Test of Wigner's Spin-Isospin Symmetry from Double Binding Energy Differences

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It is shown that the anomalously large double binding energy differences for even-even $N = Z$ nuclei are a consequence of Wigner's SU(4) symmetry. These, and similar quantities for odd-mass and odd-odd nuclei, provide a simple and distinct signature of this symmetry in $N \simeq Z$ nuclei.

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In the supermultiplet model of nuclei it is assumed that nuclear forces are independent of isospin as well as spin [1–3]. Nuclear states can then be characterized by the quantum numbers of the spin-isospin or SU(4) symmetry, giving rise to simple predictions concerning nuclear β-decay rates and masses. The former arise because the Fermi as well as Gamow-Teller operators are generators of SU(4), and as such β transitions can only occur between states belonging to the same supermultiplet; predictions of nuclear binding energies are obtained in a lowest-order approximation from the permutational symmetry of the *orbital* part of the many-body wave function which determines the degree of spatial overlap between the nucleons. Since the original work by Wigner [1] and Hund [2], it has become clear that SU(4) symmetry is badly broken in the majority of nuclei because of the increasing importance with mass of the spin-orbit term in the nuclear mean-field potential. Nevertheless, it remains a useful ansatz for studying global properties of p- and sd-shell nuclei from a simple perspective. Moreover, as will be shown in this Letter, it may have a particular and renewed relevance in the study of the heavier $N \simeq Z$ nuclei from ^{56}Ni to ^{100}Sn, a declared experimental goal of many of the current proposals for new facilities based on accelerated radioactive beams [4].

The most conclusive test of SU(4) symmetry is through a comparison with realistic shell-model calculations which can be readily performed for nuclei up to ^{40}Ca. The goodness of SU(4) symmetry in the ground state is then obtained by taking the overlap between the shell-model wave function and the favored SU(4) representation. This approach is followed, for example, for sd- and pf-shell nuclei by Vogel and Ormand [5]. The overall conclusion of such studies is that in nuclei heavier than ^{16}O significant departures from SU(4) symmetry occur, especially in midshell regions [6].

To obtain a test of the goodness of SU(4) symmetry directly from masses is more difficult. Franzini and Radicati [7] suggested the use of a ratio $R(T_z)$ of ground-state energy differences involving four isobaric nuclei with different isospin projections T_z and showed that the values agree rather well with the SU(4) predictions for nuclei with masses up to $A \approx 110$. However, it was demonstrated subsequently [8] that this ratio $R(T_z)$ is not very sensitive to SU(4) symmetry mixing.

In this Letter we point out that a sensitive test of SU(4) symmetry can be made by using double binding energy differences which also provide information concerning the strength of the neutron-proton (np) interaction, which is known to play a pivotal role in the structure of nuclei [9]. Recently, the quantity

$$\delta V_{np}(N,Z) \equiv \tfrac{1}{4}\{[B(N,Z) - B(N-2,Z)] - [B(N,Z-2) - B(N-2,Z-2)]\}, \quad (1)$$

where $B(N,Z)$ is the (negative) binding energy of an even-even nucleus with N neutrons and Z protons, was used by Brenner *et al.* [10] to extract the empirical interaction strength of the last neutron with the last proton. A notable outcome of this analysis was the occurrence of particularly large interaction strengths for $N = Z$ nuclei. Although this feature is consistent with both schematic and realistic shell-model calculations [10], a simple interpretation of this result is still lacking. It is the purpose of this Letter to show that the $N = Z$ enhancements of $|\delta V_{np}|$ are an unavoidable consequence of Wigner's SU(4) symmetry and that the degree of the enhancement provides a sensitive test of the quality of the symmetry itself.

