Quantum Effects in the Resistivity of Percolation Metal Networks

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We have studied the quantum corrections to the resistance of two-dimensional (2D) percolation networks of quasi-one-dimensional (1D) wires in which the ratio between the percolation correlation length ξ and the quantum lengths L_φ, L_T could be controllably changed over a wide range. We have shown that the percolation networks behave like homogeneous 2D conductors with respect to the quantum corrections if $\xi < L_\varphi, L_T$. Close to the percolation threshold ($\xi \gg L_\varphi, L_T$) a crossover to 1D quantum corrections is observed.

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A conductor can possess both "microscopic" disorder due to impurity scattering, giving rise to quantum interference effects (see, for example, [1]), and "macroscopic" disorder due to long range fluctuations of potential which leads to the breakdown of conductivity in the percolation limit. These fluctuations affect the quantum corrections to the resistance if their typical length scale exceeds the length scales characteristic of the quantum effects [the phase coherence length L_φ for weak localization (WL) and the thermal diffusion length L_T for electron-electron interaction (EEI)] [1]. On the metallic side of the percolation metal-insulator transition (MIT) a sample can be considered to be macroscopically uniform at scales larger than the percolation correlation length ξ [2]. Hence, if the quantum lengths L_φ and L_T are much larger than ξ, then a system is homogeneous with respect to the quantum effects, and the corrections to the conductivity should depend only on such macroscopic parameters as the dimensions of the sample and its total resistance. This averaging is no longer valid if ξ exceeds the quantum lengths. As ξ diverges with the approach to the percolation threshold, a modification of the quantum corrections is inevitable.

This Letter describes an experiment where networks of very small wires are systematically made more disordered in order to probe the crossover from weak to strong disorder. Nonuniversal behavior of the quantum corrections to the resistance in the vicinity of the MIT has been already observed in experiments on three-dimensional (with respect to the quantum effects) granular films [3] and two-dimensional (2D) semicontinuous films [4–6]. However, the geometry of the infinite conducting cluster in those systems was poorly defined. In addition, 2D percolation films usually suffer from nonlinear effects in the resistance because of non-Ohmic contacts between grains. In our experiments we used thin-film networks composed of square cells with randomly deleted bonds (a classic specimen for the study of the bond problem in percolation theory), similar to the percolating networks used to study the effect of disorder on the superconducting transition [7–9]. The unique feature of these networks is the stringent size requirements, since the unit cell size has to be smaller than the quantum lengths at low temperatures. These networks have a well-defined and controllably changed correlation length and are free of the intrinsic drawbacks of semicontinuous films. We find that, in spite of the intricate structure of the samples, the quantum corrections to their resistivity are universal and two-dimensional provided the macroscopic disorder is averaged over distances larger than the quantum lengths. A crossover between the two-dimensional continuum behavior of a percolation structure (with respect to the quantum interference effects) far from the percolation threshold and a quasi-one-dimensional behavior close to the percolation threshold has been clearly demonstrated.

Networks of total length 340 μm and width 35 μm were formed by strips of width $W = 0.05$ μm and contained more than 70 000 square unit cells of size $a = 0.4 \pm 0.05$ μm. During the e-beam writing process each side of a cell was omitted with a probability x fixed for each sample. All samples with different x values were patterned from the same gold film with the following parameters: thickness $t = 20$ nm, mean free path $l = 23$ nm, diffusion constant $D = 163$ cm^2/s, and resistance per square $R_\square^{\text{micro}} = 1.87$ Ω. The values of x varied from 0 (a regular network) to 0.5 (the percolation threshold x_c for the two-dimensional bond problem in a square lattice [2]). A section of the sample with $x = 0.47$ is shown in the inset in Fig. 1.

The parameters of the samples are listed in Table I. The values of the macroscopic sheet resistance measured across the entire sample at 4.2 K are denoted here as $R_\square(\xi)$. The measured resistance of sample a (a regular network) is in good agreement with $R_\square(L \gg a)$ calculated from

$$R_\square(L \gg a) \equiv [e^2 D(L \gg a) n(L \gg a) t]^{-1} = \left(\frac{a}{W} - 0.5\right) R_\square^{\text{micro}}, \quad (1)$$

