Extraction of V_{ub} from the Decay $B \to \pi l \nu$

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(Received 26 July 1994)

We develop the perturbative QCD formalism for semileptonic B meson decays, which includes Sudakov suppression on the spatial extent of a heavy meson containing a light valence quark. We show that the perturbative calculation for the spectrum of the decay $B \to \pi l \nu$ is reliable for the energy fraction of the pion above 0.3. Combining predictions from soft pion theorems, we obtain an upper limit of the matrix element $|V_{ub}|$ of roughly 3.5×10^{-3}.

PACS numbers: 13.20.He, 12.15.Hh, 12.38.Bx

Exclusive semileptonic meson decays, which provide information of the mixing angles in the Cabibbo-Kobayashi-Maskawa matrix of the standard model, have been studied intensively. For the heavy-to-light transition $B \to \pi l \nu$, which gives a reliable estimation of the matrix element $|V_{ub}|$ [1], there are not yet appropriate theories. The well-known chiral symmetry [2] and heavy quark symmetry [3] cannot be applied to this process. Recently, a perturbative QCD (PQCD) analysis of the decay $B \to \pi l \nu$, including Sudakov effects, has been proposed [4], which makes possible the direct extraction of $|V_{ub}|$. However, it leads to results which are too small compared to current experimental data. The smallness is due to the subtraction of an on-shell virtual heavy quark propagator from the hard scattering subdiagram, in which all the particles are supposed to be far off shell.

In this Letter we shall develop a modified PQCD approach to heavy meson decays, which also includes Sudakov effects that are formulated in a different (transverse) direction [5,6]. We associate transverse momentum $\mathbf{k}_T$ with the valence quarks in the B meson, which was not considered in Ref. [4], and derive the Sudakov factor describing the evolution of the B meson wave function in $\mathbf{k}_T$. We emphasize that we shall perform a new resummation of large radiative corrections to the system containing both heavy and light valence quarks, which is different from all the previous studies of Sudakov resummation that have concerned only light hadrons. The Sudakov factor suppresses long-distance contribution and has extended to the applicability of PQCD down to the few GeV scale in the study of elastic hadron form factors [6]. With $\mathbf{k}_T$, the virtual heavy quark in the hard scattering is always off shell and needs not to be subtracted. Our predictions for the differential decay rate then turn out to be larger than those in Ref. [4] by about three orders of magnitude. We shall show that our PQCD approach is proper for the decay $B \to \pi l \nu$, at least when the pion is energetic, and gives predictions comparable to those from soft pion theorems and to experimental data.

The amplitude of the considered process is written as

$$A(P_1, P_2) = \frac{G_F}{\sqrt{2}} V_{ub} \overline{\nu} \gamma_\mu (1 - \gamma_5) l \langle \pi(P_2) | \overline{u} \gamma^\mu b | B(P_1) \rangle , \tag{1}$$

where G_F is the Fermi coupling constant and P_1 and P_2 are the B meson and pion momenta, respectively. We start with the lowest-order factorization for the matrix element $M^\mu = \langle \pi(P_2) | \overline{u} \gamma^\mu b | B(P_1) \rangle$, in which the b quark carries the momentum $P_1 - k_1$, and its light partner carries k_1. These momenta satisfy the on-shell conditions $(P_1 - k_1)^2 \approx m_b^2$, $P_1^2 = m_B^2$, and $k_1^2 \approx 0$, m_b and m_B being the b quark and B meson masses, respectively. We choose the Breit frame such that $P_1^+ = P_1^- = m_B/\sqrt{2}$, $P_2^+ = \eta m_B/\sqrt{2}$ and all other components of P_i's vanish, where η is related to the energy fraction of the pion by $P_2^0 = \eta m_B/2$, $0 \le \eta \le 1$. k_1 has a minus component, defining the momentum fraction $x_1 = k_1^-/P_1^-$ in the B meson wave function, and a small amount of transverse components $\mathbf{k}_{1T}$. The light valence quark of the B meson, after absorbing the hard gluon, goes into the pion with the momentum fraction x_2 and transverse momenta $\mathbf{k}_{2T}$.

