Effects of Ambient Gases on Granular Materials under Vertical Vibration

H. K. Pak, E. Van Doorn, and R. P. Behringer

Department of Physics and Center for Nonlinear and Complex Systems, Duke University, Durham, North Carolina 27708-0305

(Received 11 April 1994)

We consider layers of noncohesive granular materials of mean height h subject to vertical vibration $z = A\cos(\omega t)$. Above $\Gamma \equiv A\omega^2/g \simeq 1$, convection begins, creating a heap of height L. We find that for particles of diameter $d < 1$ mm the pressure P of the surrounding gas plays a key role: $L = b\tanh(KP) + c$, where b and K depend on h, the gas viscosity, d, Γ, and A. Convection is significantly suppressed if the shaker sidewalls are permeable, showing that gas trapped in the granular material is a major source of heaping. We consider criteria to determine when gas effects are relevant.

PACS numbers: 46.10.+z, 47.20.–k

Noncohesive granular materials have both solidlike and fluidlike behavior and exhibit unusual phenomena [1] such as size segregation [2], density waves [3], convective transport [4], and anomalous sound propagation [5]. The dynamics of granular materials are important in many industrial applications [6,7]. However, due to the complexities of these materials, there is not yet an acceptable general theory for granular flows. Hence, quantitative experiments are a key tool for developing further understanding.

Recently [8–15], as well as historically [4], there has been great interest in the physics of vibrated noncohesive granular materials. Assuming vertical sinusoidal oscillations, $z = A\sin(\omega t)$, the dimensionless acceleration $\Gamma = A\omega^2/g$ (g is the acceleration of gravity) is the primary control parameter. Above $\Gamma = \Gamma_c$, the surface of the granular material becomes unstable, and a rich variety of patterns occurs depending, in particular, on h/d, where h is the mean depth of the granular mass and d is the size of a particle. If $h/d \gg 1$ and $\Gamma > \Gamma_{c-h}$, steady convective motion evolves in the form of two counterrotating rolls leading, typically, to a single heap making an angle θ_d with the horizontal (a dynamic angle of repose). Upward convective flow carries material to the apex of the heap, and downward avalanches occur along the upper surface. If h/d is moderate, a parametric instability to standing waves at half the shaker frequency occurs [9,15] for $\Gamma > \Gamma_{c-p}$. Typically, $\Gamma_{c-h} \simeq 1.2$ and $\Gamma_{c-p} \simeq 4$.

The focus of this work is the heaping instability. Three physical mechanisms have been identified as possible causes of heaping: friction between the walls and particles [12], an analog of acoustic streaming [8] if the shaking is nonuniform, and gas pressure effects. At issue here is the role played by the last of these. The forces which are present include gravity, interparticle forces, normal and frictional forces at the walls, and forces from the surrounding gas. All but gas effects have been included in molecular dynamics (MD) simulations which yield an instability to convective flow, but little if any heaping [13]. Clément, Duran, and Rajchenbach [12] have shown that wall friction can generate convective flows. However, there are conflicting observations concerning the role of gas in the heap formation [4,10,11]. Laroche, Douady, and Fauve [10] observed no heaping when the surrounding gas was evacuated to a pressure $P = 10^{-5}$ Torr, whereas Evesque [11] noted heaping for P down to 4 Torr. Gutman [6] has studied the levitation of a layer of grains by gas, although not heaping.

We present here a systematic experimental study of the role of surrounding gas on the heaping. We find, as did Laroche and co-workers, that heaping either ceases or is significantly reduced for $P = 0$. Moreover, we find that the heap is regulated by a number of factors, including P, h, d, and the gas viscosity μ.

We used a mechanical shaker driven by a dc motor (inset, Fig. 2), as explained elsewhere [14]. We used annular cells with narrow gaps; gravity and the direction of vibration are parallel to the cylinder axis. This geometry eliminates one pair of lateral boundaries (and their accompanying friction). The cell was made of two concentric cylinders. For most of this work, both cylinders were Plexiglas; the diameter of the inner cylinder

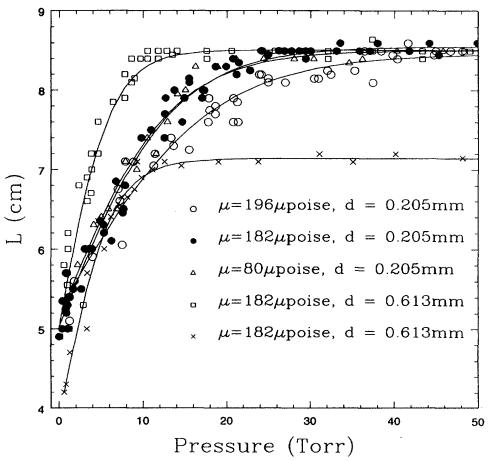

FIG. 1. L vs P for different gases and sand grain diameters, as noted. The solid lines are fits by $L = b\tanh(KP) + c$. The crosses correspond to a smaller h than the other data.

