Origin of Pseudospin Symmetry

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A many-particle operator that affects a transformation to the pseudospin basis in heavy nuclei is identified. Both mean-field and many-particle estimates demonstrate that in the helicity-transformed representation the nucleons move in a finite-depth nonlocal potential with a reduced spin-orbit strength. Because of the close relation between the helicity and chirality operations, the results suggest that the pseudospin symmetry in heavy nuclei yields to the chiral symmetry of massless hadrons in the high energy region.

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(1) Introduction.—The pseudo spin-space concept in nuclear theory [1,2] refers to a division of the single-particle total angular momentum into *pseudo* ($\mathbf{j} = \tilde{\mathbf{l}} + \tilde{\mathbf{s}}$) rather than *normal* ($\mathbf{j} = \mathbf{l} + \mathbf{s}$) orbital and spin parts. Such a division is favored by the observed approximate degeneracy of pairs of single-particle states, $\{(l-1)_j, (l+1)_{j+1}\}$ with $j = l - \frac{1}{2}$, within the major shells of heavy ($A \geq 100$) nuclei, where the spin-orbit interaction is known to be strong [3,4]. [The latter stands in contrast to the relatively weak splitting of normal spin-orbit doublets, $\{l_j, l_{j+1}\}$ with $j = l - \frac{1}{2}$, in light ($A \leq 28$) nuclei.] By assigning new $\tilde{l}_j$ and $\tilde{l}_{j+1}$ labels to the $(l-1)_j$ and $(l+1)_{j+1}$ states (for example, a $\{d_{5/2}, g_{7/2}\}$ pair is assigned new $\{\tilde{f}_{5/2}, \tilde{f}_{7/2}\}$ "pseudo" labels), an advantage is gained because these paired states can then be interpreted as members of a weakly split pseudospin doublet. Indeed, the pseudospin and pseudospace quantum numbers appear to be reasonably well conserved, supporting a picture of heavy nuclei as systems with weakly broken (dynamical) pseudospin symmetry. The oscillator shell model of all pseudospin doublets within a major shell, augmented with a quadrupole-quadrupole residual interaction, leads to the many-particle pseudo-SU(3) theory for heavy deformed nuclei [1,4,5]. The decoupling of the pseudospace and pseudospin degrees of freedom has also been suggested as a possible explanation for the existence of identical bands in superdeformed nuclei [6].

Good pseudospin symmetry in heavy nuclei, while experimentally well corroborated and successfully used in numerous theoretical applications (see [5] for references), still lacks a sound microscopic justification. The usual understanding is based on the single-particle Hamiltonian of the oscillator shell model, namely, on the "accidental" result that deviations from the oscillator spectrum approximately follow a $2j(j+1) - l(l+1)$ dependence, which transforms into $\tilde{l}(\tilde{l}+1)$ under the *normal* → *pseudo* relabeling. Relativistic mean-field estimates were presented [7] in support of such a dependence in the limit of large nucleon numbers. Also, a unitary operator was proposed [8] which acts on the spin and angle variables and accomplishes the *normal* → *pseudo* relabeling within a given shell. Later this approach was revisited [9] and resulted in the introduction of another operator which is specifically designed for shell-model applications, being unitary only within the normal-parity subspace of the oscillator [4,8].

This paper shows that the pseudospin symmetry, which reveals itself on the single-particle (mean-field) level, has a microscopic origin which is related to the nature of the internucleonic forces, perhaps with roots in chiral symmetry. A microscopic transformation that is different from those mentioned above [8,9] while reducing to them when restricted to a single major shell, is shown to fulfill key requirements for the pseudospin transformation when applied to the nucleus as a whole.

