Conductivity between Luttinger Liquids in the Confinement Regime and c-Axis Conductivity in the Cuprate Superconductors

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We calculate the interliquid conductivity for Luttinger liquids, within the so-called "confinement" regime where interliquid hopping is completely incoherent. We argue that the interliquid conductivity behaves as $\sigma_\perp(\omega) \sim \omega^{4\alpha}$, where α is the Luttinger liquid exponent. We discuss the effect of finite temperature, which is found to introduce a coherent weight into the conductivity. These results are in good agreement with experimental measurements of the frequency dependent normal state c-axis conductivity in the high-temperature superconductors $YBa_2Cu_3O_{7-\delta}$ and $La_{2-x}Sr_xCuO_4$.

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One of the more important yet less frequently discussed anomalies observed in the normal state of the cuprate-based high-temperature superconductors (HTSC's) is the c-axis conductivity $\sigma_c(\omega, T)$. By comparison, a great deal more attention has been given to understanding the anomalous behavior of the in-plane conductivity $\sigma_{ab}(\omega, T) \sim (\omega^{-1}, T^{-1})$. It is a remarkable fact, given that the structures and chemistry of HTSC's can vary a great deal outside of the common CuO_2 planes, that $\sigma_{ab}(\omega, T)$ is an almost universal function over the HTSC family. In striking contrast to this, there is no single expression for $\sigma_c(\omega, T)$ with wide applicability. Various empirical fits for the interplane dc resistivity, $\rho_c(T)$, involving combinations of functions proportional to 1, T, T^{-1}, T^{-p}, and $e^{\Delta/T}$ have been made, the fit differing from material to material. It might be argued that this is not to be entirely unexpected since ρ_c probes motion of electrons in the interplane direction, the nature of which, as stated above, varies considerably over the HTSC family. The challenge to such an attitude, however, is to demonstrate how the interplanar chemistry and/or structure can be such as to lead to c-axis conductivities well below the minimum metallic conductivity given by the Mott-Ioffe-Regel limit [1] and, more alarmingly, a *positive* derivative in temperature, $d\sigma_c/dT > 0$, observed in many HTSC's over a wide temperature range above T_c. Were it not for the intervention of superconductivity, one would have a situation in which $\sigma_{ab}(T)/\sigma_c(T)$ would increase, apparently without bound, as $T \to 0$. Within a Fermi-liquid picture, anisotropy alone cannot account for such a situation. However, the in-plane properties of HTSC's are not those of a Fermi liquid. One is thus led to the almost inescapable conclusion that the unusual behavior of σ_c is intimately connected to the non-Fermi-liquid properties of the 2D electron fluid in the cuprate planes. This is the starting point of the "confinement" hypothesis advanced several years ago [2–4].

Considerably more information about c-axis transport can be obtained by going beyond the measurement of dc conductivities. Recently, experimental determinations of the frequency dependence of the c-axis conductivity on the eV scale have been made in $YBa_2Cu_3O_{7-\delta}$ [5] and $La_{2-x}Sr_xCuO_4$ [6]. These experiments demonstrate unambiguously the absence of any Drude-like term in the electronic contribution to $\sigma_c(\omega)$ [7]. There is an enormous loss of spectral weight at low frequencies. In the samples studied, $\sigma_c(\omega)$ is a very slowly *increasing* function of ω over a wide frequency range. Again, most samples exhibit a dc conductivity below the Mott limit, when the data are extrapolated back to $\omega = 0$.

The absence of a Drude-like term in $\sigma_c(\omega)$ vindicates the confinement hypothesis of the normal state of HTSC's: single particle hopping in the interplane direction is completely incoherent. There is no band formation in the c direction, and as a consequence there is the potential for interplanar pair tunneling to give rise to a large kinetic energy gain and drive a mechanism for a high T_c [2,4,8].

The challenge to any candidate theory of the normal state of HTSC's is to explain how confinement can come about. In a recent paper [9], we have presented a strong argument for the existence of a coherent-incoherent (or "confinement-deconfinement") transition for single particle hopping between spin-charge separated Luttinger liquids. In this Letter we shall further this work to calculate the interliquid conductivity in the confinement regime. The connection to the HTSC's is made via the hypothesis that the electronic low energy physics of the cuprate planes is described by a 2D spin-charge separated tomographic Luttinger liquid [4]. In order to be able to perform definitive calculations we shall here restrict attention to the analogous problem in one dimension where the electron Green's functions are rigorously known.