A representative sample of the data is shown in Fig. 1(a) which gives $-\delta V_{np}(N,Z)$ (where known) for the sd shell. While for $N \neq Z$ the np interaction strength is roughly constant and of the order of -1 MeV, the dramatic enhancement of $|\delta V_{np}|$ occurring for $N = Z$ is clearly evident. This prominent feature can be understood from the simple perspective of Wigner's supermultiplet theory. Wigner's scheme in a harmonic-oscillator shell with degeneracy $\omega = \sum(2l+1)$ implies the classification

$$U(4\omega) \supset (\mathrm{U_{orb}}(\omega) \supset \cdots \supset \mathrm{O_{orb}}(3)) \otimes (\mathrm{U}_{ST}(4) \supset \mathrm{SU}_{ST}(4) \supset \mathrm{SU}_S(2) \otimes \mathrm{SU}_T(2)). \quad (2)$$

The dots refer to an appropriate labeling scheme for the orbital part of the fermion wave function, such

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as Elliott's SU(3) scheme [12]. The total M-fermion wave function transforms antisymmetrically under $U(4\omega)$ and is decomposed into an orbital part, behaving as $[M_1, M_2, M_3, M_4]$ under $U_{\rm orb}(\omega)$, and a spin-isospin part. To ensure overall antisymmetry the latter by necessity transforms under $U_{ST}(4)$ as the conjugate representation $[\tilde{M}_1, \tilde{M}_2, \tilde{M}_3, \tilde{M}_4]$ (i.e., rows and columns of the Young tableau interchanged) and determines the supermultiplet $SU_{ST}(4)$ representation $(\lambda\mu\nu)$ ($\lambda = \tilde{M}_1 - \tilde{M}_2$, $\mu = \tilde{M}_2 - \tilde{M}_3$, and $\nu = \tilde{M}_3 - \tilde{M}_4$). From the $SU_{ST}(4) \supset SU_S(2) \otimes SU_T(2)$ reduction the possible values of S and T follow.

The short-range character of the residual nuclear interaction favors maximal spatial overlap between the fermions which is achieved in the most symmetric $U_{\rm orb}(\omega)$ representation. Antisymmetry of the overall wave function then requires the least symmetric $U_{ST}(4)$ representation or, equivalently, the one where the eigenvalue of the quadratic Casimir operator of $SU_{ST}(4)$,

$$g(\lambda\mu\nu) \equiv \langle(\lambda\mu\nu)|C_2[SU_{ST}(4)]|(\lambda\mu\nu)\rangle = 3\lambda(\lambda + 4) + 3\nu(\nu + 4) + 4\mu(\mu + 4) + 4\mu(\lambda + \nu) + 2\lambda\nu\,, \quad (3)$$

is minimal.

For even-even nuclei the favored SU(4) representation is $(0T0)$, where T is the isospin of the ground state. In

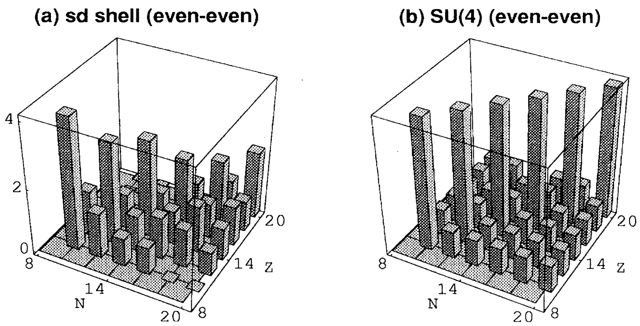

FIG. 1. Barchart representation of double binding energy differences (a) as observed in even-even sd-shell nuclei (in MeV) and (b) as predicted by Wigner SU(4). The data are taken from [11]; an empty square indicates that data are lacking. The x and y coordinates of the center of a cuboid define N and Z and its height z defines $-\delta V_{np}(N,Z)$ in MeV.

lowest order [i.e., assuming unbroken SU(4) symmetry and neglecting orbital contributions] the binding energy is then $a + bg(0T0)$ with b positive. The coefficients a and b depend smoothly on mass number [7]. Assuming constant coefficients for the four nuclei in (1), a simple expression is found for δV_{np} that depends on b only. (In fact, the analysis presented below remains valid if a and b depend *linearly* on mass number.) The result is

$$\delta V_{np}(N,Z)/b = \begin{cases} \frac{1}{4}[g(000) - g(010) - g(010) + g(000)] = -10, & N = Z\,, \\ \frac{1}{4}[g(0T0) - g(0,T-1,0) - g(0,T+1,0) + g(0T0)] = -2, & N \neq Z\,. \end{cases} \quad (4)$$

Wigner's supermultiplet theory in its *simplest* form (i.e., without symmetry breaking—dynamical or otherwise—in spin and/or isospin) therefore predicts $|\delta V_{np}|$ to be 5 times larger for $N = Z$ than for $N \neq Z$. This result is displayed in Fig. 1(b).