We then consider how to group radiative corrections into the basic factorization by locating their leading momentum regions, from which important contributions to loop integrals arise. The important corrections are characterized by large single logarithms of the form $\ln(m_B/k_T)$, which are either collinear or soft. These two regions may overlap and give double logarithms. It is known that single logarithms can be summed to all orders using renormalization group (RG) methods, while double logarithms must be organized by the resummation technique [7], which has been developed in axial gauge $n \cdot A = 0$ for the light hadron case, n being the gauge vector and A the gauge field.

A careful analysis shows that reducible corrections on the pion side produce double logarithms with soft ones canceled in the asymptotic region $b \to 0$ [6], b being the conjugate variable to $\mathbf{k}_T$. Hence, they are dominated by collinear enhancements and can be absorbed into the pion wave function, giving its evolution described by the Sudakov factor. Reducible corrections on the

 0031-9007/95/74(22)/4388(4)$06.00

B meson side also give double logarithms, but the soft ones do not cancel and the collinear ones are suppressed by the B meson wave function. These corrections can be absorbed into the B meson wave function, which is also dominated by soft dynamics. The resummation of large corrections to such a heavy-light system is our main concern below. Irreducible corrections, with an extra gluon connecting the pion and the B meson, give only soft divergences, which cancel asymptotically. They are then absorbed into the hard scattering amplitude. Another type of irreducible correction with the extra gluon attaching to the hard gluon, which was considered in Ref. [4] and summed into the Sudakov factor describing the evolution in k_1^-, is in fact not important as argued in Ref. [8]. Hence, the factorization picture holds after radiative corrections are included.

The factorization formula for M^μ is then written as

$$M^\mu = \int_0^1 dx_1\, dx_2 \int \frac{d^2\mathbf{b}_1}{(2\pi)^2}\frac{d^2\mathbf{b}_2}{(2\pi)^2}\, \mathcal{P}_\pi(x_2,\mathbf{b}_2,P_2,\mu) \times \tilde{H}^\mu(x_1,x_2,\mathbf{b}_1,\mathbf{b}_2,m,\mu) \times \mathcal{P}_B(x_1,\mathbf{b}_1,P_1,\mu)\,, \qquad (2)$$

with $\mathcal{P}_\pi$ and $\mathcal{P}_B$ the pion and B meson wave functions, respectively, and $\tilde{H}^\mu$ the Fourier transform of the hard scattering amplitude to b space. μ is the factorization and renormalization scale. Both $\mathcal{P}_\pi$ and $\mathcal{P}_B$ contain double logarithms, which will be summed up below. The approximation $m_b \approx m_B = m = 5.28$ GeV has been made to simplify the analysis.

We outline the resummation procedure employed for $\mathcal{P}_\pi$ [5,6]. If the double logarithms are grouped into an exponential $\mathcal{P} \sim \exp[-\ln m \ln(\ln m/\ln b)]$, the problem will become simpler by considering the derivative $d\mathcal{P}/d\ln m = C\mathcal{P}$, where the coefficient C contains only single logarithms and can be treated by RG methods. Because of the scale invariance of n in the gluon propagator,

$$N^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{n^\mu q^\nu + q^\mu n^\nu}{n\cdot q} + n^2\frac{q^\mu q^\nu}{(n\cdot q)^2}\right), \qquad (3)$$

$\mathcal{P}_\pi$ depends only on the ratio $\nu_2^2 = (P_2\cdot n)^2/n^2$. It is then possible to relate the derivative $d\mathcal{P}_\pi/d\ln P_2^+$ to $d\mathcal{P}_\pi/dn$, which can be easily computed using the relations $dN^{\mu\nu}/dn_\alpha = -(N^{\mu\alpha}q^\nu + N^{\nu\alpha}q^\mu)/q\cdot n$. The momentum q appearing at both ends of the differentiated gluon line hints at the application of the Ward identity. After adding together all the diagrams with different differentiated gluon lines, we obtain an equation graphically described by Fig. 1(a), in which the square vertex represents $gT^a n^2 P_2^\alpha/(P_2\cdot n\; q\cdot n)$, T^a being related to the Gell-Mann matrices λ^a by $T^a = \lambda^a/2$.

