Determination of the S-Wave Scattering Length in Pionic Deuterium with a High Resolution Crystal Spectrometer

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The pionic deuterium $3P-1S$ x-ray transition was measured with a quartz crystal spectrometer in combination with a cyclotron trap and charge coupled device detectors. The strong interaction shift and total decay width of the $1S$ level are ϵ_{1S} (shift) = 2.48 ± 0.10 eV (repulsive), Γ_{1S} (width) = 1.02 ± 0.21 eV, where the statistical and systematic errors were added linearly. They yield the total pionic deuterium S-wave scattering length: $a_{\pi^- d} = -0.0264(\pm 0.0011) + i0.0054(\pm 0.0011)m_\pi^{-1}$.

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Pionic deuterium is a simple and interesting strongly interacting three-body system and serves as a tool to study the pion-deuteron interaction at zero energy. The S-wave scattering length $a_{\pi^- d}$ may be related to the strong interaction shift (ϵ_{1S}) and the total decay width (Γ_{1S}) of the $1S$ level in pionic deuterium through the Deser formula [1]

$$-\epsilon_{1S} + i\frac{\Gamma_{1S}}{2} = \frac{4E_{1S}}{r_B} a_{\pi^- d}, \qquad (1)$$

where E_{1S} is the point Coulomb electromagnetic binding energy of the π^- in the $1S$ orbit and r_B the corresponding Bohr radius. An accurate measurement of a nP-$1S$ transition energy and width in pionic deuterium atoms therefore yields $a_{\pi^- d}$ in a straightforward way. Shift and width of the nP state are negligible. In pionic deuterium the electromagnetic corrections seem to be small [2] and have been omitted.

The strong interaction shift in pionic deuterium has been measured twice before. The two corresponding values are $4.8^{+1.6}_{-2.0}$ eV [3] and 5.5 ± 0.8(stat) ± 0.5(syst) eV [4]. The strong interaction broadening has never been determined experimentally.

The present experiment is part of our program to determine hydrogen and deuterium S-wave scattering lengths directly from measured $1S$ level shifts and widths. A strong interaction shift measurement in hydrogen was published in 1991 [5]. Our effort culminates in high resolution ϵ_{1S} and Γ_{1S} determinations with high statistics. Here we present our deuterium results, whereas the new hydrogen results will be reported separately [6].

The experiment was carried out at the Paul Scherrer Institute (PSI), at the new πE5 beam tuned to deliver 85 MeV/c π^- with an intensity of approximately 10^8 s^{-1} for a primary proton beam of 600 μA. The setup is presented in Fig. 1. It consists of a cooled deuterium target inside the cyclotron trap, a high resolution cylindrically focusing quartz crystal spectrometer of the reflection type, and position sensitive charge coupled device (CCD) x-ray detectors. Target, crystals, and detectors are located on a circle (Rowland's circle). As the details of the spectrometer were already described in [7], only modifications will be mentioned here.

Quartz crystals.—The resolution of the crystal spectrometer was improved further by replacing the spherically bent Si(111) crystals with cylindrically bent quartz (110) crystals. The energy resolution was obtained from the measurement of the $4F$-$3D$ pionic transition in beryllium at 2.84 keV which has a negligible natural width [see Fig. 2(a)]. The energy resolution is now approximately 0.7 eV FWHM (as compared to 4 eV with the Si crystals and 24 eV in the experiment of Ref. [4]). In order to increase the count rate, a second spectrometer plane was added on top and tilted with respect to the bottom plane (see Fig. 1).

New CCDs.—The three standard CCDs were replaced by four new deep depleted devices with surfaces enlarged fourfold, increasing the detection surface to approximately 17 cm^2. The two main advantages of the deep depleted CCDs are a 20% increase of efficiency at 3 keV and a further reduction of background.

