Collective Flow from the Intranuclear Cascade Model

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The phenomenon of collective flow in relativistic heavy ion collisions is studied using the hadronic cascade model ARC. Comparison is made to data for Au + Au at $p = 1.7$ GeV/c, and for Ar + Pb at $p = 1.4$ GeV/c. Collective flow is well described quantitatively without the need for explicit mean field terms to simulate the nuclear equation of state. Pion collective flow is in the opposite direction to nucleon flow, as is that of antinucleons and other produced particles. Pion and nucleon flow are predicted at BNL Alternating Gradient Synchrotron energies also, where, in light of the higher baryon densities achieved, we speculate that equation of state effects may be observable.

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Collective flow [1] in relativistic heavy ion collisions has long been a subject of interest, since it was felt the phenomenon might carry information about the nuclear matter equation of state [2]. In this Letter, we will be concerned only with the so-called "sideward flow."

Theories which have been used to describe the heavy ion collisions fall into two broad classes: those based on macroscopic thermodynamical and hydrodynamical considerations [3], and those which attempt a more microscopic description of the ion-ion collision. Among the microscopic models one can distinguish pure cascade models [4,5], which include only the elementary 2-, or, in principle, n-body collisions of the constituents. These models take as their main input the experimentally measured cross sections (and angular distributions) for hadron-hadron $\sigma(hh \rightarrow X)$ in free space, and then carry out the ion-ion collision by Monte Carlo methods. The model, ARC (a relativistic cascade) [5], which we will use to discuss flow, is such a pure hadronic cascade model. Additionally, there are microscopic models [6,7] which include mean field, collective, or in-medium effects in some fashion, as well as treating the elementary hadron-hadron collisions.

Our treatment of flow using ARC will neglect mean fields entirely, and this is based on the assumption that the mean field U satisfies $U \ll T$, a typical kinetic energy involved in the cascade. However, the mean field, although not dominant, may not be completely negligible in late or very soft collisions or for comoving spectators in the projectile and target.

Indeed, we shall see that ARC, using unmodified free space cross sections and no mean field, is adequate to the task of describing sideward flow in Au + Au at LBL Bevalac energies. Given the prior extensive success of ARC [8] in predicting and describing inclusive data for heavy ion collisions at BNL Alternating Gradient Synchrotron (AGS) energies, our strong theoretical prejudice would then be that mean fields need not be included over this range of energies ($1-15$ GeV/c). In-medium effects should, we think, be included by modifying the elementary interactions. Nevertheless, the high baryon densities apparently achieved during massive ion collisions at AGS energies [5] may still manifest themselves through traditional equation of state effects [2] such as enhanced strangeness production.

Our specific concern will be with the protonlike sideward flow measured at the Bevalac in Au + Au collisions at lab momentum $p = 0.96-1.9$ GeV/c. Such data have already been measured using the Plastic Ball spectrometer [9], and new experiments with better immunity to detector distortions were recently carried out using the EOS time projection chamber (TPC); flow results from the EOS collaboration are expected to be available soon [10]. We compare with preliminary flow data from the TPC, which has been discussed already in conference proceedings [11]. We consider protonlike flow, since ARC does not as yet dynamically include production of nuclear fragments larger than single nucleons. Coalescence calculations for deuterons and tritons [12] from ARC have been carried out at AGS energies though, and are in good agreement with data.

We calculate sideward flow *à la* Danielewicz and Odyniec [13] by first defining a reaction plane for the ion + ion collision, neglecting pions, using the beam direction and a vector defined as a weighted average of outgoing transverse momenta

$$\mathbf{Q} = \sum_i w(y_i)\mathbf{p}_T^i\,, \tag{1}$$

with $w(y)$ selected as in Ref. [13]. The essential point is that w is an odd function of $y_{\mathrm{c.m.}}$. The sideward flow curve is then defined by projecting protonlike momenta into the reaction plane, and averaging:

$$\langle P_x(y)\rangle = \frac{1}{N(y)}\left\{\sum_i(\mathbf{p}_i \cdot \hat{\mathbf{Q}}_i)\right\}. \tag{2}$$

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Here $N(y)$ is the number of protons detected in a bin of width dy around y. Kinematic cuts are placed on the ARC calculation. These are a spectator cut ($p_{\text{proj}} >$ 250 MeV/c), and a forward rapidity cut ($y_{\text{c.m.}} > 0$). The target in the EOS detector is located upstream of the main tracking chamber, which optimizes acceptance for $y_{\text{c.m.}} >$ 0 but compromises it for $y_{\text{c.m.}} < 0$. This combination of cuts ensures that our flow calculation is made in a region where detector distortions should be minimal [11]. The charge multiplicity M is then defined as the number of protons surviving the cuts.