0031-9007/95/74(23)/4643(4)$06.00

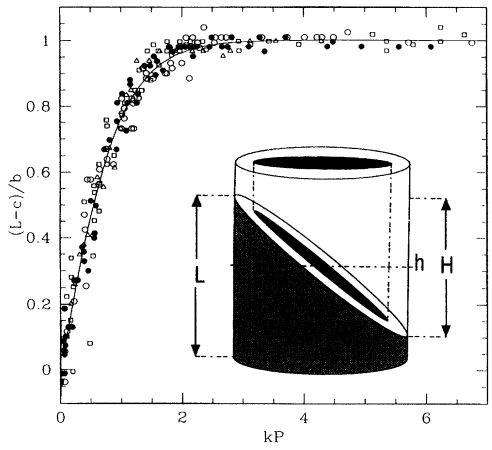

FIG. 2. $(L - c)/b$ vs KP for the data of the previous figure. The solid line is $\tanh(KP)$. The symbols used here are the same as in the previous figure. Inset: schematic of the apparatus, and definitions of H, h, and L.

was ~10 cm, and the radial gap size was ~1.0 cm. To probe the effect of surrounding gas, the annular gap of an all Plexiglas cell was sealed and the pressure inside controlled over $10^{-2} < P < 10^{3}$ Torr. The experiments were made at $\Gamma = 1.30$ where the convective heaping is strong. Values of A were a few millimeters. To guarantee that the observations corresponded to steady state conditions, it was necessary to wait from 30 min to an hour or more. In one experiment, the outer cylinder was made of a thin porous stainless steel sheet with a regular grid of ~0.02 cm diam holes, and an open area fraction of ~0.4. The inner Plexiglas cylinder had a diameter of 7.6 cm and the gap was ~1.0 cm. The granular materials consisted of sand (smooth Ottawa sand with $d = 0.065$ cm and two rough sands with $d = 0.02$ and 0.06 cm, respectively) and approximately monodisperse glass spheres with $0.01 \leq d \leq 0.2$ cm.

We characterize the size of the heap by two related measures: H, the vertical distance between the top of the heap and the valley which forms in the opposite part of the annulus, and L, the distance between the base of the shaker and the top of the heap (inset, Fig. 2).

When P is decreased, the physical shape of the heap does not change until P falls below ~10–20 Torr. Then, at fixed Γ, H and L decrease monotonically with P. Below $P \approx 1$ Torr, the heap reaches an asymptotic state which is nearly flat. There still may remain at $P \simeq 0$ weaker convection, with a radial sense of flow, which leads to a relatively small amount of heaping. This is presumably because of friction between the grains and the wall [12], and is more pronounced for low A. We will explore this issue elsewhere.

Figure 1 shows L vs P for air, helium, propane ($\mu =$ 182, 196, and 80 μP, respectively), and various types of materials. The data were obtained at $\Gamma = 1.3$ with $A = 0.534$ cm and $\omega/2\pi = 7.78$ Hz using all Plexiglas side walls. The solid lines are nonlinear least-squares fits by $L = b\tanh(KP) + c$, where b, c, and K are fitting parameters which clearly depend on μ, h, and d. Saturation of L with increasing P, as indicated by the data, must occur because of the increasing efficiency of avalanches as the dynamic angle of repose θ_d becomes large. If the data of Fig. 1 are plotted as $(L - c)/b \propto H$ vs KP, they collapse onto a single curve, Fig. 2, within experimental resolution; the P dependence is universal for a considerable range of d. These data are representative of all our results obtained with $d < 1$ mm. For the larger d studied here, $d \geq 0.13$ cm, heaping was not observed. The range $0.1 < d < 0.13$ cm is an interesting crossover regime which we will describe elsewhere.

