Crossover from Non-Fermi-Liquid to Fermi-Liquid Behavior in the Two Channel Kondo Model with Channel Anisotropy

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We present a solution of the two channel Kondo model with channel anisotropic exchange couplings. The solution provides an analytic description of the crossover from a high temperature regime, dominated by the non-Fermi-liquid behavior of the isotropic two channel Kondo model, to a low temperature regime characterized by the standard Fermi-liquid behavior of the single channel Kondo model.

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Among the known realizations of non-Fermi-liquid behavior which are currently the subject of many theoretical and experimental investigations, an important role is being played by a class of generalized Anderson impurity models. Their common feature is a non-Fermi-liquid regime due to a degeneracy of the impurity lowest energy levels which cannot be completely lifted by the coupling to conduction electrons. This leads to various anomalous properties, for instance, a nonanalytic temperature and external field dependence of the impurity contribution to the free energy. Such a behavior is evidently unstable to any perturbation which eliminates the residual degeneracy. In this case the low temperature regime should show standard Fermi-liquid properties. Nevertheless, if the magnitude of the energy scale associated with such a perturbation is sufficiently small, a slow crossover from the non-Fermi- to the Fermi-liquid regime is expected to occur as the temperature is lowered.

In this Letter we give a detailed analytic description of such a crossover in the prototype of these non-Fermi-liquid impurity models, the two channel Kondo model.

The general multichannel Kondo model has been worked out to describe the low temperature behavior of a magnetic impurity with orbital structure embedded in a metal (see Ref. [1], and references therein). Besides dilute magnetic alloys, many other physical realizations of the multichannel Kondo model have been proposed including two level systems in metals [2], heavy fermion compounds [3], and high T_c superconductors [4].

The model is described by the Hamiltonian

$$\sum_{a=1}^{N}\left[H_0(\psi_{a\sigma}, \psi_{a\sigma}^\dagger) + \sum_{i=x,y,z} J_i S^i \psi_{a\alpha}^\dagger(\mathbf{0}) \sigma_{\alpha\beta}^i \psi_{a\beta}(\mathbf{0}) \right], \quad (1)$$

which represents an N-times degenerate conduction band with kinetic energy

$$H_0(\psi, \psi^\dagger) = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger \psi_{\mathbf{k}}\,, \quad (2)$$

coupled, for simplicity at the origin, to a magnetic impurity spin $\vec{S}$ via an antiferromagnetic coupling J_i. In what follows we consider a spin-1/2 impurity.

A perturbative approach supplemented by a renormalization group (RG) analysis shows that the conduction electrons tend to screen the magnetic impurity, since the system does not like to sustain the impurity spin degeneracy. This tendency shows up through the exchange flowing towards strong coupling under RG process. The complete screening is, however, possible only for the single channel ($N = 1$) model, where the single conduction electron can indeed form a singlet with the impurity, thus freezing the impurity spin degree of freedom. This corresponds, from the RG point of view, to the case in which the model flows to the infinite exchange fixed point. The behavior of the system around this point is well understood [5,6]. The impurity contribution to the magnetic susceptibility χ_i and to the ratio of the specific heat to the temperature $C_{V,i}/T$ are both finite at zero temperature, as in a usual Fermi-liquid.

On the contrary, two (and more) channels are unable to lift the degeneracy (they tend to overscreen the impurity spin). The effective exchange flows in this case towards an intermediate coupling fixed point. Its existence has been proven and, in the case of large number of channels, analyzed in Ref. [1]. The proof relies on the fact that both the zero and the infinite exchange fixed points are unstable for more than one channel, hence a stable fixed point at intermediate exchange and non-Fermi-liquid behavior. The properties of the model around this fixed point for any number of channels has been analyzed by various techniques [7]. For $N = 2$, which is the case we are interested in, it was found (see also Refs. [8,9]) that $\chi_i(T) \sim C_{V,i}(T)/T \sim \ln(1/T)$ at low temperature, and that the system has a residual entropy $\ln(2)/2$, as if half of the impurity spin degrees of freedom are decoupled from the conduction electrons.

It is important to notice that this behavior occurs only if there is no criteria for the system to prefer one of the channels or one of their linear combinations [1]. In the opposite case, the system will always choose the channel with the strongest exchange to screen the impurity, and the usual Fermi liquid behavior of the single channel

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model will finally take place at zero temperature. In order to study how this happens in detail let us introduce a channel anisotropy by adding to the Hamiltonian (1) for $N = 2$ the term

$$\lambda_i J_i S^i \Big[\psi^\dagger_{1\alpha}(\mathbf{0})\sigma^i_{\alpha\beta}\psi_{1\beta}(\mathbf{0}) - \psi^\dagger_{2\alpha}(\mathbf{0})\sigma^i_{\alpha\beta}\psi_{2\beta}(\mathbf{0})\Big], \tag{3}$$

with the dimensionless parameters $0 \le \lambda_i \le 1$ controlling the strength of the anisotropy: $\lambda_i = 0$ corresponds to the symmetric two channel Kondo model, $\lambda_i = 1$ to the single channel model.