ARC was designed, of course, for much higher energy heavy-ion collisions at the AGS. However, we felt it was of interest to compare flow from the Plastic Ball and the forthcoming EOS data, especially the recent Au + Au runs at maximum Bevalac energy, with an unmodified version of ARC. We do not include comparisons of inclusive spectra, because such data have not been published for the Plastic Ball, and are not yet published in a journal in the case of EOS. We simply comment that the agreement with mid-rapidity spectra in Ref. [10] seems very good, considering that the present beam energy is a factor of 10 below that for which ARC was designed. Differences between theoretical and experimental spectra can be expected to show up at forward rapidities due to a simplified treatment of Fermi motion which does not completely respect conservation of energy. This is a negligible effect at the AGS energies.

Our flow comparison for the Plastic Ball is based on the slope of the $\langle P_x \rangle$ curve near mid-rapidity. This observable is minimally distorted by the Plastic Ball acceptance, as judged by simulations. The measured slope for Plastic Ball events with a multiplicity selection comparable to that placed on ARC (the upper 50% of events in the multiplicity spectrum are accepted) is shown as a dotted line in Fig. 1(a). The Plastic Ball slope is corrected for dispersion in the estimated reaction plane; for the calculation of ARC $\langle P_x \rangle$ we use the ideal reaction plane.

In Fig. 1(b) flow from ARC is compared with the preliminary result from EOS. The EOS curve is that measured for $\langle P_x/A \rangle$, for events having $0.6M_{\max} < M <$ $0.9M_{\max}$, with $M_{\max}$ the maximum protonlike multiplicity observed in the TPC [11]. No spectator cut is made in this data, and only fragments up to charge $Z = 2$ are included in the flow curve. However, the TPC does not identify very heavy nuclear fragments ($Z > 7$). To compare this flow with ARC, we eliminate all spectator (noninteracting) protons in events having a large number of spectators ($N_{\text{spec}} > 25$), assuming that in such peripheral events large fragments are likely to remain. This produces a multiplicity distribution close to the observed one, and the same selection as above is then made on M for the ARC generated events. The $\langle P_x \rangle$ agrees very well over the whole range of y measured. We stress that $\langle P_x \rangle$ up to $y \sim 0.5$ is insensitive to the spectator cut; it is flow only near the projectile rapidity that is significantly affected.

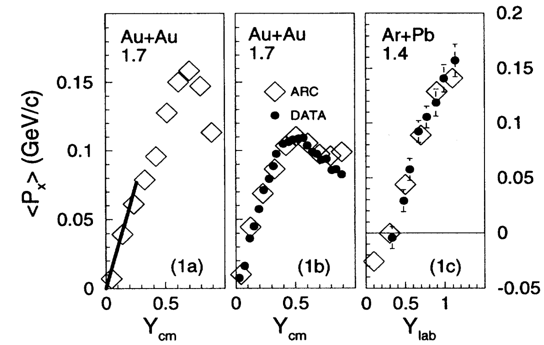

FIG. 1. ARC vs experiment: Beam momentum per nucleon, in GeV/c, is given in each figure. (a) Plastic Ball vs ARC. Solid line indicates Plastic Ball slope, corrected for dispersion in the estimated reaction plane. ARC $\langle P_x \rangle$ is calculated in the ideal event plane. A spectator cut, $p >$ 0.25 GeV/c, is imposed. The Plastic Ball slope is not extended past mid-rapidity since the acceptance filter becomes important there. (b) EOS vs ARC Preliminary TPC data are compared to the ARC calculation, as described in the text. (c) Streamer Chamber data for the asymmetric system Ar + Pb are compared to ARC. Impact parameters $b < 5.5$ fm are included in the ARC calculation, to correspond to the "semicentral" experimental cut [14].