Calibration.—For both energy calibration and stability, we again used 2.95 keV argon $K\alpha$ x rays obtained by the fluorescence of argon gas in the target by an x-ray generator. However, this time with the improved energy

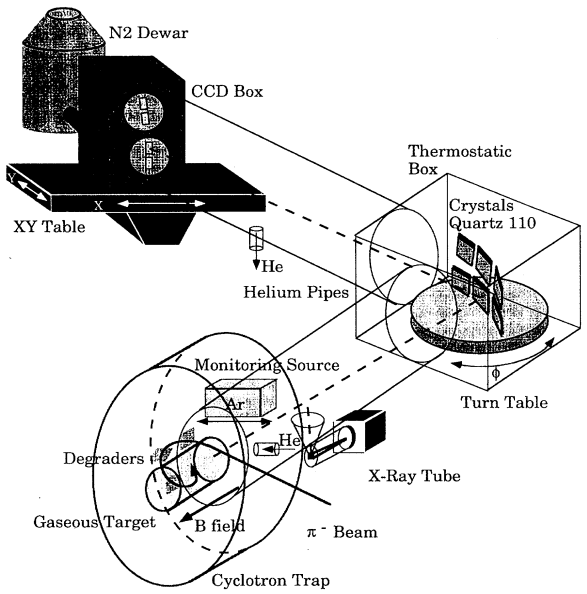


FIG. 1. Artist's view of the experimental setup. The target cell inside the cyclotron trap is filled with saturated D_2 vapor at 1 atm ($15\rho_{STP}$) or argon at 0.1 atm for calibration. During the run another argon target, irradiated with an x-ray tube, was inserted periodically for stability checks. The crystal table can be turned and the CCD table moved in both x and y directions. The crystals and CCDs are described in the text. The spectrometer elements are connected with pipes containing helium. The CCDs are shielded from the target with cement, paraffin, and lead. The spectrometer system is inside a temperature stabilized house.

resolution the two lines $K\alpha_1$ and $K\alpha_2$ are partially resolved [see Fig. 2(b)]. This doublet was recently remeasured at NIST with a precision for the $K\alpha_1$ energy of 6 ppm: $E_{K\alpha_1} = 2957.685 \pm 0.019$ eV [8]. Furthermore, the electronic gallium $K\alpha$ x rays were used in the third order of diffraction as an independent calibration check. In particular, the Ga $K\alpha_2$ line, corresponding to 3075 eV, is so close to the pionic deuterium energy that no spectrometer movement was necessary when changing lines.

Data acquisition.—New software was used for fully automated data acquisition, spectrometer control (through VME), and on-line analysis.

The CCDs were read out every 30 min. The event rate was about 6 h^{-1} compared to a background rate of approximately 3 s^{-1}. The cuts for the event selection correspond to our standard treatment of the CCD pictures described in [9] and allow an almost complete background suppression, resulting in a signal/background ratio of 6. After a correction for the tilt and the shift of the top plane and the projection of all CCD matrices on top of each other, the spectra of Fig. 2 were obtained. Figure 2(c) presents the measured pionic deuterium $3P$-$1S$ position spectrum. The $3P$-$1S$ transition at 3.07 keV was chosen rather than the $2P$-$1S$ transition at 2.59 keV because the CCD efficiency and x-ray transmission are increased at the higher energy. Yield measurements at $15\rho_{STP}$ showed that the $3P$-$1S$ yield was sufficient [10]. After taking into account the movement of the CCD table and the rotation of the crystal table (see Fig. 1), the comparison of the argon and deuterium peak positions gives the $3P$-$1S$ transition energy, which is shown in Table I (first line), together with the calculated electromagnetic energy value including radiative corrections and finite size effects. The difference between the two is the strong interaction shift ϵ_{1S}:

$$\epsilon_{1S} = 2.48 \pm 0.05(\text{stat}) \pm 0.05(\text{syst}) \text{ eV} \quad (\text{repulsive}). \tag{2}$$

This new shift value is over an order of magnitude more precise and nearly a factor of 2 smaller than the previous experimental results quoted above. It is compatible within 1.2σ with the value of Ref. [3] and disagrees by 2.3σ with the value given in Ref. [4].

