$3\alpha + 3\alpha$ and $3\alpha + {}^{12}C$ Configurations in ^{24}Mg

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The recently observed resonance ($E_{c.m.} = 32.5$ MeV) in the 0_2^+-0_2^+ channel of $^{12}C + {}^{12}C$ collisions is predicted to be a weakly coupled structure of two three-α-particle states (0_2^+ state of ^{12}C). At the same time a well-known resonance ($E_{c.m.} = 29$ MeV) in the 0_1^+-0_2^+ channel is explained as of three α particles plus ^{12}C core structure. The band crossing model is shown to be valid also for inelastic scatterings to α-cluster states. A strong resonance is predicted in the 3_1^--0_2^+ channel in the same energy region. A relation to the linear chain configuration of six α particles is discussed.

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Recently, a broad resonance has been observed in inelastic scatterings to $^{12}C(0_2^+) + {}^{12}C(0_2^+)$ channel at $E_{c.m.} = 32.5$ MeV [1], which appears to indicate an existence of six-α-particle configurations in high excitation of ^{24}Mg, because the 0_2^+ state of ^{12}C is known to be of three-α-particle structure [2]. The enhancement at 90° observed around the resonance energy has been explained by the assumption of coherent resonant contributions of many partial waves, which is motivated by a possible existence of the rotational band with six-α-particle linear-chain configuration [3]. In this paper, however, a new interpretation is proposed, which predicts also six-α-particle configuration but with weakly coupled two three-α-particle configurations, which is apparently different from the linear chain one. As a byproduct, it explains a resonance in $^{12}C(0_1^+) + {}^{12}C(0_2^+)$ automatically, which was observed at $E_{c.m.} = 29$ MeV but has long been in a mist [4]. The latter resonance state is now predicted to have three-α-particle state plus ^{12}C core structure.

Analyses are made by extending the band crossing model (BCM) [5] to the dinucleus molecular configurations with intrinsic excitations of structure changes such as from the compact ground state to the spatially extended three-α-particle state of 0_2^+ in ^{12}C. The BCM was extremely successful in systematically explaining the resonances prominently observed in the inelastic scatterings to the collective 2^+ and 3^- states. A characteristic prediction of the model is enhancements of spin alignments correlating to the resonance peaks in the inelastic channels, which has been experimentally confirmed [6]. The BCM, however, is solely based on crossings of the molecular rotational bands due to the intrinsic spins of excited nuclei, whose stretched coupling to the orbital angular momentum give rise to an effective change of moment of inertia of the molecular band, as schematically shown in Fig. 1(a). One may naturally think that no such mechanism applies to cases of excitations with intrinsic spin zero, such as the 0_2^+ state in ^{12}C or ^{16}O, so one cannot expect any prominent resonance in the relevant channels. This is true if the moment of inertial of the excited molecular band is the same as that of the elastic one, which is approximately the case in the collective excitations. However, we know that the 0_2^+ state of ^{12}C is of a spatially extended three-α-particle structure, though not exactly with the linear chain configuration [2]. Therefore, interactions between ^{12}C nuclei with one or both in the 0_2^+ state are expected to be different from that between the ground state nuclei. More precisely, the range is longer than the latter's. The moments of inertia of the excited molecular bands thus are larger than that of the elastic one, as remarked a long time ago [5]. Hence, we can expect that the excited bands cross with the elastic one as in the cases with nonzero spins, and thereby resonances occur in the relevant inelastic channels.

Since the crossings are due to the changes in the interaction potentials, not to the angular momentum couplings in the cases with nonzero spins, a simple prediction of crossing spin is not at hand. We have to calculate carefully the interaction potentials between ^{12}C nuclei in their ground state as well as in their excited states. For this purpose, we calculate diagonal and coupling potentials for the ^{12}C-^{12}C system by the double folding model:

$$V_{ij,kl}(\mathbf{R}) = \int \rho_{ij}^{(^{12}C)}(\mathbf{r}_1)\rho_{kl}^{(^{12}C)}(\mathbf{r}_2) \times v_{NN}(E,\rho,\mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2)d\mathbf{r}_1\, d\mathbf{r}_2, \tag{1}$$

where $\rho_{ij}^{(^{12}C)}(\mathbf{r})$ represents the diagonal ($i = j$) or transition ($i \neq j$) nucleon density of ^{12}C. As for the internucleon interaction v_{NN}, we employ an effective nucleon-nucleon force DDM3Y (density-dependent Michigan 3 range Yukawa potential) [7] which is defined by

$$v(E,\rho,\mathbf{r}) = g(E,\mathbf{r})f(E,\rho),$$
$$f(E,\rho) = C(E)[1 + \alpha(E)e^{-\beta(E)\rho}], \tag{2}$$

where $g(E,\mathbf{r})$ is the original M3Y interaction [8]. Here, E is the bombarding energy per nucleon and r is the internucleon separation. We take $\rho = \rho_1(\mathbf{r}_1) + \rho_2(\mathbf{r}_2)$ for