The ambient gas plays a demonstrably important role in generating a heap; the question is why. Two effects seem potentially relevant: viscous drag and pressure gradients which may result from gas trapped in the material. For the following estimates, we use a standard set of parameters: $\Gamma = 1.3$, $A = 0.3$ cm (so that $v_s \equiv A\omega =$ 20 cm/s), $h = 10$ cm, $d = 0.02$ cm, $\mu = 200$ μP, and $P = 1$ atm. We use $\rho_g = 3$ g/cm^3 for the density of the grain material. We can estimate the effect of viscous drag via the Stokes force, $F_s = 3\pi d v_s \mu$. (Strictly, the actual Reynolds number for the flow past a grain is probably somewhat higher [16] than that for which Stokes flow applies, but the order of magnitude is probably right.) The viscosity of a gas shows very little pressure dependence, and F_s is small compared to the weight $W = g\pi\rho_g d^3/6$ of a grain [16]; $F_s/W = 0.06$ for the standard conditions. Hence we conclude that viscous drag is not important for these experiments.

An alternative explanation is that gas flows beneath the grains during the part of the shaker cycle when the grains lift off the base of the shaker, creating a gap of height δ between grains and base [6,10]; during this process, the grains tend to dilate, making the inflow of gas easy. Later in each cycle, the grains compact, and the gas between the grains and base is compressed. This provides an upward force on the grains. This effect has been studied by Gutman [6] and Laroche, Douady, and Fauve [10]. The former carried out extensive calculations and experiments to characterize the effect of gas on δ, assuming a granular layer of uniform height. These calculations show that the additional upward force can significantly affect δ, although no consideration was given to heaping. Laroche, Douady, and Fauve [10] have shown experimentally that the pressure rises as the heap collides with the base.

In order to test the role of gas trapping on the heaping process, we used the shaker with the porous outer wall, substantially reducing the possibility of trapping the gas. For the various materials studied here, rough and smooth sand and glass spheres, we found that at $\Gamma = 1.3$ either no heap formed (all but one material) or a small heap formed (smooth sand). When we sealed the holes

from the outside with plastic film or tape, leaving the surface in contact with the granular material unchanged, nearly normal heaping was observed. We conclude that gas trapped and compressed by the material plays an important role in the heaping process

It is useful to have a measure of how strong the pressure effect can be. The calculations by Gutman [6] provide a starting point. But, they are specific to given grain sizes, pressures, etc., and they assume a 1D motion which is not truly appropriate when there is heaping. Nevertheless, it is possible to proceed along the same lines as Gutman to produce a first-order estimate of the strength of the gas effects which can be easily applied to the present experiments.

We frame the issue by assuming that gas has become trapped between the layer of granular material and the bottom of the shaker. The process by which this happens is complex when there is heaping, but it clearly occurs. As the gas is compressed during each cycle, it leaks out through the material, of mean height h, which is a porous medium. As the gas flows out towards the top of the medium, there is an upward pressure gradient which tends to lift the material and leads to heaping. To model this process, we assume that (a) the system is 1D; (b) the flow of gas follows Darcy's law and is characterized by a velocity v of the gas relative to the grains; (c) the gas leaks out relatively fast, so that the flow properties through the medium can be assumed static as the gas pressure decays; (d) the characteristic initial velocity of gas relative to grains is set by the characteristic velocity of the shaker, $v_s = A\omega$; (e) the gas is compressible; and (f) the slowest relaxational mode of the gas leaking through the medium is the most important and sets the scale for the upward force of gas pressure on the grains. We find the slowest mode of a boundary value problem in which the gas velocity is $v \simeq v_s$ initially. The details of the boundaries at the top and bottom of the porous granular layer are taken as no gas flux at the bottom, and fixed pressure at the top, but these details are not important for the scaling argument developed here. We estimate the average upward force on a grain by the time average of the resulting pressure gradient from this mode.

With these assumptions, the gas flow and pressure are modeled by Darcy's law [17]: $(\rho/\phi)\partial v/\partial t = B[-\nabla P - (\mu/\gamma)v]$, and the continuity equation: $\partial\rho\phi/\partial t = -\nabla\cdot(\rho v)$. (The time derivative in Darcy's law is not important here because the flow is slow relative to acoustic scales; hence $B \simeq 1$ need not be known.) Variations in ρ and P are related by the compressibility κ: $\delta\rho = \rho\kappa\delta P$. We take [6] $\kappa = P^{-1}$, and assume a random densely packed material with porosity $\phi \simeq 0.4$. The permeability of the medium is well represented [17] by the Ergun relation: $\gamma = d^2\phi^3/150(1-\phi)^2$.