The RG flow diagram in the (J, λ) space is sketched in Fig. 1 showing that any finite λ flows to unity and, therefore, the system asymptotically has single channel properties. The smaller the λ, the slower is the crossover from two to single channel behavior. Apart from these scaling arguments, not much is known on the channel anisotropic Kondo model.

Recently, Emery and Kivelson (EK) [8] realized that for a particular value of J_z, but arbitrary $J_x = J_y = J_\perp$, the symmetric two channel Kondo model maps onto a resonant level model. We will show that the channel anisotropy (3) can be accounted for by introducing another term in the effective resonant level model, which nevertheless remains solvable. We use the approach originally developed by Yuval and Anderson [10] for the single channel case. This method is similar to the bosonization technique of EK, but it is more rigorous not relying on a linearized electron spectrum.

Let us start from the simple case of $\lambda_z = 0$ but $\lambda_\perp \neq 0$, and, as a first step, diagonalize the Hamiltonian (1) keeping only the z component of the exchange J_z. The remaining transverse exchange is then treated by perturbation theory. For instance, at second order in the transverse exchange one has to calculate expressions of the kind

$$\langle\uparrow|\, S^+(t)\sigma^-_a(t)S^-(0)\sigma^+_a(0)\,|\uparrow\rangle,$$

where $|\uparrow\rangle$ is the ground state of the conduction electrons with the impurity spin being up, and $\sigma^i_a = \psi^\dagger_{a\alpha}(0)\sigma^i_{\alpha\beta}\psi_{a\beta}(0)$ $(a = 1, 2)$. Yuval and Anderson showed that such correlation functions could be calculated making use of the solution of the x-ray edge singularity [12]. Following them, we find the contribution $\sim J_\perp^2$:

$$\langle\uparrow|\, S^+(t)\sigma^-_a(t)S^-(0)\sigma^+_a(0)\,|\uparrow\rangle \sim \frac{1}{t^\eta}, \tag{4}$$

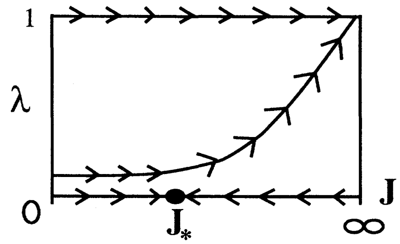

FIG. 1. Qualitative (J, λ) flow diagram for the channel anisotropic two channel Kondo model: $\lambda = 0$ is the channel isotropic case, $\lambda = 1$ corresponds to the single channel model.

with

$$\eta = 2\left(1 - 2\frac{\delta}{\pi}\right)^2 + 8(N - 1)\left(\frac{\delta}{\pi}\right)^2, \tag{5}$$

where the phase shift $\delta = \tan^{-1}(\pi\nu_0 J_z/4)$. ν_0 is the one spin, one channel density of states of the conduction electrons at the Fermi energy, and N the number of channels. For $N \le 2$ there exists a value of J_z such that the dimension of the spin flip operator $\eta = 1$. In particular, for $N = 1$ this value corresponds to the phase shift $\delta = \pi(1 - 1/\sqrt{2})/2$, while, for $N = 2$, $\delta = \pi/4$. For $N > 2$ there is no J_z such that $\eta = 1$, therefore the method does not work so simply.

On the other hand, let us consider a single band of conduction electrons Ψ coupled to an impurity d via single particle operators of the form $h[\Psi^\dagger(\mathbf{0})d + \text{H.c.}]$ and $\Delta[\Psi^\dagger(\mathbf{0})d^\dagger + \text{H.c.}]$. The impurity energy level is located at the Fermi energy. A perturbation expansion in powers of these single particle operators generates terms like

$$\langle 1|d^\dagger(t)\Psi(t)\Psi^\dagger(t)d|1\rangle \sim \frac{1}{t},$$

where $|1\rangle$ is the ground state with the impurity level occupied. This correlation function behaves as (4) for $\eta = 1$.

The nontrivial step is to show that the two perturbation expansions, for the Kondo and the resonant level model, coincide not only in the second order, as we have just shown, but in any order of perturbation expansion (provided the parameters h and Δ are appropriately chosen).

