast ions (high energy beam ions or fusion ions) affect these MHD modes. It is interesting to note that the observed 4/3 and 3/2 modes are often preceded by some perturbation which may create a trigger island, e.g., fishbone activity ($m/n = 1/1$ mode), small β collapse, impurity influx event, neutral beam power drop, etc. Analysis based on the Mirnov coil data shows that the triggered initial island width is usually $\gtrsim 1 - 2$ cm, compared with a noise level island $\lesssim 0.5$ cm. The triggering mechanism seems to be quite complex. Further work is needed to understand why the most commonly observed tearing-type modes are $m/n = 4/3$, $3/2$, and sometimes $5/4$, instead of $5/3, 6/5, 6/4, \ldots$, and why the $m/n = 2/1$ modes are so stable in high performance supershots.

In conclusion, good agreement has been found in this Letter between the single-helicity magnetic island evolution ($m/n = 3/2$, $4/3$, or $5/4$) observed in TFTR supershot plasmas and the nonlinear neoclassical ∇p-driven tearing mode theory. This agreement implies that TFTR supershot plasmas are stable to the classical current-driven tearing modes. This observation should have an important impact on the design of future tokamaks which involve large bootstrap current fractions. Further work needs to be carried out to explore the threshold island width, triggering mechanism, and multimode interactions in both the experimental and theoretical studies.

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