The $4F$-$3D$ pionic beryllium line was fitted with a double Voigt function giving the instrumental line profile. The deuterium spectrum was deconvoluted with this profile and the remaining Lorentzian yielded the line width $\Gamma_{3P\text{-}1S} = 1.06 \pm 0.17$ eV. After applying a small correction and adding a systematic error for a potential Doppler broadening due to Coulomb deexcitation [11], we can present for the first time a measured value for the total decay width of the $1S$ state,

$$\Gamma_{1S} = 1.02 \pm 0.21 \text{ eV}, \tag{3}$$

where the error is mainly statistical. From the values (2) and (3) we can, with Deser's formula, determine the $\pi^- d$ S-wave scattering length

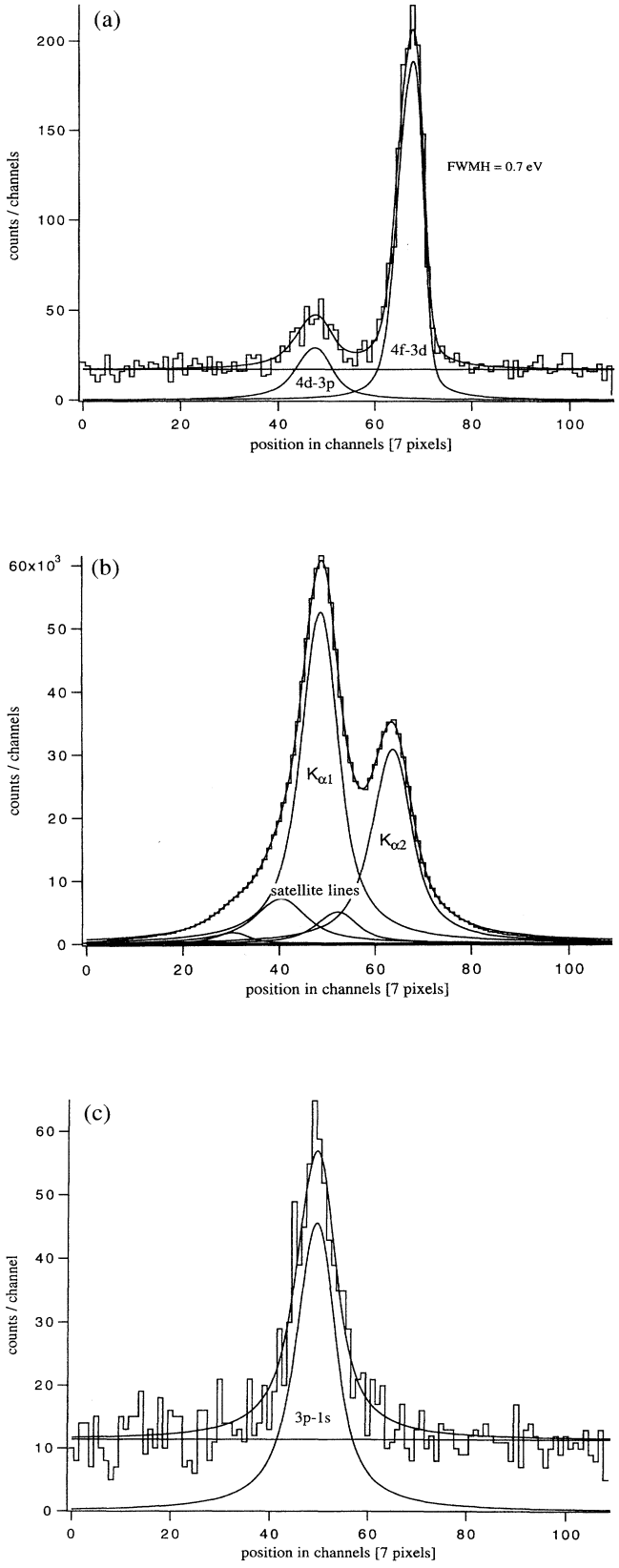
$$a_{\pi^- d} = -0.0264(\pm 0.0011) + i0.0054(\pm 0.0011) m_\pi^{-1}. \tag{4}$$

The S-wave scattering length of pionic deuterium provides an important link between the elementary πN interaction and the isoscalar π-nucleus interaction at threshold: The real part of $a_{\pi^- d}$ is related to the πN scattering lengths and the imaginary part leads to a direct determination of the π^- absorption amplitude on a proton-neutron pair.