For $N = 1$ and $\delta = \pi(1 - 1/\sqrt{2})/2$ the resonant level model corresponds to the well-known Toulouse limit [13] (see Ref. [5])

$$H = H_0(\Psi, \Psi^\dagger) + \frac{J_\perp}{2}\cos^2(\delta)\sqrt{\nu_0\xi_0}\,[\Psi^\dagger(\mathbf{0})d + d^\dagger\Psi(\mathbf{0})], \tag{6}$$

while for $N = 2$, $\delta = \pi/4$ and in the presence of channel anisotropic transverse exchange the equivalent resonant level model reads [11]

$$H = H_0(\Psi, \Psi^\dagger) + \frac{J_\perp}{2}\cos^2(\delta)\sqrt{\nu_0\xi_0}\,[\Psi^\dagger(\mathbf{0}) + \Psi(\mathbf{0})](d - d^\dagger) + \lambda_\perp\frac{J_\perp}{2}\cos^2(\delta)\sqrt{\nu_0\xi_0}\,[\Psi^\dagger(\mathbf{0}) - \Psi(\mathbf{0})](d + d^\dagger), \tag{7}$$

where ξ_0 is a high-energy cutoff related to the bandwidth. In the channel symmetric case $(\lambda_\perp = 0)$, the Hamiltonian (7) reduces to that discussed by EK [8].

The mapping can still be performed in the general case $\eta \neq 1$ and $\lambda_z \neq 0$. First we define the scattering phase shifts δ and δ' by

$$\tan(\delta \pm \delta') = \frac{\pi \nu_0 J_z}{4}(1 \pm \lambda_z), \tag{8}$$

so that finite λ_z implies finite δ'. In order to account for a $\delta' \neq 0$, one has to add to (7) the interaction term

$$V\left(d^\dagger d - \frac{1}{2}\right)\Psi^\dagger(\mathbf{0})\Psi(\mathbf{0}), \tag{9}$$

where $V = 2/(\pi\nu_0)\tan(2\delta')$. The deviations from $\delta = \pi/4$ can be accounted for by adding to (7) another interaction term

$$\tilde{V}\left(d^\dagger d - \frac{1}{2}\right)\Psi_s^\dagger(\mathbf{0})\Psi_s(\mathbf{0}), \tag{10}$$

where

$$\tilde{V} = \frac{2}{\pi\nu_0}\cot(2\delta), \tag{11}$$

and Ψ_s is a new free Fermi field with a kinetic Hamiltonian $H_0(\Psi_s, \Psi_s^\dagger)$ [see Eq. (2)] [14]. Above we have outlined the mapping of the two channel Kondo model (1)–(3) to the interacting resonant level model (7)–(11); the detailed derivation of this mapping, as well as the generalization of the Yuval-Anderson approach to the case of arbitrary N, is described in Ref. [11].

We would like to point out an interesting outcome of the Yuval-Anderson method applied to the two channel Kondo model which is out of reach of bosonization techniques. It is easy to see that, in the channel isotropic case, the model is symmetric under $\delta \to \pi/2 - \delta$ [this is clear from the definition of η, Eq. (5)]. This extends the result of Ref. [1] that the two channel Kondo model behaves similarly around $J_z = 0$ (i.e., $\delta = 0$) and around $J_z = \infty$ (i.e., $\delta = \pi/2$). By symmetry the fixed point should be exactly at $\delta = \pi/4$.

We now analyze the model for $\lambda_z = 0$ and $\delta = \pi/4$. (One can show that a small $\lambda_z \neq 0$ does not modify essentially the solution; it only makes the crossover faster.) In the case originally considered by EK ($\lambda_i = 0$), the combination $d + d^\dagger$ was decoupled from the conduction electrons [see Eq. (7)]; this caused the residual entropy. The anisotropy $\lambda_\perp \neq 0$ provides a coupling between this combination and the conduction electrons. In the Nambu representation $D^\dagger = (d^\dagger, d)$, the impurity Green function

$$\hat{G}_d(t) = -i\langle T[D(t)D^\dagger(0)]\rangle$$

is a 2×2 matrix. Its Fourier transform can easily be evaluated. For ω much smaller than the bandwidth, we find

$$\hat{G}_d(\omega) = \frac{1}{2}\,\frac{\hat{\tau}_0 - \hat{\tau}_x}{\omega + i\Gamma \operatorname{sgn}\omega} + \frac{1}{2}\,\frac{\hat{\tau}_0 + \hat{\tau}_x}{\omega + i\lambda_\perp^2\Gamma\operatorname{sgn}\omega}, \tag{12}$$