In this Letter then we shall consider the problem of determining the interchain conductivity for a system of weakly coupled Hubbard chains. The low energy physics of a 1D Hubbard model, with parameters ($t_\parallel$, U) as usually defined, is that of a spin-charge separated Luttinger liquid. Denoting the Hamiltonian for the Luttinger liquid on

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chain i by $H_{\mathrm{LL}}^{(i)}$ we have

$$H = \sum_i H_{\mathrm{LL}}^{(i)} + t_\perp \sum_{i,x} \{c_{i\sigma}^\dagger(x) c_{i+1\sigma}(x) + \mathrm{H.c.}\}. \quad (1)$$

We have previously argued [9] that in a system governed by (1), and for sufficiently small $t_\perp$, electron motion in the interchain direction is completely *incoherent*. As such, the Fermi surface retains a one-dimensional form and there is no coherent interchain electron velocity. The underlying reason for this is that the interchain hopping term in (1) hops *real* electrons, however, the exact eigenstates of H_{LL} are *not* electronlike. Instead of exhibiting a sharp delta-function peak, the electron spectral function $\rho(k,\omega)$ of (1) has power law singularities at $\omega = \pm v_c k, v_s k$, a result of spin-charge separation and the infrared orthogonality catastrophe induced upon insertion of a single electron into the liquid [10]. As a result, in contrast to a noninteracting electron gas, in which all interchain hops are elastic, electron hopping between Luttinger liquids is a mixture of elastic and inelastic processes, the latter involving virtual transitions over an energy range $\Delta E \sim 1/t$. Of course, even for the case of single electron hopping between Fermi liquids there will be inelastic processes due to the incoherent part of the electron spectral function. However, in a Fermi liquid the coherent "quasiparticle" part of the electron Green's function $G(k,t)$ has a lifetime $\sim (k - k_F)^2$. As shown, e.g., in [11], in this case the quasiparticle part long outlives the incoherent contribution to $G(k,t)$ as $k - k_F \to 0$. Coherent interchain hopping results. In marked contrast to this, for coupled Luttinger liquids, and sufficiently small $t_\perp$, spin-charge separation implies a width of the electron spectral function $\sim (k - k_F) \gg (k - k_F)^2$, and this, in combination with interference between inelastic and elastic processes due to the orthogonality catastrophe, leads to the destruction of any signal of coherent interchain hopping [9]. Coupled with the Luttinger liquid hypothesis for HTSC's, this offers a natural explanation for the anisotropic transport observed in those materials, and significantly elucidates the proposal of "confinement" originally made on the basis of such transport data. We believe that such a regime will be ubiquitous to *any* sufficiently anisotropic, strongly correlated system, not just the cuprate superconductors. Indeed, we have argued elsewhere [12] that the anomalously rapid 3D to 2D crossover observed in the magnetoresistance of the (highly anisotropic) organic superconductor $(TMTSF)_2PF_6$, as a function of applied magnetic field, is a manifestation of the coherence-incoherence transition discussed in [9].

In the light of this result, we proceed to calculate the interliquid conductivity in the confinement regime. As in [9] we introduce the probability $P(t)$ that at time $t > 0$ the system is in the $t < 0$ ground state,

$$P(t) = \left| \left\langle \prod_i O_i | e^{-iHt} | \prod_i O_i \right\rangle \right|^2, \quad (2)$$

where $|O_i\rangle$ is the Luttinger liquid ground state of $H_{\mathrm{LL}}^{(i)}$, $t_\perp = 0$ for $t < 0$; hence the $t < 0$ ground state is $|\prod_i O_i\rangle$. It is simple to show that, to $O(t_\perp^2)$, the effective rate of hopping out of a given chain is given by

$$-\frac{dP(t)}{dt} \equiv \Gamma(t) = 4t_\perp^2 L \mathrm{Re} \int_0^t dt' \left\{ \int dx\, G_e(x, t - t') G_h(x, t - t') \right\}, \quad (3)$$

where $G_e(x,t) \equiv \langle c(x,t) c^\dagger(0,0)\rangle$ and $G_h(x,t) \equiv \langle c^\dagger(x,t) c(0,0)\rangle$. Equivalently,

$$\Gamma(t) = t_\perp^2 L \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin \omega t}{\omega} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \rho(k,\omega') \rho(k, \omega' + \omega) \Theta(-\omega') \{1 - \Theta(-(\omega' + \omega))\}, \quad (4)$$

where $\rho(k,\omega)$ is the Luttinger liquid spectral function.

$P(t)$ and consequently $\Gamma(t)$ may readily be generalized to finite temperature via the usual Boltzmann weighting, the Θ functions in (4) being replaced by Fermi functions. The advantage of (4) over the space-time formalism of (3) is that it gives a more physical expression in terms of the electron spectral function and is simpler to use at finite temperature.