(2) Microscopic pseudospin transformation.—To incorporate both the single-particle and many-particle aspects of the pseudospin picture, a microscopic operator that accomplishes the *normal* → *pseudo* transformation should be of the form

$$U_{\text{total}} = \prod_{i=1}^{A} U(\mathbf{r}_i, \mathbf{p}_i, \boldsymbol{\sigma}_i), \qquad (1)$$

where $\mathbf{r}_i$ stand for the position, $\mathbf{p}_i$ for the momentum, and $\boldsymbol{\sigma}_i$ for the Pauli spin matrices of the individual nucleons. The structure of $U(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma})$ is fixed by the following constraints: (a) $\tilde{\mathbf{l}}^2 = U\mathbf{l}^2U^{-1} = \mathbf{l}^2 + 2\mathbf{l}\cdot\boldsymbol{\sigma} + 2 = 2\mathbf{j}^2 - \mathbf{l}^2 + \frac{1}{2}$(this sets the transformation rule [8]); (b) $[U, \mathbf{j}] = 0$(rotational invariance); (c) $[U, \mathcal{P}] = [U, \mathcal{T}] = 0$(parity and time-reversal symmetry); (d) $UU^\dagger = U^\dagger U = 1$(unitarity and conservation of observables); and (e) $[U, \mathbf{p}] = 0$(translational invariance).

Once constraints (a), (b), (c), and (d) are applied, only three distinguishable choices for U remain:

$$U = (d \cdot d^\dagger)^{-1/2} d,$$
$$d = (\cos\theta\, r_0\mathbf{p} + i \sin\theta\, \mathbf{r}/r_0) \cdot \boldsymbol{\sigma}, \qquad (2)$$

where r_0 is a characteristic length, and due to the option of rescaling r_0, the value of θ can be fixed at $\pm\frac{\pi}{4}$, 0, or $\frac{\pi}{2}$. The first choice yields the boson annihilation $(+\frac{\pi}{4})$/creation $(-\frac{\pi}{4})$ operator form that is specifically designed for shell-model applications [9]. However, these two operators are unitary only within a subspace of normal

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parity states while in the unique parity subspace their action is undefined. When global unitarity is required, only two possibilities remain. The case $\theta = \frac{\pi}{2}$ corresponds to the $U_r = i\boldsymbol{\sigma}\cdot\mathbf{r}/r$ operator (henceforth called the r helicity) that was proposed in [8]. The $\theta = 0$ choice is the p helicity $U_p = \boldsymbol{\sigma}\cdot\mathbf{p}/p$. This is the only form that is *compatible with the constraint of translational invariance* and thus consistent with a realistic many-particle theory that is not confined to the shell-model approach. Additional arguments in favor of this choice are given below.

(3) Single-particle Hamiltonian and wave functions.— If (1) accomplishes the pseudospin transformation, then in addition to satisfying general constraints, it should decouple the spin and orbital degrees of freedom in the heavy nuclei. The applicability of the mean-field approach allows for a reasonable direct check on this by considering transformed single-particle Hamiltonians and wave functions. Corrections for center-of-mass motion are relatively small in heavy nuclei and therefore not expected to worsen such an argument.

For simplicity a spherically symmetric field is considered. In this case the Hamiltonian and its wave functions are given by

$$H = \frac{\mathbf{p}^2}{2M} + V(r) + W(r)\mathbf{l}\cdot\boldsymbol{\sigma}, \tag{3}$$

$$\psi_{njlm}(\mathbf{r}) = i^l R_{njl}(r)(\mathbf{Y}_l \otimes \chi)_{jm}, \tag{4}$$

where n is the radial quantum number (number of nodes), $\mathbf{Y}_l$ is a spherical harmonic, and χ is a Pauli spinor ($s = \frac{1}{2}$).