The relation between $\text{Re}a_{\pi^- d}$ and the πN scattering lengths has been investigated by many authors [12–15]. $\text{Re}a_{\pi^- d}$ may be expressed as

$$\text{Re}a_{\pi^- d} = 4 \frac{m_N + m_\pi}{2m_N + m_\pi} b_0 + SS + DS + HC + AB, \tag{5}$$

where the first term describes the free scattering of the π^- on the two nucleons of the deuteron. The mass ratio arises from a πN to πd c.m. transformation. $b_0 = \frac{1}{3}(a_1 + 2a_3)$ is the isoscalar πN scattering length. The remaining terms describe the single scattering correction, double



scattering, higher order, and absorption corrections. Since b_0 is small (it vanishes in the chiral limit [16]), the contribution of the correction terms to $\mathrm{Re}a_{\pi^-d}$ is important, and their nonabsorptive part is determined mainly by the isovector πN scattering length $b_1 = \frac{1}{3}(a_3 - a_1)$ [14,15]. Based on Faddeev calculations and using the πN scattering lengths of Bugg, Carter, and Carter [17], Thomas and Landau [15] obtained $\mathrm{Re}a_{\pi^-d} = (-0.042 \pm 0.004)m_\pi^{-1}$. If one scales their result [18] with the frequently used πN scattering lengths of Koch and Pietarinen [19], one obtains $\mathrm{Re}a_{\pi^-d} = (-0.052 \pm 0.007)m_\pi^{-1}$. Comparison with our result [Eq. (4)] shows that this value is in total disagreement with experiment. From this we conclude that either our theoretical understanding of the pion-deuteron scattering length is incomplete or the uncertainties assigned to the πN scattering lengths in Ref. [19] are greatly underestimated—always assuming isospin symmetry to hold. Implications of this in combination with our results from the pionic hydrogen experiment [6] are discussed in Ref. [20] in the context of a test of isospin symmetry of the strong interaction.

The imaginary part of a_{π^-d} is related through the optical theorem to the π^-d total cross section at c.m. momentum $q \approx 0$. The correction for elastic scattering is negligible. By introducing the ratio of the partial decay widths for the ground state of pionic deuterium, $R = \Gamma(\pi^-d \to nn)/\Gamma(\pi^-d \to nn\gamma)$, the following relation can be established:

$$\mathrm{Im}a_{\pi^-d} = \tfrac{1}{3}k_n^3|g_0|^2(1 + 1/R), \qquad (6)$$

where g_0 is the amplitude for π^- absorption on a nucleon pair with isospin zero and $k_n = 2.607m_\pi$ the neutron momentum for the $\pi^-d \to nn$ decay at rest. Using the measured value $R = 2.83 \pm 0.04$ [21], we determine from Eq. (6) with our $\mathrm{Im}a_{\pi^-d}$ result (4) the isoscalar amplitude for π^- absorption at rest on a nucleon pair

$$|g_0| = (2.6 \pm 0.3) \times 10^{-2}m_\pi^{-2}. \qquad (7)$$

This determination is model independent. The g_0 values quoted in the literature are based on measured total