Because of the factor $1/q\cdot n$ in the new vertex and the nonvanishing of n^2, the leading regions of q are soft and ultraviolet, in which Fig. 1(a) can be factorized according to Fig. 1(b) to lowest order of α_s. The part on the left-hand side of the dashed line is exactly $\mathcal{P}_\pi$, and that on the right-hand side is assigned to the coefficient C. We

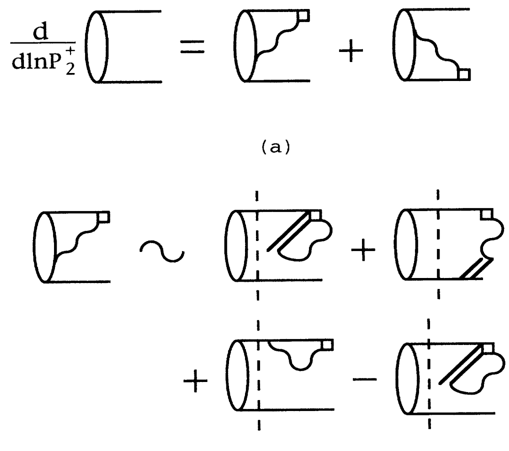

FIG. 1. Graphic representation of Eq. (4).

introduce a function $\mathcal{K}$ to organize the soft enhancements in the first two diagrams of Fig. 1(b) and $\mathcal{G}$ for the ultraviolet divergences in the other two diagrams. The soft subtraction in $\mathcal{G}$ is to avoid double counting. We then derive the differential equation,

$$\frac{d}{d\ln P_2^+}\mathcal{P}_\pi = \{2\mathcal{K}(b_2\mu) + \tfrac{1}{2}\mathcal{G}(x_2\nu_2/\mu) + \tfrac{1}{2}\mathcal{G}[(1-x_2)\nu_2/\mu]\}\mathcal{P}_\pi\,, \qquad (4)$$

where the functions $\mathcal{K}$ and $\mathcal{G}$ have been calculated using RG methods [5]. Solving Eq. (4), we obtain the solution

$$\mathcal{P}_\pi(x_2,b_2,P_2,\mu) = \exp\left[-\sum_{\xi=x_2,1-x_2} s(\xi,b_2,\eta m)\right] \times \overline{\mathcal{P}}_\pi(x_2,b_2,\mu)\,. \qquad (5)$$

The explicit expression of the exponent $s(\xi,b,Q)$ has been obtained in Ref. [6] and will not be shown here due to its complexity.

The function $\overline{\mathcal{P}}_\pi$ still contains single logarithms from ultraviolet divergences, which need to be summed using RG methods [5]. The large-b behavior of $\mathcal{P}_\pi$ is then written as

$$\mathcal{P}_\pi = \exp\left\{-\sum_{\xi=x_2,1-x_2} s(\xi,b_2,\eta m) - 2\int_{1/b_2}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q[g(\bar{\mu})]\right\} \times \phi_\pi(x_2,1/b_2)\,, \qquad (6)$$

$\gamma_q = -\alpha_s/\pi$ being the quark anomalous dimension in axial gauge. The argument $1/b_2$ in the initial condition ϕ_π of the RG equation indicates that the scale μ in the wave function has evolved to this lower scale.