where the resonance width $\Gamma = \pi\nu_0^2\xi_0 J_\perp^2\cos^4(\delta)$, $\hat{\tau}_i$ being the Pauli matrices, and $\hat{\tau}_0$ the unit matrix. The impurity spectral function is

$$\hat{A}(\omega) = \frac{1}{2}(\hat{\tau}_0 - \hat{\tau}_x)\frac{\Gamma}{\omega^2 + \Gamma^2} + \frac{1}{2}(\hat{\tau}_0 + \hat{\tau}_x) \times \frac{\lambda_\perp^2\Gamma}{\omega^2 + \lambda_\perp^4\Gamma^2},$$

and it is therefore equally shared by two Lorentzians with different widths Γ and $\lambda_\perp^2\Gamma$. In the channel isotropic case $\lambda_\perp \to 0$, one of the two Lorentzians tends to $\delta(\omega)$, representing the impurity degree of freedom which is decoupled from the conduction band in this particular limit [8].

The impurity contribution to the free energy can be calculated in a standard way by integration over the coupling constant. The result is

$$F = F_0 + \int \frac{d\omega}{2\pi} f(\omega)\left[\tan^{-1}\left(\frac{\Gamma}{\omega}\right) + \tan^{-1}\left(\frac{\lambda_\perp^2\Gamma}{\omega}\right)\right],$$

where F_0 is the free energy in absence of coupling between the impurity and conduction electrons, $f(\omega)$ is the Fermi distribution function and the integral should be limited to the conduction bandwidth. The entropy can be calculated by $S(T) = -\partial F/\partial T$. By defining

$$S_\gamma(T) = \frac{\gamma}{2\pi T}\left[\psi\left(\frac{1}{2} + \frac{\gamma}{2\pi T}\right) - 1\right] - \ln\left[\frac{\Gamma(\frac{1}{2} + \gamma/2\pi T)}{\sqrt{\pi}}\right],$$

where $\psi(z)$ is the psi function and $\Gamma(z)$ the gamma function, the entropy turns out to be $S(T) = \ln(2) + S_\Gamma(T) + S_{\lambda_\perp^2\Gamma}(T)$; see Fig. 2. We see that $S(0) = 0$, as expected since no degeneracy is left at $T = 0$, but there is a region of temperatures (the wider the smaller $\lambda_\perp$ is) where the entropy is close to that of the symmetric two channel model.

Another quantity of physical interest is the longitudinal impurity susceptibility. It has been shown that, exactly on the EK line, $\chi_i^{zz} = 0$ and one has to consider deviations from this line in order to account for a finite impurity susceptibility [9]. In our case, this amounts to adding to the Hamiltonian (7) a term [11]

$$\delta H = \mu_{\rm eff} h_z\left(d^\dagger d - \frac{1}{2}\right),$$

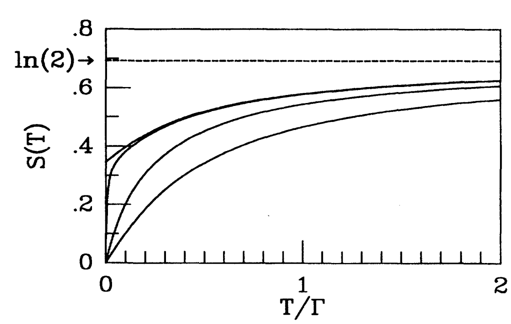

FIG. 2. Entropy $S(T)$ for various values of the anisotropy $\lambda_\perp$: from the top $\lambda = 0, 0.1, 0.5, 1$.

where h_z is the infinitesimal longitudinal magnetic field and $\mu_{\rm eff} = 4\delta/\pi - 1$. For the impurity susceptibility we find

$$\chi_i^{zz} = \frac{\mu_{\rm eff}^2}{\pi\Gamma(1-\lambda_\perp^2)} \times \left[\psi\left(\frac{1}{2} + \frac{\Gamma}{2\pi T}\right) - \psi\left(\frac{1}{2} + \frac{\lambda_\perp^2 \Gamma}{2\pi T}\right)\right].$$

In the case $\lambda_\perp \ll 1$ the susceptibility shows the same kind of crossover behavior as the entropy:

$$\chi_i^{zz} \propto \begin{cases} \frac{2}{\pi\Gamma}\frac{1}{1-\lambda_\perp^2}\ln\left(\frac{1}{\lambda_\perp}\right), & T \ll \lambda_\perp^2\Gamma, \\ \frac{1}{\pi\Gamma}\frac{1}{1-\lambda_\perp^2}\ln\left(\frac{\Gamma}{T}\right), & \lambda_\perp^2\Gamma \ll T \ll \Gamma, \\ \frac{1}{4T}, & T \gg \Gamma. \end{cases}$$

As expected the magnetic susceptibility saturates at low temperature, although at intermediate temperatures it shows the logarithmic behavior of the two channel Kondo model.