We have also made calculations for beam momenta of 0.96 and 1.9 GeV/c. The agreement between theory and experiment remains equally good at these higher and lower energies.

We also compare [Fig. 1(c)] ARC calculation to Streamer Chamber data [14] for Ar + Pb, (p = 1.4 GeV/c), showing ARC generates sufficient flow in an asymmetric system. We see no reason *a priori* why the cascade should fail to describe flow in light + heavy systems, given it succeeds in heavy + heavy ones, over a range of energies. Other works, though, have suggested that a particular sensitivity to mean field effects exists in asymmetric systems [15].

It is commonly asserted that cascade models produce little or no collective flow, though it has been demonstrated that cascade models can produce significant flow [16]. Two salient features of ARC and other cascade models with respect to flow are the treatment of angular momentum conservation in two bodily collisions and the choice of repulsive versus attractive orbits. Other authors have discussed this question [17].

If justification can be found in quantum mechanics for the cascade model, it is in part from the eikonal approximation to potential scattering, valid when $U \ll T$, and for small angles. Generalizing the eikonal approximation to N bodies with pairwise interactions one would retain the notion of a classical orbit, for the whole system. Both impact parameter and reaction plane then exist for the 2-body system. Since angular momentum is classically conserved one need consider only orbits for the whole system which preserve the 2-body reaction planes. Allowing for quantum mechanics in the 2-body collisions could lead to indeterminacy in the planes, but angular momentum is still conserved. So, we expect a spread in angle of the plane: $\Delta\theta \sim (2l + 1)^{-1}$. In the calculations we retain as a first approximation a well-defined plane for all collisions with 2-body final states, which at Bevalac energies is in effect almost all collisions. If the plane is instead randomized, ~20% reduction in flow ensues.

A simple estimate can be made of the number of partial waves present here. For the beam momentum and a typical cross section one finds $l = kb \sim 5$ [$b = (\sigma/\pi)^{1/2}$] for first collisions. Certainly we expect that not much of the scattering is s wave at $p_L \sim 1.7$ GeV/c, even for subsequent collisions.

Secondly, again at Bevalac energies, one might expect that the *NN* scattering involves largely repulsive forces. If such a condition is imposed here by allowing only repulsive orbits in the cascade, a smaller, ~8% increase in flow results. This is clearly a 2-body potential effect, indicating only a small sensitivity to such effects, but will of course alter the effective equation of state. At higher energies where the scattering would seem to be more definitely diffractive, one must probably return to orbits randomized with respect to repulsion and attraction, but not plane.

One further issue of concern is whether Fermi motion, which as we pointed out above could be handled more correctly, is in fact somehow responsible for the flow exhibited by the cascade. One finds that, if anything, the flow increases when Fermi motion is turned off. We expect that some small adjustment of the theory may result when consistent Fermi motion, smearing of the 2-body plane, and impact-parameter-dependent choice of repulsive or attractive orbits are introduced, but certainly there would not appear to be any need at this point for large mean field terms, and collective flow at these energies can, it seems, come from a pure cascade model.

We calculated flow at AGS energies, however without using cuts, in the absence of comprehensive data. For all particle types we define a sideward flow, based on the reaction plane already determined only from the protons. A graph of the flow curves for pions and protons obtained in Au + Au at $p = 11.6$ GeV/c is shown in Fig. 2. It is seen that the pion flow is in the opposite direction to the proton flow at these energies, and is numerically considerably smaller. The smaller pion $\langle P_x \rangle$ likely results from a cancellation between prompt pions and the *bremmstrahlung* pions produced by resonance decay and following the nucleon flow.