FIG. 2. (a) Position spectrum of $4F$-$3D$ and $4D$-$3P$ pionic beryllium lines. As the $4F$-$3D$ natural line width is very small, this spectrum was used to determine the energy resolution of our crystal spectrometer. (b) Argon $K\alpha$ x-ray position spectrum used for calibration. The two lines, $K\alpha_1$ and $K\alpha_2$, are separated by 2.13 eV. On the x axis, one channel corresponds to seven pixels = 157.5 μm. The fit function is a sum of six Voigt profiles ($K\alpha_1$, $K\alpha_2$, and satellite lines) with the resolution function obtained from the $4F$-$3D$ pionic Be line (a) folded in. The positions and intensities of the satellite lines were taken from the NIST spectrum. (c) Pionic deuterium $3P$-$1S$ x-ray position spectrum. After taking into account the spectrometer movements, the comparison of the peak position with the argon $K\alpha_1$ peak position of (b) yields the transition energy. The total decay width Γ_{1S} is obtained after deconvoluting the instrumental resolution function of (a).

TABLE I. Measured pionic deuterium $3P$-$1S$ transition energy, calculated electromagnetic value, and deduced strong interaction shift of the $1S$ state.

Pionic deuterium (eV)	
E_{3P-1S}, measured	3075.47 ± 0.05(stat) ± 0.05(syst)
E_{3P-1S}, calculated	
Point nucleus (Klein-Gordon)	3074.69[a]
Vacuum polarization $\alpha(Z\alpha)$	3.72
Nuclear and π^- finite size	−0.51[b]
Higher order radiative corrections	+0.04
Relativistic recoil	−0.02
Nuclear polarization	+0.03[c]
Total calculated	
Electromagnetic transition energy	3077.95 ± 0.01
Strong interaction shift ϵ_{1S}	2.48 ± 0.10

[a]Based on pion mass $m_\pi = 139.569\,95(35)$ MeV/c^2 [24].
[b]Radius of the deuteron $\langle r^2\rangle^{1/2} = 2.106(11)$ fm [25], charged pion radius $\langle r^2\rangle^{1/2} = 0.657(12)$ fm [26].
[c]Estimate by [27].

cross sections and hence must be extrapolated to zero energy. The recent $\pi^+ d \to pp$ experiment by Ritchie *et al.* [22] leads after Coulomb corrections to the value $g_0 = (2.3 \pm 0.1) \times 10^{-2} m_\pi^{-2}$ [23]. The result derived from the $np \to d\pi^0$ experiment of Hutcheon *et al.* [22] via detailed balance is not substantially different. All determinations are compatible with our result.

In conclusion, we have carried out a high precision x-ray experiment on pionic deuterium from which we determine the strong interaction level shift and (for the first time) the level width of the $1S$ ground state. The real and imaginary parts of the $\pi^- d$ S-wave scattering length $a_{\pi^- d}$ are deduced. The theoretical values of $\mathrm{Re}\, a_{\pi^- d}$, quoted in the literature, disagree with our result by about a factor of 2. From $\mathrm{Im}\, a_{\pi^- d}$ we determine the isoscalar amplitude for π^- absorption.

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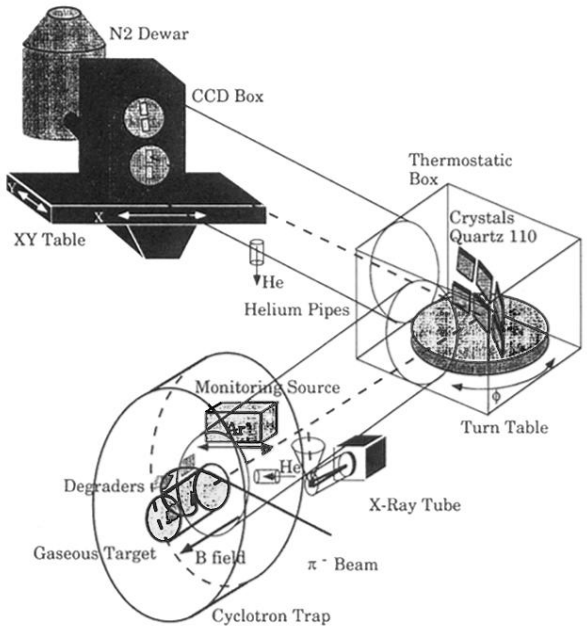

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