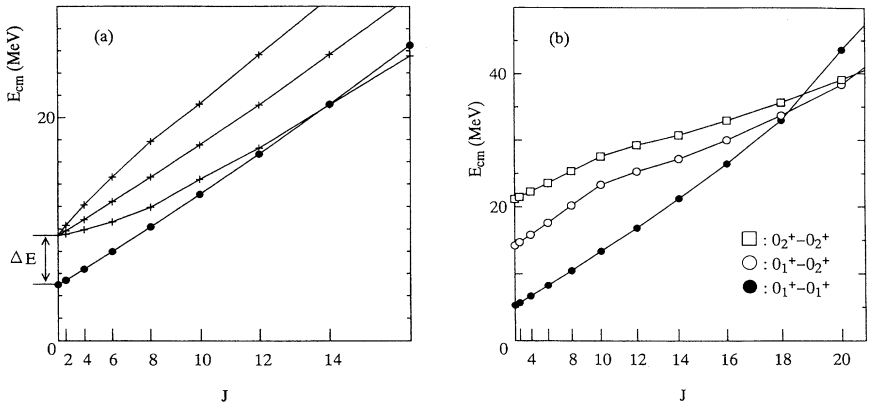

FIG. 1. (a) Schematic band crossing diagram corresponding to the collective 2^+ excitation, i.e., the moments of inertia of the excited bands are the same as that of the elastic molecular band, but the crossing occurs due to stretched angular momentum coupling. (b) Calculated molecular bands by the folding model with DDM3Y for the elastic and excited molecular bands with one or both ^{12}C in the 0_2^+ state. The crossings occur due to the changes of the moments of inertia in the excited bands by the spatial extended density distribution of the 0_2^+ state.

a nucleon at $\mathbf{r}_1$ in a projectile nucleus interacting with a nucleon at $\mathbf{r}_2$ in a target nucleus [8]. The inclusion of the density dependence is very important in such an extended state as the 0_2^+ one. We also employ realistic density distributions of ^{12}C nucleus in the ground state as well as in the excited states, together with transition densities between them, which are obtained by the resonating group method for three-α clusters and are shown to reproduce available electromagnetic properties of ^{12}C [9]. In the folding model, we neglect the nucleon exchange effects between ^{12}C nuclei except the so-called knock-on exchange which is represented by the zero-range term with the energy dependent strength in $g(E,\mathbf{r})$ [8]. This would be permissible in the present subject, at least not so crucial, because we are interested in high spin molecular states which locate in the tail region where the other exchange contributions are negligibly small. In Fig. 1(b), the calculated elastic molecular band is shown together with the excited ones with one or both ^{12}C excited to the 0_2^+ state. We readily see that the mutual-excited band crosses with the elastic one around the spin 18. Accordingly, we can expect a resonance with spin 18 correlatingly in the elastic and mutual-excited channels. It is also interesting to notice that the single-excited molecular rotational band crosses with the elastic one at a little lower energy than the mutual-excited band. Thus, we expect resonance to be observed also in the single-excited channel, but a little lower than 32.5 MeV, where the resonance is observed in the mutual-excited channel [1]. Actually, a resonance has been known at 29 MeV for a long time, but not yet well understood [4].

The members of the bands in Fig. 1(b) are potential resonance energies obtained by single-channel calculations; i.e., they are the zeroth order energies, which are a few MeV higher than the experiments. In order to get real resonance energies and resonance cross sections, we have to take into account coupling among the molecular configurations with proper scattering boundary conditions. We have performed coupled-channel calculations with the above realistic interactions and a phenomenological imaginary potential which is supposed to take into account other reaction processes, such as compound nucleus formation, not included explicitly in the present calculations. In the present calculations, we include the following channels: 0_1^+-0_1^+, 0_1^+-2_1^+, 0_1^+-3_1^-, 0_1^+-0_2^+, 0_1^+-2_2^+. We thus have no free parameters in the real part, but three parameters of Saxon-Woods form in the imaginary part of the optical potential. The parameters used here are $W_0 = 12.0$ MeV, $a = 0.40$ fm, and $R = 5.0$ fm at the observed peak energy ($E_{\text{c.m.}} = 32.5$ MeV), and at the other energies a and R are slightly and smoothly changed which give a global better fitting. Results are compared with the experiments in Fig. 2 for the resonances in the mutual-excited and single-excited channels. It is a great surprise that the resonances observed in two inelastic channels are well reproduced at the same time without adjusting any real interactions. Furthermore, a resonance is predicted in the same energy region and with the same total spin in the 0_2^+-3_1^- channel, which is shown in the bottom panel of Fig. 2 [10]. This is consistent with the band crossing diagram shown in the top panel. It is noticed that the resonance energies obtained in the coupled channel calculations are lower than the zeroth order energies given in the top panel. This is naturally expected to be so due to the couplings among the channels or among the resonance states. Analyses of dynamic polarization potentials in the channels are now being made, which are expected to give a quantitative explanation to the shifts of resonance energies. Another noticeable feature of the results is a possible coupling between two channels, 0_1^+-0_2^+ and 0_2^+-0_2^+. An enhancement is seen at 32.5 MeV in the 0_1^+-0_2^+ channel, correlating with the resonance in the 0_2^+-0_2^+ channel, and some weak enhancement is seen at 29 MeV in the 0_2^+-0_2^+ channel, correlating with the resonance in the 0_1^+-0_2^+ channel. With the same mechanism, a slight enhancement is obtained at 32.5 MeV in the 0_2^+-0_3^+ channel, shown in the bottom panel. It is interesting to investigate these correlations experimentally.