Following standard analytical procedures [16], we linearize these equations for small amplitude fluctuations in P and v. For the conditions envisioned here, the coupled equations can be reduced to a diffusion equation with diffusion coefficient $D = \gamma/\phi\kappa\mu$. Hence all modes decay. Solutions, $(v, \delta P) = (v_0, P_0)\exp(-t/\tau + ikz)$, depending on the vertical coordinate z, must satisfy $\tau^{-1} = Dk^2$. The pressure and velocity amplitudes are related by $P_0 = (k\tau/\phi\kappa)v_0 = (\mu/\gamma\kappa)v_0$. Implementing the boundary conditions, we find that the space-time-varying part of the pressure consists of modes $P = P_0\exp(-t/\tau_k)\cos(kz)$, where the smallest k (corresponding to the slowest decay) is $k = \pi/2h \simeq h^{-1}$. The decay time of the slowest mode is $\tau = (2h/\pi)^2/D \simeq h^2/D$. For cases relevant here, the relaxation times τ are relatively short compared to the shaker period.

The upward pressure force is found by integrating ∇P over a grain, which is assumed to be a sphere. Because the relaxation times τ are relatively short compared to the shaker period, a time average of the gradients must be carried out. An estimate of the ratio of the average upward pressure force on a grain and its weight is given by $\langle F_p/W\rangle \simeq 1.2P\Gamma/(\pi kD)^2\rho_g \simeq 4.3\times 10^4(h\mu/d^2)^2(\Gamma/P\rho_g)$. For the standard conditions, $\langle F_p/W\rangle = 0.46$.

This analysis suggests several points. First, gas effects depend strongly on grain size; specifically, the mean force falls off as d^{-4}. This is borne out by measurements of H as a function of d, Fig. 3, for $P = 1$ atm. H is a strong function of d: $H/d \sim d^{-1.63}$. For $d = 0.10$ cm, $h = 10$ cm, there was a clear if weak variation of H with P. No pressure effect was found for $d = 0.13$ cm, $h = 10$ cm. A second point is that heaping should be suppressed if h/d is small. The characteristic diffusion times depend on h^2; for short layers, pressure variations relax quickly compared to the shaker period. Data for H vs h shown in Fig. 4 confirm this expectation. Note that H saturates with increasing h and approaches a roughly constant value H_s.

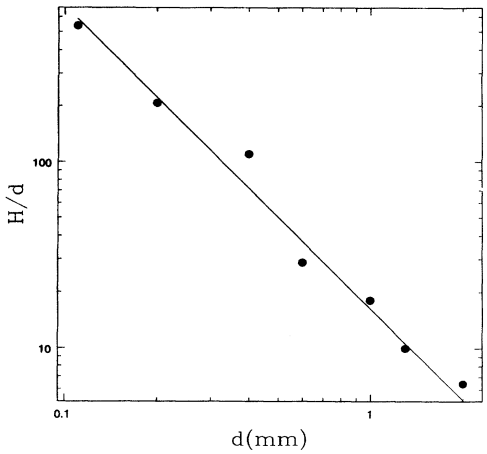

FIG. 3. Scaled saturation height H/d vs d for glass spheres in air at 1 atm. The data for H/d are described reasonably well by a power law in d with an exponent of -1.63 (solid line).

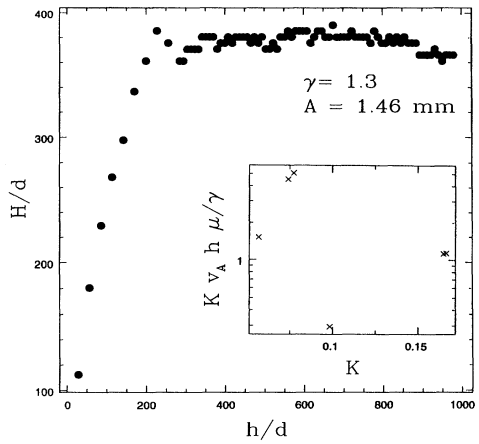

FIG. 4. Data for H/d vs h/d for $d = 0.02$ cm, rough sand. Data for other materials are qualitatively similar. Inset: The dimensionless measure $Kv_A\mu h/\gamma$, where we use $k \simeq 1/h$ for the slowest mode wave number.