As to $\mathcal{P}_B$, the resummation of the double logarithms is subtler. The self-energy correction to the massive b quark, giving only soft single logarithms, should be excluded. On the other hand, $\mathcal{P}_B$ involves the invariants such as P_1^2, which cannot be related to n, so that the technique of replacing d/dm by d/dn fails. However, the above difficulties can be removed by applying the eikonal approximation to the heavy quark line. In the collinear region with the loop momentum q parallel to k_1 and in the soft region, the b quark line can be replaced by an eikonal line:

$$\frac{(\not{P}_1 - \not{k}_1 + \not{q} + m)\gamma^\alpha}{(P_1 - k_1 + q)^2 - m^2} \approx \frac{(P_1 - k_1)^\alpha}{(P_1 - k_1)\cdot q} + R\,, \quad (7)$$

where the remaining part R is less important. The involved physics is that a soft gluon or a gluon moving parallel to k_1 cannot explore the details of the b quark, and its dynamics can be factorized.

The first difficulty is then resolved, because self-energy diagrams of an eikonal line are excluded by definition [9]. With the scale invariance of $P_1 - k_1$ as shown in Eq. (7), which is equivalent to the flavor symmetry in heavy quark effective theory, $P_1 - k_1$ does not lead to a large scale, and the remaining large scale is only k_1^-. Furthermore, an explicit lowest-order investigation shows that $\mathcal{P}_B$ depends only on the single ratio $\nu_1^2 = (k_1\cdot n)^2/n^2$ [8], and thus d/dk_1^- can be replaced by d/dn now.

Following the similar procedures to those for the pion, we obtain the differential equation

$$\frac{d}{d\ln k_1^-}\mathcal{P}_B = [\mathcal{K}(b_1\mu) + \tfrac{1}{2}\mathcal{G}(\nu_1/\mu)]\mathcal{P}_B\,. \quad (8)$$

It can be shown that the functions $\mathcal{K}$ and $\mathcal{G}$ for the B meson are exactly the same as those in Eq. (4) [8]. It is then straightforward to derive the solution

$$\mathcal{P}_B = \exp\left[-s(x_1,b_1,m) - 2\int_{1/b_1}^{\mu}\frac{d\bar\mu}{\bar\mu}\gamma_q[g(\bar\mu)]\right] \times \phi_B(x_1,1/b_1)\,. \quad (9)$$

The evolution of the functions ϕ_π and ϕ_B in $1/b$ will be neglected below. At last, the RG analysis of $\tilde H^\mu$ gives

$$\tilde H^\mu(x_i,b_i,m,\mu) = \exp\left[-4\int_{\mu}^{t}\frac{d\bar\mu}{\bar\mu}\gamma_q[g(\bar\mu)]\right] \times \tilde H^\mu(x_i,b_i,m,t)\,, \quad (10)$$

where t is taken as the largest mass scale associated with the hard gluon, $t = \max(\sqrt{x_1x_2\eta}\,m, 1/b_1, 1/b_2)$. Having factorized all the large logarithms into the exponents, we can then compute $\tilde H^\mu$ to $O(\alpha_s)$.

Substituting Eqs. (6), (9), and (10) into Eq. (2), we obtain the factorization formula for $M^\mu = f_1P_1^\mu + f_2P_2^\mu$, where the form factors f_1 and f_2 are given by

$$f_1 = 16\pi C_F m^2\int_0^1 dx_1\,dx_2\int_0^\infty b_1\,db_1\,b_2\,db_2\,\phi_B(x_1,b_1)\phi_\pi(x_2)x_1\eta h(x_2,x_1,b_2,b_1,m)\exp[-S(x_i,b_i,m)] \quad (11)$$

and

$$f_2 = 16\pi C_F m^2\int_0^1 dx_1\,dx_2\int_0^\infty b_1\,db_1\,b_2\,db_2\,\phi_B(x_1,b_1)\phi_\pi(x_2)[(1 + x_2\eta)h(x_1,x_2,b_1,b_2,m) - x_1h(x_2,x_1,b_2,b_1,m)]\exp[-S(x_i,b_i,m)]\,, \quad (12)$$

respectively, with

$$h(x_1,x_2,b_1,b_2,m) = \alpha_s(t)K_0(\sqrt{x_1x_2\eta}\,mb_1)[\theta(b_1 - b_2)K_0(\sqrt{x_2\eta}\,mb_1)I_0(\sqrt{x_2\eta}\,mb_2) + \theta(b_2 - b_1) \times K_0(\sqrt{x_2\eta}\,mb_2)I_0(\sqrt{x_2\eta}\,mb_1)]\,. \quad (13)$$