Let the chains be stacked a distance d apart along the z axis. We wish to calculate the interchain current induced by an electric field E parallel to $\hat{z}$. For $E = 0$ there is of course no net current, Eq. (4) describing incoherent motion of equal magnitudes into and out of a given chain. To determine the current in nonzero field we first observe that in the confinement regime interchain motion will be completely *diffusive*. We may write a constitutive equation for the interchain diffusion current J_D generated by a particle density gradient

$$J_D(z,t) = -\int_0^t dt'\, D(t - t') \frac{\partial}{\partial z} n(z,t'). \quad (5)$$

Here $n(z,t)$ is the particle density on the chain at z, and $D(t - t')$ is the diffusion kernel to be determined.

To zeroth order in $t_\perp$, if two adjacent chains differ in particle number by ΔN, then their Fermi momenta differ by $\Delta k = \pi\, \Delta N/L$. However, as was shown in [9] even for $\Delta N \sim O(L/a)$, where a is an in-chain length scale proportional to the lattice spacing, the additional terms introduced into $\Gamma(t)$ are rendered incoherent. Now, any *physical* particle number gradient will have ΔN only $O(1)$. We therefore believe that it is correct to argue that, to

$O(t_\perp^2)$, *all* hopping occurs at the incoherent rate given by (3) and the diffusion kernel is therefore given by

$$D(t - t') = 4t_\perp^2 d \,\mathrm{Re} \int_0^t dt' \left\{ \int dx\, G_e(x, t - t') G_h(x, t - t') \right\}, \tag{6}$$

which has the long time asymptotic behavior $D(t) \sim 1/t^{1+4\alpha}$.

Thus for a density gradient fluctuating at frequency ω, $J_D(\omega) = D(\omega)\nabla n(\omega)$ with $D(\omega) \sim \omega^{4\alpha}$. Utilizing the appropriate Einstein relation to connect $\sigma_c(\omega)$ to $D(\omega)$ we finally arrive at the following expression for the interchain conductivity:

$$\sigma_\perp(\omega) \sim \alpha \frac{e^2}{\hbar} ad\left(\frac{t_\perp^2}{v_c v_F}\right)\left(\frac{\omega}{\Lambda v_c}\right)^{4\alpha}. \tag{7}$$

In writing (7) we have taken $v_c \gg v_s$ for simplicity, where $v_{c,s}$ refers to the charge and spin velocity, respectively. This corresponds to the limit $U/t \gg 1$ of the Hubbard model. Generically, we may replace the factor $t_\perp^2/v_c v_F$ by $\sim a^{-2} t_\perp^2/t_\parallel^2$.

For the 1D Hubbard model, $0 < 4\alpha < \frac{1}{4}$ so that the power law behavior of $\sigma_\perp(\omega)$ is very weak—almost frequency independent. Within the Luttinger liquid hypothesis we expect this weak power law behavior to be applicable to the normal state interplane conductivity of the HTSC's. Indeed a very flat frequency dependence of the c-axis conductivity has been observed in $YBa_2Cu_3O_{7-\delta}$ [5] and $La_{2-x}Sr_xCuO_4$ [6]. In fact, a power law fit to the intermediate frequency data for $YBa_2Cu_3O_7$ yields a 2D Luttinger liquid exponent $\frac{1}{4} \lesssim 4\alpha \lesssim \frac{1}{3}$ which, while perhaps being a little larger than what one might expect, is not at all unreasonable. A similar fit to the electronic component [13] of $\sigma_c(\omega)$ in $La_{1.9}Sr_{0.1}CuO_4$ from [6] yields a similar exponent. Moreover, substitution of typical parameters $t_\perp$, $t_\parallel$, etc. for these HTSC's into the 2D generalization of (7) yields conductivities in reasonable *quantitative* agreement with those observed experimentally [14].

The expression (7) should be approximately valid over a range of intermediate frequencies $t_\perp \lesssim \omega \lesssim t_\parallel, U$. Our perturbative calculation cannot say anything precise about the low frequency region, $\omega \ll t_\perp$, but we expect on self-consistency grounds that $\sigma_\perp(\omega)$ would approach a small, but nonzero, value as $\omega \to 0$. At very high frequencies $\omega \gtrsim t_\parallel, U$ we expect on very general grounds that $\sigma_\perp(\omega)$ must eventually cross over to an ω^{-2} behavior.