To our knowledge, the two channel Kondo model is most convincingly realized by two level systems in metal alloys [2]. This has recently been experimentally confirmed thanks to the development of the point contact spectroscopy [15]. In these systems, the role of the spin is played by some orbital degree of freedom, while the physical spin plays the role of the channel index. Thus the model is by construction channel isotropic. However, an external magnetic field breaks the channel symmetry and generates an effective λ proportional to the curvature of the conduction electron band times the magnetic field B. Consequently, B causes the crossover to a Fermi-liquid behavior at low temperature as observed in Ref. [15]. As to the physical magnetic susceptibility, it is the second derivative of the free energy with respect to λ, and can be shown to be $\chi \propto \ln[\mathrm{Max}(B^2, T)]$.

In conclusion, we have presented the solution of the two channel Kondo model in the presence of a channel anisotropic transverse exchange for a particular value of the longitudinal exchange. The most interesting feature of this model is that for sufficiently small anisotropy it shows a slow crossover from the non-Fermi-liquid behavior, characteristic for the isotropic two channel model, to the Fermi liquid behavior of the single channel case. The main goal of our solution is to give an analytic description of this crossover [16].

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[1] P. Nozières and A. Blandin, J. Phys. (Paris) **41**, 193 (1980).
[2] K. Vladár, G. Zimányi, and A. Zawadowski, Phys. Rev. Lett. **56**, 286 (1986Phys. Rev. Lett. **56**, 286 (1986); A. Muramatsu and F. Guinea, *ibid.* **57**, 2337 (1986).
[3] D.L. Cox, Phys. Rev. Lett. **59**, 1240 (1987Phys. Rev. Lett. **59**, 1240 (1987); C.L. Seaman *et al., ibid.* **67**,2882 (1991); B. Andraka and A.M. Tsvelik, *ibid.* **67**, 2886 (1991).
[4] D.L. Cox *et al.,* Phys. Rev. Lett. **62**, 2188 (1989Phys. Rev. Lett. **62**, 2188 (1989Phys. Rev. Lett. **62**, 2188 (1989); T. Giamarchi *et al., ibid.* **70**, 3967 (1993); V.J. Emery and S. Kivelson, *ibid.* **71**, 3701 (1993).
[5] P.W. Anderson, G. Yuval, and D.R. Hamann, Phys. Rev. B **1**, 4664 (1970).
[6] K.G. Wilson, Rev. Mod. Phys. **47**, 773 (1975); P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).
[7] N. Andrei and C. Destri, Phys. Rev. Lett. **52**, 364 (1984); A.M. Tsvelik and P.B. Wiegmann, Z. Phys. B **54**, 201 (1984); I. Affleck and A.W.W. Ludwig, Nucl. Phys. **B360**, 641 (1991).
[8] V.J. Emery and S. Kivelson, Phys. Rev. B **47**, 10812 (1992).
[9] D.G. Clarke, T. Giamarchi, and B.I. Shraiman, Phys. Rev. B **48**, 7070 (1993Phys. Rev. B **48**, 7070 (1993); A.M. Sengupta and A. Georges, *ibid.* **49**, 10020 (1994).
[10] G. Yuval and P.W. Anderson, Phys. Rev. B **1**, 1522 (1970).
[11] M. Fabrizio, A.O. Gogolin, and Ph. Nozières (to be published).
[12] P. Nozières and C. De Dominicis, Phys. Rev. **178**, 1097 (1969).
[13] G. Toulouse, Phys. Rev. B **2**, 270 (1970).
[14] From Eq. (8) we see that if $\lambda_z \to 1$ the solvable limit $\delta = \pi/4$ is moved towards $J_z \to \infty$.
[15] D.C. Ralph and R.A. Buhrman, Phys. Rev. Lett. **69**, 2118 (1992Phys. Rev. Lett. **69**, 2118 (1992); D.C. Ralph, A.W.W. Ludwig, Jan von Delft, and R.A. Buhrman, *ibid.* **72**, 1064 (1994).
[16] Recently we became aware that this model was independently solved by Bethe-ansatz methods with qualitatively similar results [N. Andrei and A. Jeres, following letter, Phys. Rev. Lett. **74**, 4507].