$C_F = \frac{4}{3}$ is the color factor, and K_0 and I_0 are the modified Bessel functions of order zero. The function h is the Fourier transform of one of the hard scattering subdiagrams, whose contribution is written as

$$\frac{\alpha_s(t)}{x_1x_2\eta m^2 + (\mathbf{k}_{1T} - \mathbf{k}_{2T})^2}\,\frac{1}{x_2\eta m^2 + \mathbf{k}_{2T}^2}\,. \quad (14)$$

Note that the second factor, coming from the virtual b quark propagator, does not involve singularity as $x_2 \to 0$ due to the existence of $\mathbf{k}_{2T}^2$.

The complete Sudakov exponent S is given by

$$S(x_i,b_i,m) = s(x_1,b_1,m) + s(x_2,b_2,\eta m) + s(1 - x_2,b_2,\eta m) - \frac{1}{\beta}\left[\ln\frac{\ln(t/\Lambda)}{\ln(1/b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{\ln(1/b_2\Lambda)}\right], \quad (15)$$

with $\beta = (33 - 2n_f)/12$, $n_f = 4$ being the number of quark flavors, and $\Lambda \equiv \Lambda_{\rm QCD} = 100$ MeV here. The Sudakov factor $\exp(-S)$ decreases quickly in the large b_i region and vanishes as $b_i > 1/\Lambda$. This behavior ensures that the main contribution to f_i is due to small b_i, for which α_s is small, no matter what x_i's are.

ϕ_π so chosen as the Chernyak-Zhitnitsky model [10], $\phi_\pi(x) = 5\sqrt{3}f_\pi x(1 - x)(1 - 2x)^2$ with $f_\pi = 93$ MeV the pion decay constant. For the B meson wave function we consider [4,11]

$$\phi_B(x) = \frac{\pi Nx(1 - x)^2}{m^2 + C(1 - x)}\,, \quad (16)$$

with $N = 1.232\ \text{GeV}^3$ and $C = -0.999\,93m^2$, satisfying the normalization $\int dx\,\phi_B = f_B/2\sqrt{3}$, $f_B = 160$ MeV being the B meson decay constant [12].

Results of $f_1 + f_2$ with b_1 and b_2 integrated up to the same cutoff b_c are shown in Fig. 2(a). We observe that at $\eta = 0.3$ approximately 50% of the contribution to $f_1 + f_2$ comes from the region with $\alpha_s(1/b_c) < 1$ ($\alpha_s/\pi < 0.32$), or equivalently, $b_c < 0.5/\Lambda$. At $\eta = 0.4$, 55% of the contribution is accumulated in this perturbative region. As $\eta = 1$, the perturbative contribution has