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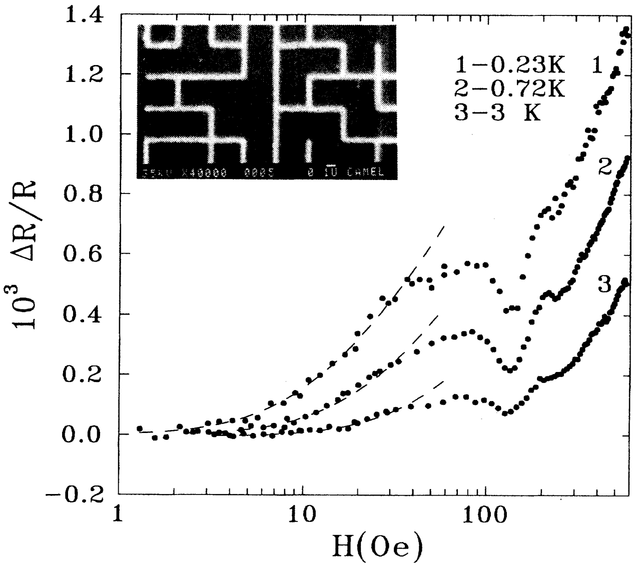

FIG. 1. $\Delta R(H)/R$ for sample e. The dashed lines are the 2D dependences calculated from Eq. (4) with the effective $R_\square^{\mathrm{MR}} = 78\ \Omega \ll R_\square(\xi)$. The inset shows a fragment of sample e.

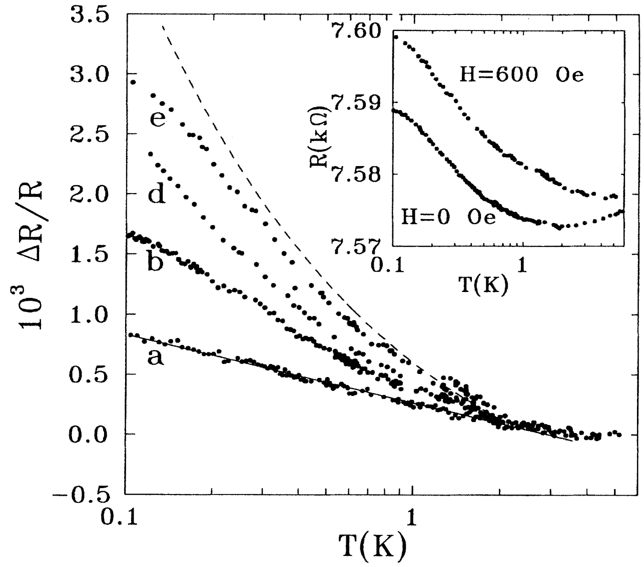

FIG. 2. $\Delta R(T)/R = [R(T) - R(4.2\ \mathrm{K})]/R(4.2\ \mathrm{K})$ for samples a, b, d, and e at $H = 600$ Oe. The inset shows $R(T)$ of sample e at $H = 600$ Oe and $H = 0$ Oe. The solid line is the 2D result calculated from Eq. (2) with $R_\square = 20\ \Omega$. The dashed line is the 1D result calculated from Eq. (3). The offset for the theoretical and experimental dependences was chosen such that all of them coincide at roughly 2 K.

where $D(L \gg a) = D/(2 - W/a)$ is the macroscopic diffusion constant [10], and $n(L \gg a) = (2W/a)n$ is the macroscopic density of electron states, which is much less than the microscopic value of n in an unpatterned film because of the small filling factor $2W/a$ of the network. With the approach to the percolation threshold, $R_\square(\xi)$ increases in agreement with the theoretical prediction for a two-dimensional percolation system $R_\square(\xi) \propto (1 - x/x_c)^{-\mu}$, where $\mu = 1.3$ [2]. The values of ξ shown in Table I were calculated from $\xi = a(1 - x/x_c)^{-\nu}$ using $\nu = 4/3$ [2]. Both scaling dependences should hold, strictly speaking, only in the vicinity of the MIT, but, similar to the results described in Ref. [9], the scaling dependences hold even far from the percolation threshold.

The temperature dependences of the resistance were measured between 10 and 0.1 K at magnetic fields $H = 0$ and $H = 600$ Oe. Figure 2 shows the data for networks with different values of x at $H = 600$ Oe. This magnetic field is strong enough to suppress the temperature dependence of the WL contribution below 10 K. Hence, the remaining temperature dependences should be considered as a result of EEI only. With increasing x the values of $\Delta R(T)/R$ initially increase proportionally to the macroscopic resistance $R_\square(\xi)$. For a regular network (sample a) this dependence is well described below 3 K by the contribution of the diffusion channel of EEI to the resistance of a 2D conductor [1]

$$\frac{\Delta R_{\mathrm{int}}(T)}{R} = -R_\square^{\mathrm{TD}} \frac{e^2}{2\pi^2\hbar} \ln(T). \qquad (2)$$