We now turn our attention to thermal effects. Intuitively, one would expect that temperature will enhance the possibility of coherent hopping since it will cut off long time orthogonalities. This is indeed the case: evaluation of (4) at nonzero temperature yields the result

$$\frac{\Gamma(t)}{L} \sim \alpha\Lambda\left(\frac{t_\perp^2}{v_c\Lambda}\right)\left(\frac{T}{v_c\Lambda}\right)^{4\alpha} Tt + \text{incoherent}. \tag{8}$$

Note that this type of term is *not* affected by spin-charge separation, and can therefore be expected to persist to long times (i.e., $t \gtrsim t_\perp$). However, the coherent peak introduced into the conductivity $\sigma_\perp(\omega, T)$ by this linear-in-t term will generally still be broadened, so that it is a nontrivial matter to deduce even the dc conductivity $\sigma_\perp(\omega = 0, T)$ from (8). We believe that the correct interpretation of (8) is that it approximately prescribes the *total coherent weight* in the sense that

$$\int \sigma_\perp^{\mathrm{coh}}(\omega, T)\, d\omega \sim \alpha \frac{e^2}{\hbar} ad\left(\frac{t_\perp^2}{v_c v_F}\right) T\left(\frac{T}{\Lambda v_c}\right)^{4\alpha}. \tag{9}$$

Thus, $\sigma_\perp^{\mathrm{coh}}(\omega = 0, T)$ will be determined by the way this weight is broadened in frequency.

Finally, we remark that Eq. (8) is strictly valid only for times satisfying $Tt \lesssim 1$. Within perturbation theory we therefore require $T \lesssim t_\perp$. However, the complementary regime $T \gtrsim t_\perp$ can be dealt with within standard incoherence theory, since the in-liquid scattering rate $1/\tau_\parallel \sim T$. One finds in this case [4,15] that $\rho_\perp(T) \sim (t_\perp/t_\parallel)^2 \rho_\parallel(T)$ so that $\rho_\perp(T)$ is *linear* in T. Such a high-temperature behavior is a common property of HTSC's. We emphasize, though, that it is completely misleading to use the term "metallic" to describe such a temperature dependence of the c-axis resistivity: it is the result of incoherent interplane transport, of the more standard type than that argued to take place within the confinement regime.

Our calculations should be most applicable to single-layer HTSC's such as $La_{2-x}Sr_xCuO_4$. In the multilayer materials the situation is complicated by the existence of two types of $t_\perp$: an intracell one, $t_\perp^{\mathrm{intra}}$, coupling the "cells" of n closely spaced CuO_2 planes, and an intercell one, $t_\perp^{\mathrm{inter}}$, which hops electrons between the cells. In the bilayers particularly, it is not inconceivable that intracell, interlayer superexchange terms are generated and may be responsible for the "spin-gap" signature in $\sigma_c(\omega)$ [16]. The point is that since the simple interliquid hopping term leads to only a very small conductivity at low frequencies (in particular, the zero temperature dc conductivity is very small, if not zero) the *observed* low frequency conductivity will be largely determined by whatever *other* interplane physics is involved in the real materials. Thus, it is not surprising that a universal behavior is not observed in the dc conductivity of the HTSC's. This is one of the few places where interplane chemistry or structure is important, but only insofar as it determines the *details* of σ_c, not the general fact of confinement.

From a pedagogical point of view it is worth remarking that there is some superficial similarity between the confinement phenomenon and the old Schrieffer theory of tunneling through an insulating barrier [17]. Indeed, Eq. (4) is strongly reminiscent of the Schrieffer tunneling formula. The crucial difference, however, is that in

insulator barrier tunneling incoherence is introduced by the barrier (transverse momentum is not conserved in the tunneling process), while in Luttinger liquid tunneling the tunneling process itself generates the incoherence. We note, too, that one corollary of our theory is that the very theory of tunneling must be rethought with respect to c-axis tunneling into a HTSC above T_c. This is because the derivation [1718] of the Schrieffer tunneling formula assumes the existence of a good momentum quantum number in the direction normal to the interface, a property which does not exist in the confinement regime.

In summary, we have addressed the issue of determining the conductivity $\sigma_\perp(\omega, T)$ between Luttinger liquids in the confinement regime. Within a model of coupled 1D liquids we have argued that, for a large range of intermediate frequencies, $\sigma_\perp(\omega, T)$ follows a weak power law behavior as given in (7). We expect such a result to generalize to higher dimensions. Indeed, this weak power law behavior is both qualitatively and quantitatively consistent with experimental observations on $YBa_2Cu_3O_{7-\delta}$ [5] and $La_{2-x}Sr_xCuO_4$ [6]. The effect of nonzero temperature is to introduce a coherent component into $\sigma_\perp(\omega, T)$ with total weight $\propto (t_\perp/t_\parallel)^2 T^{1+4\alpha}$. For temperatures $T \gtrsim t_\perp$ the system enters the conventional incoherence regime in which the in-liquid scattering rate exceeds the interliquid hopping rate, and one finds $\rho_\perp(T)/\rho_\parallel(T) \sim (t_\perp/t_\parallel)^2$.

Finally, we remark that in the light of these arguments and those presented in [9,12] it would be of great interest to examine the conductivity along the weakest hopping direction in the quasi-1D and -2D organic conductors.

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