It has recently been pointed out that one can analyze odd-mass nuclei in a similar way [13]. The empirical interaction strength of the last neutron with the last proton is, in the case with N even and Z odd,

$$\delta V_{np}(N,Z) \equiv \tfrac{1}{2}\{[B(N,Z) - B(N-2,Z)] - [B(N,Z-1) - B(N-2,Z-1)]\}, \quad (5)$$

and similarly for odd-neutron nuclei. This quantity is displayed in Figs. 2(a) and 2(c) for odd-mass sd-shell nuclei and again shows an enhancement, this time for $N = Z - 1$ when N is odd and $N = Z + 1$ when Z is odd. The favored SU(4) representation for odd-mass nuclei is $(0, T - \frac{1}{2}, 1)$ if $\frac{1}{2}M + T$ is even and $(1, T - \frac{1}{2}, 0)$ if $\frac{1}{2}M + T$ is odd, both possibilities having the same eigenvalue of $C_2[SU_{ST}(4)]$, since $g(\lambda\mu\nu) = g(\nu\mu\lambda)$. From this one obtains, in the odd-proton case,

$$\delta V_{np}(N,Z)/b = \begin{cases} \frac{1}{2}[g(100) - g(100) - g(010) + g(000)] = -10, & N = Z + 1\,, \\ \frac{1}{2}[g(1,T-\frac{1}{2},0) - g(1,T-\frac{3}{2},0) - g(0,T+\frac{1}{2},0) + g(0,T-\frac{1}{2},0)] = -2, & N > Z + 1\,, \\ \frac{1}{2}[g(1,T-\frac{1}{2},0) - g(1,T+\frac{1}{2},0) - g(0,T-\frac{1}{2},0) + g(0,T+\frac{1}{2},0)] = -2, & N < Z + 1\,, \end{cases} \quad (6)$$

and similarly for odd-neutron nuclei. One thus finds enhancements in $|\delta V_{np}|$ which coincide with the observed ones, as is illustrated in Figs. 2(b) and 2(d).

Finally, for nuclei with N and Z odd, the empirical np interaction can be defined as

$$\delta V_{np}(N,Z) \equiv [B(N,Z) - B(N-1,Z)] - [B(N,Z-1) - B(N-1,Z-1)], \quad (7)$$

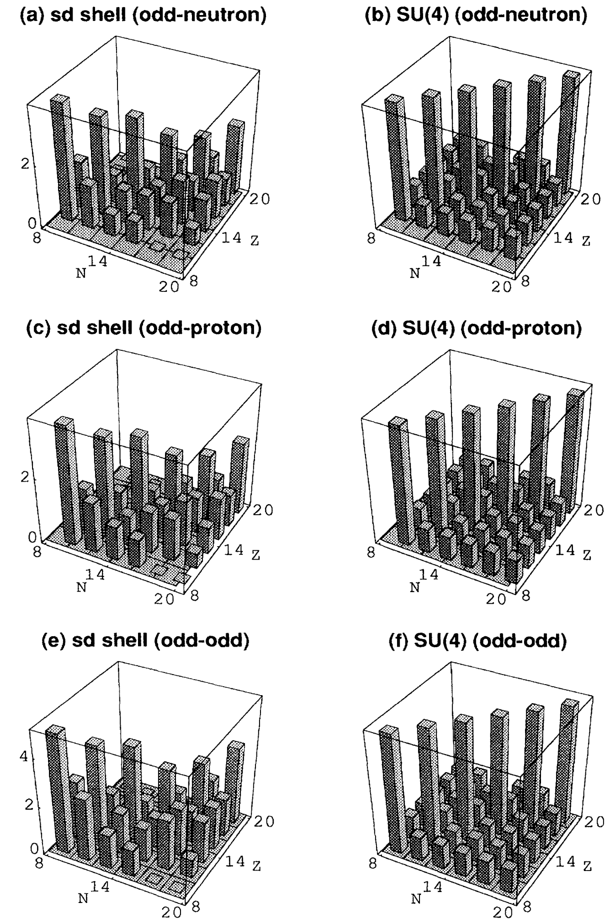

FIG. 2. Same as Fig. 1 but now for observed double binding energy differences in (a) odd-neutron, (c) odd-proton, and (e) odd-odd *sd*-shell nuclei and the corresponding quantity in Wigner SU(4) in (b), (d), and (f), respectively.