In a coordinate representation the p helicity has the following operator form:

$$U_p = -iK(\hat{l} - \Lambda - 1)^{-1} r(\boldsymbol{\sigma}\cdot\nabla), \tag{5}$$

where

$$K = \Gamma\left(\frac{\hat{l} + \Lambda + 2}{2}\right)\Gamma\left(\frac{\hat{l} - \Lambda}{2}\right) \times \left[\Gamma\left(\frac{\hat{l} + \Lambda + 3}{2}\right)\Gamma\left(\frac{\hat{l} - \Lambda - 1}{2}\right)\right]^{-1}$$

is unitary, $\hat{l} = \frac{1}{2}[(1 + 4\mathbf{l}^2)^{-1/2} - 1]$ has the orbital momentum as its eigenvalues, $\Lambda = \mathbf{r}\cdot\nabla = r\partial/\partial r$ generates shear, and $\Gamma(x)$ denotes the gamma function. The unitarity of K follows from the conjugation rules $\Lambda^\dagger = -(\Lambda + 3)$ and $\hat{l}^\dagger = \hat{l}$. Then (3) and (4) transform into

$$H_p = U_p H U_p^\dagger = \frac{\mathbf{p}^2}{2M} + \tilde{V}(r) + \tilde{W}(r)\mathbf{l}\cdot\boldsymbol{\sigma}, \tag{6}$$

$$\psi^{(p)}_{nj\tilde{l}m}(\mathbf{r}) = U_p \psi_{njlm}(\mathbf{r}) = i^{\tilde{l}} \tilde{R}_{nj\tilde{l}}(r)(\mathbf{Y}_{\tilde{l}} \otimes \chi)_{jm}, \tag{7}$$

where $\tilde{V}(r)$ and $\tilde{W}(r)$ are now *strongly nonlocal* functions given by

$$\tilde{V}(r) = K[V(r) - 2W(r) - (\hat{l} + 1)v(r)]K^\dagger, \tag{8}$$

$$\tilde{W}(r) = K[v(r) - W(r)]K^\dagger \mathbf{l}\cdot\boldsymbol{\sigma}, \tag{9}$$

with

$$v(r) = (\hat{l} - \Lambda - 1)^{-1}[rV'(r) - (\hat{l} + 2)rW'(r)] \times (\hat{l} + \Lambda + 2)^{-1}$$

(primes denoting derivatives) and

$$\tilde{R}_{nj\tilde{l}}(r) = -\Gamma\left(\frac{l + \Lambda + 3}{2}\right)\Gamma\left(\frac{\tilde{l} - \Lambda}{2}\right) \times \left[\Gamma\left(\frac{\tilde{l} + \Lambda + 3}{2}\right)\Gamma\left(\frac{l - \Lambda}{2}\right)\right]^{-1} R_{njl}(r).$$

Although (6) in its general form does not provide incontrovertible evidence for a reduction in the magnitude of the spin-orbit splitting (see below), the latter is likely to hold at low $\tilde{l}$ within the nuclear surface region so long as the effective value of the Λ operator exceeds $\tilde{l} - 1$.

To qualitatively understand the behavior of $\tilde{R}_{nj\tilde{l}}(r)$, observe that the U_p transformation involves three consecutive operations: a Fourier transform, a switch from l to $\tilde{l}$, and an inverse Fourier transform, which together determine the mapping for the radial function. This mapping generates the following universal behavior:

$$\tilde{R}_{nj\tilde{l}}(r) \propto \begin{cases} r^{\tilde{l}}, & r \to 0, \\ r^{-(l+3)}, & r \to \infty. \end{cases} \tag{10}$$

The standard $r^{\tilde{l}}$ dependence in the interior region follows because deep in the bulk of a heavy nucleus $\tilde{V}(r)$ is not expected to deviate significantly from the flat behavior of $V(r)$. The $r^{-(l+3)}$ asymptotic behavior means a more diffuse surface. This and strong nonlocalities in the surface region come with the p-helicity transformation.

(4) Dirac-Brueckner approach and the helicity transformation.—A relativistic extension of the Brueckner theory (see [10] for references) provides parameter-free microscopic predictions for both infinite and finite nucleon systems. While this approach gives a good description of nuclear matter, the gross features of finite nuclei (especially light species) are reproduced less well, but nonetheless much better than in nonrelativistic theories [11]. For this reason, results of Dirac-Brueckner nuclear matter calculations are used below for examining the p-helicity transformed two-body nuclear interaction, as well as the mean field, in heavy nuclei.