Here $R_\square^{\mathrm{TD}}$ is an effective resistance per square used as a fit parameter. We obtained $R_\square^{\mathrm{TD}} = 20\ \Omega$, which is reasonably close to macroscopic resistance of this sample $[R_\square(L \gg a) = 14\ \Omega]$. Hence, sample a can be considered as a *homogeneous* 2D conductor with microscopic disorder at scales characteristic of EEI. For the quantum correction to the resistance due to EEI, this scale is given by the thermal diffusion length $L_T = (\hbar D/k_B T)^{1/2}$. *A priori* one might expect this behavior only for the case $L_T \gg \xi, a$. The actual values of L_T, calculated for $D = 163\ \mathrm{cm}^2/\mathrm{s}$ (the diffusion constant for a single strip) are larger than a only for $T \le 0.8$ K, but, apparently, two-dimensional behavior starts well before L_T becomes much larger than a. This is reminiscent of the superconducting percolating networks described in Ref. [8], where two-dimensional behavior was observed well into the inhomogeneous regime when the superconducting coherence length was much smaller than the percolation coherence

TABLE I. Parameters of the samples. x is the probability of broken bonds, ξ is the calculated percolation correlation length, $R_\square(\xi)$ is the macroscopic resistance per square measured at 4.2 K, $R_\square^{\mathrm{MR}}$ is the effective resistance per square extracted from the magnetoresistance fits, and $R_\square^{\mathrm{TD}}$ is the effective resistance per square extracted from the 2D EEI temperature dependence fits.

Sample	x	ξ (μm)	$R_\square(\xi)$ (Ω)	$R_\square^{\mathrm{MR}}$	$R_\square^{\mathrm{TD}}$
a	0	0.4	14	14	20
b	0.3	1.4	50	38	~50
c	0.4	3.4	130	47	—
d	0.45	8.6	527	51	—
e	0.47	17	1039	78	—

length. The values of $\Delta R(T)/R$ for sample b are still in reasonable agreement with the 2D result [Eq. (2)] calculated using the macroscopic $R_\square(\xi)$.

Closer to the percolation threshold the temperature dependences of the resistance of the samples approach a limiting behavior in spite of the large differences in $R_\square(\xi)$. In other words, $\Delta R_{\rm int}(T)/R$ does not scale with the macroscopic resistance per square $R_\square(\xi)$ but instead approaches the temperature dependence of the resistance of a single one-dimensional strip. The dashed line in Fig. 2 represents the temperature dependence of the EEI correction to the resistance of a single strip, which is one-dimensional (1D) with respect to EEI ($W \ll L_T$) [1]:

$$\frac{\Delta R_{\rm int}(T)}{R} = \frac{e^2}{2\hbar}\frac{R_\square^{\rm micro}}{W} L_T \propto T^{-1/2}. \qquad (3)$$

The face that $\Delta R(T)/R$ for the samples closest to the MIT approach $\Delta R(T)/R$ for a single strip indicates that in these samples the infinite percolation cluster is one-dimensional at scales characteristic of EEI. However, even for the most disordered sample e, the values of $\Delta R(T)/R$ are still smaller than those of a single strip. We believe this is due to the presence of dead ends and intersections of the branches of the backbone of the infinite percolation cluster. Douçot and Rammal [11] showed that localization effects are reduced when 1D wires have intersecting branches because of the reduction of electron backscattering at points of coordination number higher than 2. Although there are no calculations, we expect similar effects for EEI, since two interacting electrons moving diffusively and close to one another have a larger probability of becoming separated at a point of large coordination number within the coherence time $\tau_\varepsilon = \hbar/k_B T$. Additional evidence of the influence of the intersections of the backbone and the dead ends is provided by the magnetoresistance (MR) of the samples, as will be shown below.

The magnetic field dependences of the resistance $\Delta R(H)/R$ for several samples at the same temperature ($T = 0.15$ K) are shown in Fig. 3. The observed positive MR is due to the suppression of weak localization effects in samples with strong spin-orbit scattering (SOS) [1]. The most noticeable feature of these dependences is the presence of Aharonov-Bohm (AB) oscillations with a period $\Delta\Phi = \Phi_0 \equiv \pi\hbar c/e$, where Φ is the magnetic flux threading one unit cell [1]. If we normalize the MR curves in Fig. 3 to have the same $\Delta R/R$ value at the highest field, we see that the amplitude of the oscillations decreases with increasing x. This is due to the fact that the number of closed loops of area a^2 decreases with increasing x. In strong magnetic fields the MR oscillations are damped because of the finite width of the strips. We have not observed harmonics of the AB oscillations, in agreement with the prediction of Douçot and Rammal for both percolation and regular normal-metal networks [11].