TABLE I. Observed binding energy differences $|\delta V_{np}|$ (in keV) in p-shell nuclei.

	$N=3$	$N=4$	$N=5$	$N=6$	$N=7$	$N=8$
$Z=3$	6554[a]	5970[a]	2479	1979	730	···
$Z=4$	5783[a]	7151[a]	1055	1585	827	···
$Z=5$	2342	1010	6771[a]	5706[a]	2867	2288
$Z=6$	1788	1491	5727[a]	5841[a]	2221	2362
$Z=7$	···	···	2574	1958	5607[a]	4132[a]
$Z=8$	···	···	1952	2090	4138[a]	3941[a]

[a]Enhanced in SU(4).

and is shown for *sd*-shell nuclei in Fig. 2(e). The favored SU(4) representation for odd-odd nuclei is (010) if $N = Z$ and $(1, T-1, 1)$ otherwise. The same result emerges as for the even-even case, namely, $\delta V_{np}(N,Z)/b = -10(-2)$ for $N = Z (N \neq Z)$.

In summary, in even-even and odd-odd nuclei enhancements are found for $N = Z$ while in odd-neutron (odd-proton) nuclei $|\delta V_{np}|$ are enhanced for $N = Z - 1$ ($N = Z + 1$). The observed double binding energy differences in p-shell nuclei are qualitatively consistent with the simple SU(4) prediction, as can be seen from Table I. Moreover, the magnitude of the extracted interaction strengths for even-even, even-odd, and odd-odd nuclei are very similar. These features can be understood intuitively if the function of the double differences is considered more closely. The first term in (1) yields the interaction of the last two neutrons with themselves, with the remaining neutrons and with Z protons; subtraction of the second term then leaves the interaction of the last two neutrons with the last two protons, and the factor of $\frac{1}{4}$ gives the average interaction between a single neutron and proton. In (5), the same reasoning leads to the average np interaction obtained from the last (odd) proton and the last two neutrons. Thus for $N = Z + 1$, the δV_{np} extracted should be the same as that for $N = Z$ in the even-even case. For $N = Z - 1$, the corresponding even-even nucleus would have $N = Z - 2$. Similarly, the interaction strength obtained for the odd-odd nucleus with $N = Z$ should correspond to that from the even-even nucleus with $N + 1 = Z + 1$. Thus the δV_{np} values should appear in quartets consisting of the even-even nucleus (N, Z), the odd-mass nuclei $(N, Z-1)$ and $(N-1, Z)$, and the odd-odd nucleus $(N-1, Z-1)$. This quartet structure is evident in the enhanced empirical values for the p-shell nuclei shown in Table I.

It is of interest to consider the relationship between the original interpretation of δV_{np} as a measure of the np interaction between the "last" nucleons and the observation of enhancements of this quantity at the $N \simeq Z$ line. In this regard it is important to remember that δV_{np} is an *average np* interaction over the last few nucleons and that this average reflects the many-body nature of the problem through the configuration mixing induced by the residual interaction. One is then led to conclude that this average is drastically different in $N \simeq Z$ nuclei. For even-even nuclei at least, this is consistent with generalized Hartree-Fock-Bogoliubov calculations incorporating both $T = 0$ and $T = 1$ pairing. Studies using this formalism [14,15] show that for $N = Z$ nuclei $T = 0$ pairing is dominant and determines the nuclear many-body ground state while for $N \neq Z$ nuclei $T = 1$ pairing is dominant, with the possible exception of $N = Z \pm 2$ nuclei where both pairing modes are competitive. This finding in even-even nuclei (which can be understood intuitively in terms of simple counting arguments [15]) agrees qualitatively with the present results obtained from SU(4). This result is also supported by shell-model calculations, since in the latter the δV_{np} enhancements disappear completely if the $T = 0$ component of the two-nucleon interaction is not consiered [10]. The interpretation involving SU(4) employs Young tableaux which describe all valence nucleons and hence suggests a many-body effect. It also provides a crucial generalization of the phenomenon to odd-mass and odd-odd systems.