For a wide range of nuclear densities, including the saturation point, the nucleon-nucleon interaction in the infinite medium is approximated perfectly by a one-boson exchange potential (OBEP) with the boson parameters fitted to the Bonn model and the density-dependent effective nucleon mass M^* calculated in a self-consistent manner [10]. And to a very good approximation, the density-dependent self-consistent field has the same Lorentz structure as the free Dirac Hamiltonian. Consequently, a single-particle Hamiltonian *in the medium* commutes with the p helicity, and the *helicity transformation does not affect the single-particle energies.*

However, the two-body interaction changes dramatically. In the representation of plane-wave Dirac spinors for nucleon states, normalized to unity, the p-helicity operation is equivalent to $i\gamma^5 S$ when acting on right (ket) states. Here γ^5 is the usual product of Dirac matrices ($\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$), and S is a formal operation for switching the sign of the effective mass [$Sf(M^*, \mathbf{p}) = f(-M^*, \mathbf{p})$]. (Thus there is no difference between the chiral and the helicity transformations in the $M^* \to 0$ limit.) Since the γ matrices change sign under the γ^5 generated chiral transformation, and the OBEP is bilinear in those matrices, the *helicity transformation of the OBEP is reduced to changing the sign of* M^* in the momentum representation. This is easily accomplished in the two-nucleon center-of-mass frame and produces strongly incident-energy dependent and therefore nonlocal interactions. Because only a rough estimate for the potentials is required for this analysis, here these potentials are converted into local approximations by averaging over allowed values of the relative momentum q with an appropriate distribution of q at a fixed momentum transfer k. The localized helicity-transformed OBEP in the momentum space is found to converge rapidly in the shortwave region ($k > 2k_F$) to the initial potential, averaged with the same distribution. The values of the localized central part of the internucleon potential also coincide at $k = 0$ before and after the transformation in accordance with the helicity-invariance of the single-nucleon energy in the infinite medium.

The localized estimates for transformed single-particle potentials in coordinate space, shown in Figs. 1 and 2, were calculated in first order perturbation theory with respect to $\delta V(k)$, the localized difference between the transformed and initial OBEP. Unperturbed potentials were taken in the standard Woods-Saxon parametrization [12] with a slight adjustment for the radial dependence which allows for a simple analytic Fourier transform along with a quantitative fit. The estimate was done analytically using a zero-order nuclear density distribution of the same parametrization but with a lesser diffuseness [3] and a Skyrme-type low momentum expansion for $\delta V(k)$. Because of the strong nonlocality of transformed OBEP, the analytic formulas for single-particle potentials are more complicated than in a conventional scheme with Skyrme forces [13]. Basic complications and approximations of this analysis are the following: (a) d and f waves of the relative motion make an impact that is on the same order of magnitude as the normally included s and p waves; (b) $\delta G(k)$, a difference between the localized G matrix in the transformed space and the physical G matrix, coincides with $\delta V(k)$ in the first order because changes in the short wavelength region are small (see previous paragraph); (c) the ratio of proton and neutron densities is taken equal for all r, and Coulomb corrections are not considered; (d) M^* and k_F are fixed at their saturation point values [10].