We observe that the flow momentum for produced particles is generally in the opposite direction to that for nucleons; flow curves for antinucleons, kaons, and pions in Fig. 2 make this evident. Though more study is needed, it seems that the magnitude of antinucleon flow is similar to that for protons, while kaon and pion flows are somewhat less. In peripheral collisions, that is to say, in collisions having appreciable sideward flow,

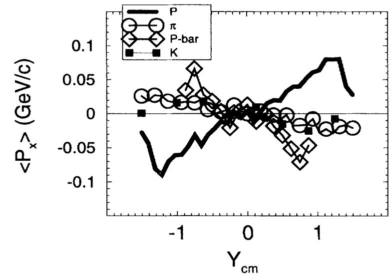

FIG. 2. Flow vs AGS Energy: Pion and nucleon flows are shown, as well as kaon and antinucleon flows, including screening. Produced particles all have flow momentum in the opposite direction to the nucleons. The magnitude of flow is sensitive to the mixture of repulsive and attractive orbits, and in fact one expects diffractive scattering at these energies. K^+ and K^- flows are averaged together to give "K" flow: In fact K^+ is positive and K^- is negative.

particle production will take place initially in the dense central region where target and projective nuclei overlap. Antinucleons trying to emerge from the interaction region in the direction of nucleon flow will encounter more target or projectile nucleons than those emerging in the opposite direction. Given the large annihilation cross section, it is natural that antinucleons should *antiflow*. Despite screening of the basic annihilation process, there is still appreciable (40%) annihilation, but also considerable antiflow. So, the observation of antiflow for antiprotons would not rule out screening of antiproton production in the medium [18]. A similar mechanism to that generating the antiproton flow may function in the case of negative kaons and pions, where absorption on nucleons is still a strong effect. However, positive kaons have a small positive flow momentum.

At the kinetic energy ($T_{\mathrm{c.m.}} \sim 3.5$ GeV) of first pp collisions, the cross section is more inelastic than at lower energies. So, production plays an important role at the higher energies. Even at the beam momentum of 1.7 GeV/c ($T_{\mathrm{c.m.}} = 450$ MeV), considered for the Bevalac Au + Au data, production is relevant. However, a new dynamical region may be entered below the threshold for pion production ($T_{\mathrm{c.m.}} \sim 200$ MeV). Then, the mechanism for flow production will involve only elastic processes, and mean fields may perhaps make significant contributions.

That a pure cascade model can produce sufficient collective flow is a surprising and interesting result. The question might be asked: Why did early cascade models fail to produce enough flow? From the outset, it should be stated that the literature has been somewhat contradictory on this questions [19]. Cascade calculations exist displaying significant flow [16,20] in some systems. Nevertheless, the general conclusion seems to have been arrived at, that the pure cascade generates too little collective flow. We would argue that this conclusion is incorrect, at least at the higher Bevalac energies and above, and that a correctly constructed cascade does in fact produce enough flow without mean fields. There remains the question of consistency with early cascade calculations. If in fact the flow generated was as high as 50–70% of the observed values, then including effects of the scattering style (due to angular momentum conservation and repulsive interactions as described above) could explain much of the seeming difference. It remains for further work to establish this correspondence more quantitatively.

One may also ask if the equation of state need not be included explicitly in the modeling to generate flow, then is any role played by the equation of state in producing flow? Certainly the cascade model, including as it does classical, relativistic kinetic processes, produces at least the thermal pressure and equation of state of an ideal relativistic gas. This thermal pressure may be all that is needed for the flow. Some potential effect has been noted, of the order of 10%, and since it arises from the use of repulsive orbits the equation of state would be van der Waals in character, due to the effective excluded volume.

At AGS energies, flow is predicted in both pions and nucleons, and it is not an inconsiderable effect. The direction of sideward flow is opposite in pions and nucleons. The success of ARC in describing flow at Bevalac energies, and inclusive data at AGS energies emboldens us to use the code to study flow at AGS energies. High energy flow predictions hopefully will be testable when the EOS detector is moved to BNL and the E895 collaboration begins to take data. Since the densities achieved are higher than at the Bevalac, we can still hope to see explicit equation of state effects, and flow may be a good observable to study in this connection. One may look for deviations from the pure cascade picture in rarer, very high multiplicity events, or at lower beam energies, where higher baryon density may be achieved. For example, we note a very strong difference between flow for K^+ and K^-, which might prove a barometer for high density effects, such as the presence of a mixed phase.

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