The most interesting regions of the magnetoresistance data are the ranges of weak and strong fields. Figure 1 shows the magnetic field dependences of the resistance for sample e at different temperatures. The low-field range corresponds to the case when then magnetic flux through a unit cell is much less than Φ_0, or, in other words, the magnetic length $L_H = (\hbar c/2eH)^{1/2}$ is much larger than the cell size. In this case the MR is mainly due to the suppression of the WL contribution to the resistance from closed loops of size larger than L_H. The low-field $\Delta R(H)/R$ dependences for all samples are well described by the 2D result of the WL theory (the dashed curves in Fig. 1), obtained for strong SOS [1]:

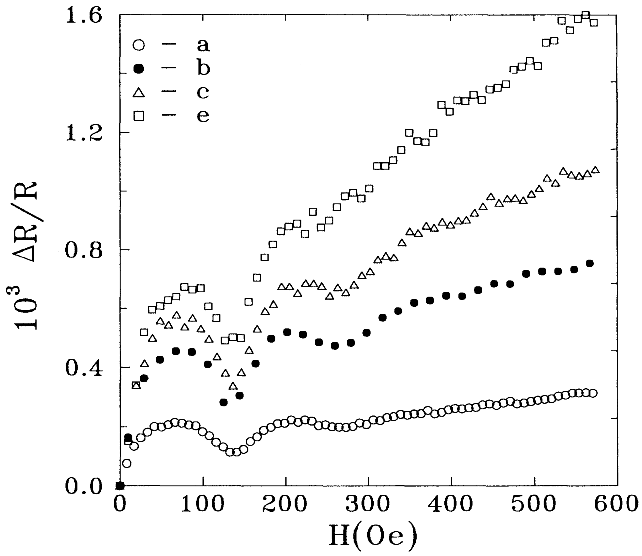

FIG. 3. The $\Delta R(H)/R$ dependences for samples a, b, c, and e at $T = 0.15$ K.

$$\frac{\Delta R(H)}{R} = R_\square^{\rm MR}\frac{e^2}{4\pi^2\hbar} f\left(\frac{4eHL_\varphi^2}{\hbar c}\right), \qquad (4)$$

where $f(y) = \psi(1/y + 1/2) + \ln(y)$, and ψ is the digamma function. For each theoretical curve, two fit parameters have been used: a temperature-independent effective resistance per square $R_\square^{\rm MR}$ (the values of $R_\square^{\rm MR}$ are listed in Table I) and the phase coherence length $L_\varphi(T)$ (see Fig. 4). For samples a and b the values of $R_\square^{\rm MR}$ are close to the macroscopic resistance $R_\square(\xi)$. For such samples L_φ exceeds ξ, and all peculiarities of the structure are averaged over the quantum scales. This is the continuum limit of the theoretical consideration of the MR of networks by Douçot and Rammal [11]. For samples $c-e$, which are closer to the MIT, the values of $R_\square^{\rm MR}$ are close to one another and become much less than $R_\square(\xi)$. This is an expected result: For these samples $L_\varphi \ll \xi$, and the quantum interference effects are insensitive to those changes in the structure occurring at distances of the order of ξ [which determines the rapid growth of $R_\square(\xi)$]. We would like to emphasize that for the samples closer to the MIT there is a difference in the dimensionalities of the quantum corrections to the temperature dependence of the resistance (due to EEI)

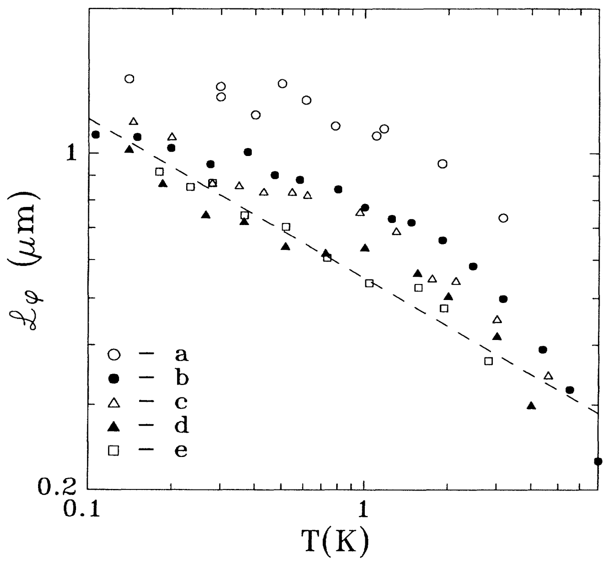

FIG. 4. The low-field values of $L_\varphi(T)$ for several samples. The dashed line is the temperature dependence $L_\varphi \propto T^{-1/3}$, which is typical for a 1D single wire.