Although the single-nucleon potentials shown in the figures are only rough local estimates for strongly nonlocal fields, they display several features that are characteristic of pseudospin symmetry. First, in accordance with Sec. 3, the transformation preserves the finite depth of the central potential and increases the surface diffuseness. And because the kinetic energy is conserved by the transformation, this in turn implies that the transformed radial wave functions associated with higher energy orbitals (which are most important for heavier nuclei) must be localized at a larger radial distance than for the corresponding conventional functions. Second, a minimum of

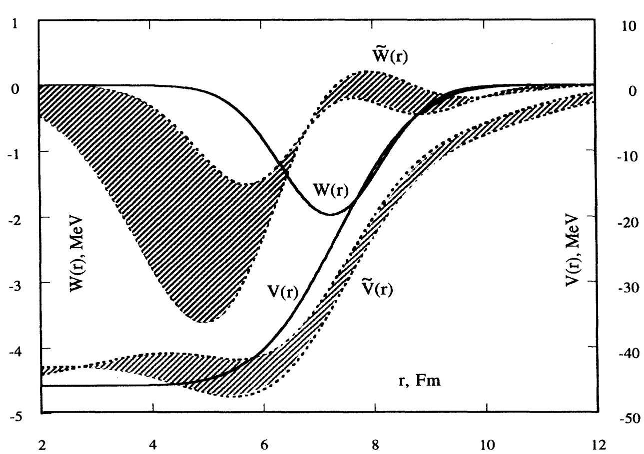

FIG. 1. Localized estimates for the neutron central and spin-orbit potentials of 208Pb before and after the helicity transformation (continuous lines and shaded areas, respectively). The two curves that define the borders of the shaded areas were determined by using different reasonable approximations for the relative momentum distribution in a finite nucleus.

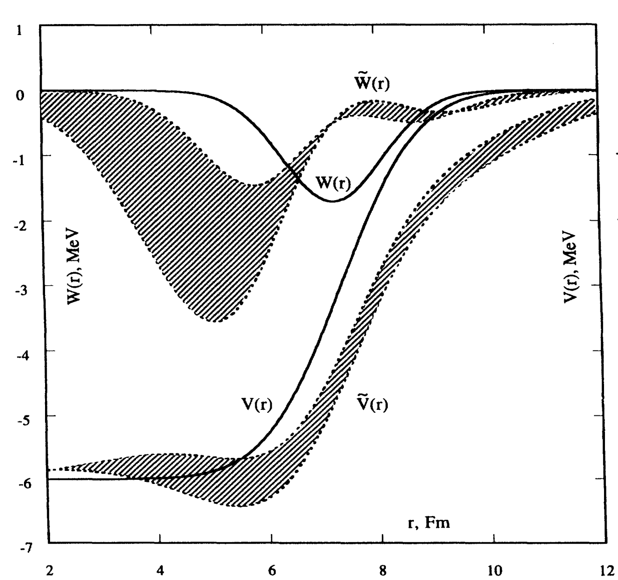

FIG. 2. The same as Fig. 1 but in this case for protons.

the spin-orbit potential, which is located in the surface region in the normal representation, gets shifted deeper into the bulk as a result of the helicity transformation. And from this it follows that the magnitude of the spin-orbit potential in the region where the wave functions are localized and which is primarily responsible for the interaction strength, exhibits a dramatic decrease. Also note that the effective *pseudo* spin-orbit interaction of the neutrons is more repulsive than one of the protons—in consonance with experiment [8].

(5) Conclusions.—The microscopic origin of the pseudospin symmetry is considered. The many-particle p-helicity operator is found to be the only one that generates the *normal* → *pseudo* relabeling of the spin and orbital momenta while satisfying all other global symmetry requirements. In addition, it is shown to transform wave functions in a physically reasonable manner and to effectively compensate for the single-particle spin-orbit interaction strength that is observed in the normal (not pseudo) picture.

The approximate independence of the single-nucleon spectrum in an infinite medium on the helicity transformation and the consistency of the microscopic estimates for the single-particle nuclear potentials with the Dirac-Brueckner calculations, is used to connect the pseudospin symmetry to the boson-exchange nature of nucleon-nucleon interaction. Based on the results of that analysis and because of the close relation (coincidence in the chiral symmetry limit) of the helicity and chirality operations, the goodness of pseudospin symmetry may be expected to increase with rising densities (or energy per particle) in hadronic systems, and actually yield to chiral symmetry in the region of asymptotic freedom.

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