Figures 1 and 2 show that substantial deviations from the simple SU(4) predictions occur for the *sd* shell. This is a reflection of the spin-orbit interaction which increases in importance with increasing mass and which destroys SU(4) symmetry, since it favors one spin direction over the other. The double energy differences in the even-even, odd mass as well as odd-odd nuclei show a systematic behavior at the $N \simeq Z$ line: a rapid decrease towards the middle and an increase near the end of the *sd* shell. While the former could be explained (at least partially) as resulting from the decrease of the overall interaction strength (i.e., of the coefficient b) with increasing mass number, it would be difficult to understand the latter, unless it is associated with a restoration of SU(4) symmetry. The results of Vogel and Ormand [5] for $N = Z$ (or $T = 0$), obtained by taking the overlap between the shell-model wave function and the favored SU(4) representation, are in qualitative agreement with what we find here from binding energy systematics in that the overlaps first decrease with mass but increase towards the end of the *sd* shell.

The sensitivity of these predictions to a small degree of symmetry breaking can be analyzed by considering admixtures of the next-favored SU(4) representation into the ground state. We illustrate this with the example of even-even nuclei, which have as next-favored SU(4) representation (101) if $N = Z$ and $(2, T - 1, 0)$ otherwise. [An exception to this rule occurs in doubly closed shell nuclei where the SU(4) representation is unique.] Proceeding as before one finds (for double binding energy differences not involving doubly closed shell nuclei)

$$\delta V_{np}(N,Z)/b = \begin{cases} -10 + 8\alpha^2, & N = Z, \\ -2 - 4\alpha^2, & N = Z \pm 2, \\ -2, & N \neq Z, Z \pm 2, \end{cases} \tag{8}$$

where α^2 is the weight of the next-favored SU(4) representation. Away from the $N = Z$ line, δV_{np} is completely insensitive to the admixture of higher SU(4) representations. For $N \simeq Z$, however, the changes are dramatic. For a 30% admixture ($\alpha^2 = 0.3$, which is a typical value obtained in realistic shell-model calculations [5]), $|\delta V_{np}(N = Z)|$ is reduced from $10b$ to $7.6b$, while $|\delta V_{np}(N = Z \pm 2)|$ increases from $2b$ to $3.2b$. Although very schematic, this result does show that double binding energy differences at the $N \simeq Z$ line are sensitive to SU(4) symmetry breaking.

As the mass of the nucleus increases, the SU(4) symmetry is increasingly broken. Along the $N = Z$ line, this is a result of two *conspiring* effects: the spin-orbit term in the nuclear mean-field potential and the Coulomb interaction, both increasing in importance with mass. So it would seem that the $N \simeq Z$ enhancements in $|\delta V_{np}|$ will gradually disappear in the heavier nuclei. However, this is not necessarily the case for the nuclei beyond ^{56}Ni where the masses which determine δV_{np} are currently lacking. Although they have strongly admixed SU(4) representations, these nuclei might exhibit a *pseudo*-SU(4) symmetry. The latter symmetry arises by treating the *pf* shell as pseudo-*sd* in the spirit of suggestions made by Arima, Harvey, and Shimizu [16] and Hecht and Adler [17] (see also [18], and references therein). The quality of the pseudo-SU(4) symmetry in the *pf* shell will depend on two *conflicting* effects: a pseudo-spin-orbit splitting which is greatly reduced and a Coulomb interaction which continues to increase with mass. A theoretical study incorporating both effects should determine whether the pseudo-SU(4) symmetry has any chance of surviving in heavier nuclei. Experimentally, the pseudo-SU(4) symmetry cannot be tested with β-decay selection rules, since the Gamow-Teller operator is not a generator of pseudo-SU(4). The measurement of masses and the determination of double binding energy differences along the $N \simeq Z$ line, on the other hand, should provide a sensitive test for the existence of a pseudo-SU(4) symmetry in nuclei.

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