and the low-field MR (due to WL). For EEI, all types of trajectories of the two electrons interacting twice within a region of size $L \leq L_T$ are important, and the main contribution to $\Delta R(T)$ comes from 1D diffusion inside a single strip. For WL, only looplike trajectories of area larger than Φ_0/H contribute.

The temperature dependences of L_φ derived from the low-field magnetoresistances for different samples are plotted in Fig. 4. For the samples closer to the MIT (d and e) the $L_\varphi(T)$ dependences are similar to the $L_\varphi(T)$ dependence of a single 1D wire [12]. For single strips of comparable parameters the dominant phase-breaking mechanism at low temperatures is quasielastic electron-electron collisions (the Nyquist phase-breaking) yielding $L_\varphi \propto T^{-1/3}$ [13]. For less disordered samples, the $L_\varphi(T)$ dependences depart from those of a single one-dimensional wire and do not follow a simple power law. In addition the magnitude of the phase-breaking length now depends on x. There are predictions for the behavior of $L_\varphi(T)$ for this case, when $\xi \sim L_\varphi$ [3,14], but our data does not afford a quantitative comparison with the theory.

The high-field magnetoresistance of the networks provides additional evidence of the effect of the intersections of the backbone and of the dead ends on the quantum corrections. Without this influence, the MR (determined by the suppression of the one-dimensional interference effects inside a single strip), would be the same for all the samples, independent of the probability x, like for percolation networks made of "local" field-dependent resistors. Although the shape of the $\Delta R(H)$ dependences for all samples resemble those for a 1D single strip [12], the values of $\Delta R(H)/R$ in strong magnetic fields (when L_H becomes much smaller than the cell size) become larger with the increase of x, as can be seen in Fig. 3. This is consistent with the fact that, with the approach to the percolation threshold, the average distance between nodes with a coordination number larger than 2 increases, and the reduction of the WL effects becomes less prominent. In addition, the ratio between the values $\Delta R(H)/R$ for samples e and a at the same magnetic field is about 5 at $T = 0.15$ K and decreases down to 2 at 3 K. This is also consistent with our picture: As the temperature increases, the quantum lengths become smaller, and the reduction of the quantum interference effects by the intersections and dead ends becomes less pronounced, making the corrections for all samples to be closer to the corrections for a single wire with no dangling branches. The role of the dead ends has also been confirmed by our recent measurements of the quantum corrections to the resistance of 1D wires with periodically spaced dangling side branches [15].

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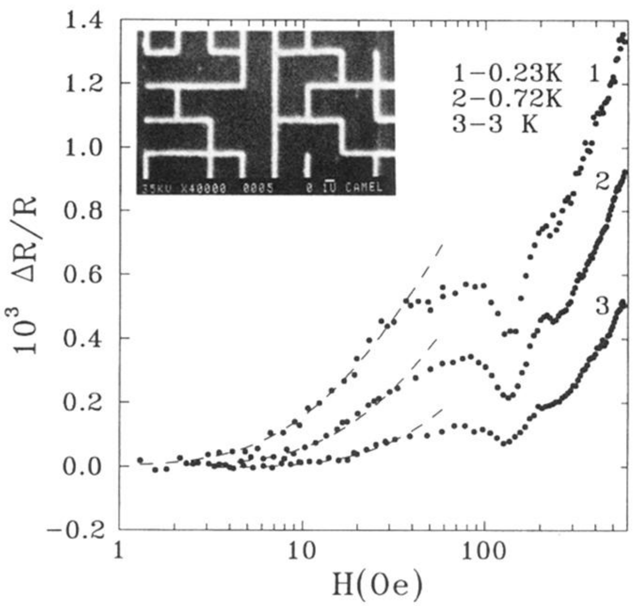

FIG. 1. $\Delta R(H)/R$ for sample *e*. The dashed lines are the 2D dependences calculated from Eq. (4) with the effective $R_{\square}^{\rm MR}$ = 78 Ω ≪ $R_{\square}(\xi)$. The inset shows a fragment of sample *e*.