Obviously, the enhancement of heaping by gas pressure cannot occur when $P = 0$, and the decrease of heaping [10] at very low P occurs because there is no longer enough gas to sustain the process. Before the limit of zero pressure is reached, the linear approximations must be replaced by a nonlinear analysis [6]. In the present model, this should occur when P becomes comparable to P_0. We might expect that the breakdown of linearity and the cessation of heaping would occur at roughly the same pressure, although that need not be rigorously true. If these two events occur at the same P, $P_0 \simeq v_A\mu/k\gamma \simeq v_A\mu h/\gamma$ should be approximately proportional to the fitting parameter K^{-1} of Fig. 1. Alternatively, P_0K should be approximately a constant. The expectation that this quantity is roughly constant is reasonably well met, Fig. 4, inset, although there is at least one point which is a notable exception. The correct prediction of K requires a much more complex calculation than considered here or by Gutman. Such a calculation must allow for the motion of grains and gas and for heap formation.

A final issue is what happens as P increases. The scaling analysis above suggests that the heaping should diminish as P grows. Indeed, this is the case. Between 20 Torr and 1 atm, H decreases by $\sim$10% for $d \simeq 0.05$ cm, and by $\sim$60% for the marginal case, $d = 0.1$ cm.

To conclude, these experiments have elucidated the role gas plays in convection and heaping through quantitative measurements of L versus important system parameters, most particularly P. A simple model indicates the effect of various parameters, but a detailed understanding of how gas enhances convection is not yet available. One reason the cited MD studies, which only include mechanical interparticle interactions, fail to show significant heaping is that these models do not account for gas effects. It is also important to note that convection and heaping can *still* be driven by other mechanisms such as wall friction or inhomogeneous forcing. The present work only addresses the role of the ambient gas. This analysis also suggests why a single heap forms: wherever h/d is locally large, gas effects will be greater, and heaping favored. The biggest heap will always win.

This work was supported by the National Science Foundation under Grant No. DMR-9017236, by the National Science Foundation and the Air Force Office of Research under Grant No. DMS-9201034, and by the Exxon Research and Engineering Corp.

[1] H. M. Jaeger and S. R. Nagel, Science **255**, 1523 (1992); R. P. Behringer, Nonlinear Science Today **3**, 1 (1993).
[2] J. C. Williams, Powder Tech. **15**, 245-251 (1976).
[3] G. W. Baxter, R. P. Behringer, T. Fagert, and G. A. Johnson, Phys. Rev. Lett. **62**, 2825 (1989).
[4] M. Faraday, Philos. Trans. R. Soc. London **121**, 299 (1831).
[5] C-h. Liu and S. R. Nagel, Phys. Rev. Lett. **68**, 2301 (1992).
[6] R. G. Gutman, Trans. Instn. Chem. Engrs. **54**, 174 (1976); **54**, 251 (1976).
[7] J. C. Howard, *Fluidized Bed Technology, Principles and Applications* (Hilger, New York, 1989).
[8] S. B. Savage, J. Fluid Mech. **194**, 457 (1988).
[9] P. Evesque and J. Rajchenbach, Phys. Rev. Lett. **62**, 44 (1989); J. B. Knight, H. M. Jaeger, and S. R. Nagel, Phys. Rev. Lett. **70**, 3728 (1993); S. Douady, S. Fauve, and C. Laroche, Europhys. Lett. **8**, 621 (1989).
[10] C. Laroche, S. Douady, and S. Fauve, J. Phys. (Paris) **50**, 699 (1989).
[11] P. Evesque, J. Phys. (Paris) **51**, 697 (1990).
[12] E. Clément, J. Duran, and J. Rajchenbach, Phys. Rev. Lett. **69**, 1189 (1992).
[13] J. A. C. Gallas, H. J. Herrmann, and S. Sokolowski, Phys. Rev. Lett. **69**, 1371 (1992); H. Herrmann (private communication); Y.-h. Taguchi, Phys. Rev. Lett. **69**, 1367 (1992); H. Herrmann (private communication); Y.-h. Taguchi, Phys. Rev. Lett. **69**, 1367 (1992); P. A. Thompson, *Computer Simulations in Condensed Matter Physics VI,* edited by D. P. Landau, K. K. Mon, and H. B. Schuttler (Springer-Verlag, Berlin, 1993).
[14] H. K. Pak and R. P. Behringer, Phys. Rev. Lett. **71**, 1832 (1993).
[15] F. Melo, P. Umbanhower, and H. Swinney, Phys. Rev. Lett. **72**, 172 (1994).
[16] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1959), pp. 66, 170, and 245.
[17] D. A. Nield and A. Bejan, *Convection in Porous Media* (Springer, New York, 1992).