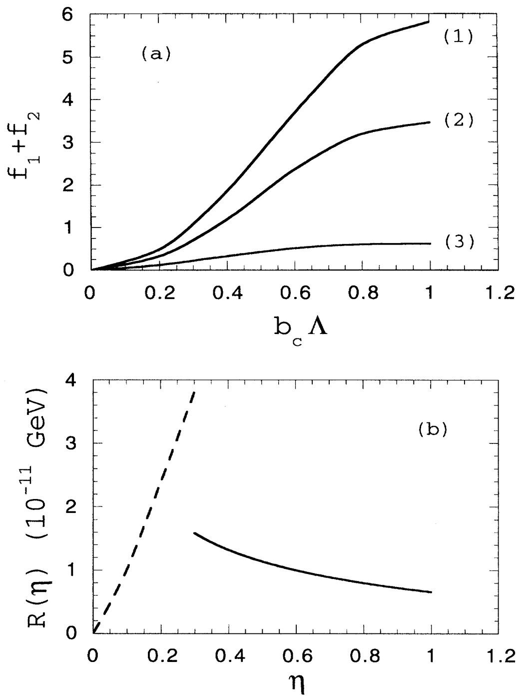

FIG. 2. (a) Dependence of $f_1 + f_2$ on the cutoff b_c for (1) $\eta = 0.3$, (2) $\eta = 0.4$, and (3) $\eta = 1.0$. (b) Dependence of $R(\eta)$ on η derived from the modified PQCD formalism (solid line) and from the soft pion theorems (dashed line).

reached 70%. It implies that the modified PQCD analysis of the decay $B \to \pi l \nu$ in the range of $\eta > 0.3$ is relatively reliable according to the criteria given in Ref. [6]. We emphasize that contributions satisfying these criteria are not completely perturbative, but perturbation theory indeed makes sense in the above region with $\eta > 0.3$. The differential decay rate for the specific case $B^0 \to \pi^- l^+ \nu$ with vanishing lepton masses is given by

$$\frac{d\Gamma}{d\eta} \equiv |V_{ub}|^2 R(\eta) = |V_{ub}|^2 \frac{G_F^2 m^5 \eta^3}{768\pi^3} |f_1 + f_2|^2. \quad (17)$$

Substituting the result of f_i into Eq. (17), we derive the behavior of $R(\eta)$ as in Fig. 2(b), which shows a slow decrease with η.

In order to have the full spectrum in η, we approximate $d\Gamma/d\eta$ in the range of $\eta < 0.3$ by the soft pion limits of f_i [13]:

$$\lim_{\eta\to 0} R(\eta) = \frac{G_F^2 m^5 f_{B^*}^2 g_{BB^*\pi}^2}{192\pi^3 f_\pi^2} \frac{\eta^3}{(1 - r + r\eta)^2}, \quad (18)$$

which shows an approximate linear relation with η. Here $r = m_B^2/m_{B^*}^2 \approx 0.98$, $f_{B^*} \approx 1.1 f_B$ [14] is the decay constant of the B^* meson, and $g_{BB^*\pi} \approx 0.75$ [15] is the $BB^*\pi$ coupling constant. We extrapolate Eq. (18) to $\eta = 0.3$ as shown in Fig. 2(b), and a fair match between the soft pion and PQCD predictions is observed. Though this extrapolation of the soft pion limit may not be reliable and the product $f_{B^*} g_{BB^*\pi}$ suffers a large uncertainty, the agreement of our calculation, which does not involve any phenomenological parameter, with nonperturbative results to order of magnitude is not trivial.

It is then possible to estimate the total decay rate Γ by combining Eq. (18) for $\eta < 0.3$ with the PQCD predictions for $\eta > 0.3$. We obtain $\Gamma \approx 1.3 \times 10^{-11} |V_{ub}|^2$ GeV, which corresponds to a branching ratio $26|V_{ub}|^2$ for the total width $(0.51 \pm 0.02) \times 10^{-9}$ MeV of the B^0 meson [16]. The current experimental limit on the branching ratio of $B^0 \to \pi^- l^+ \nu$ is 3.3×10^{-4} [17]. We then extract the matrix element $|V_{ub}| < 3.5 \times 10^{-3}$, close to the value 0.003 given in the literature [16]. In fact, $|V_{ub}|$ can be extracted directly by comparing our perturbative predictions with the spectrum of the decay $B \to \pi l \nu$, once it is available. On the other hand, our work provides a nontrivial test of PQCD in decay processes involving heavy mesons.

We thank G. L. Lin, M. Neubert, G. Sterman, and Y. P. Yao for helpful discussions. This work was supported by the National Science Council of the Republic of China under Grants No. NSC84-2112-M194-006 and No. NSC84-211-M001-034.

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