In the present model, the dominant configuration of the resonance at 32.5 MeV is a weakly coupled two ^{12}C

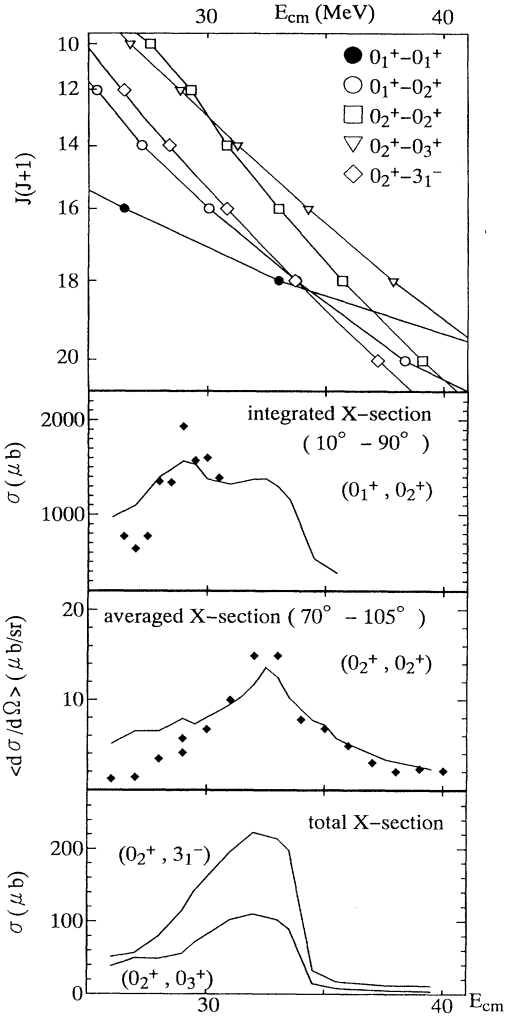

FIG. 2. The top panel shows the calculated molecular bands, including also 0_3^+ and 3_1^- excitations. The second panel gives a comparison of the present results with the experimental excitation function of the 0_1^+-0_2^+ inelastic scattering. In the third panel, the calculated excitation function of the 0_2^+-0_2^+ channel is compared with the experiment averaged over 70°–105°. Calculated angle-integrated excitation function of 0_2^+-3_1^- and 0_2^+-0_3^+ channels are given, though there are no data available to compare directly with them. As discussed in the text, the resonance positions are shifted down from those of single-channel calculations (the top panel), though the shift is small in the 0_2^+-3_1^- channel.

nuclei both in the 0_2^+ state, and that at 29.5 MeV is one ^{12}C nucleus in the 0_2^+ state weakly coupled with the other in the ground state. The orbital angular momentum l between two ^{12}C nuclei is equal to total spin J in the dominant component. In this sense, the state that the present model gives at 32.5 MeV seems to be different from a linear chain configuration, which is a superposition of all partial waves. Furthermore, the predicted resonance in the 0_2^+-3_1^- channel has a structure of three α particles plus 3^- excited ^{12}C core with the stretched angular momentum coupling, i.e., l being equal to $J - 3$, which is of no relevance to the linear chain configuration.

As is seriously discussed in Ref. [3], angular distributions are generally sensitive to reaction mechanisms, so we have calculated them in the present model. Examples of the results are shown in Fig. 3. In the top panel, the elastic scattering is compared with the experiment at the energy nearest to the resonance $E_{c.m.} = 32.5$ MeV [11]. In the middle panel, the angular distribution of the single-excited channel at $E_{c.m.} = 29.5$ MeV, nearly at the resonance $E_{c.m.} = 29$ MeV in the single-excited channel [4]. In the bottom panel, the angular distribution of the mutual-excited channel at the resonance energy $E_{c.m.} = 32.5$ MeV [12]. In these calculations the same parameters of the imaginary potential are used as in the excitation function calculations shown in Fig. 2. It is surprising that the angular distributions are well reproduced in all these three channels, though they are not so precisely reproduced as in the χ^2 fitting [3]. Those in off-resonance energies are also reproduced in a similar quality or in a slightly less quality. From the upper panel, one may infer that the present grazing partial wave is a little larger than the experimental one. Anyhow, a general trend is extremely good, including the enhancement at 90° in the mutual-excited channel, from

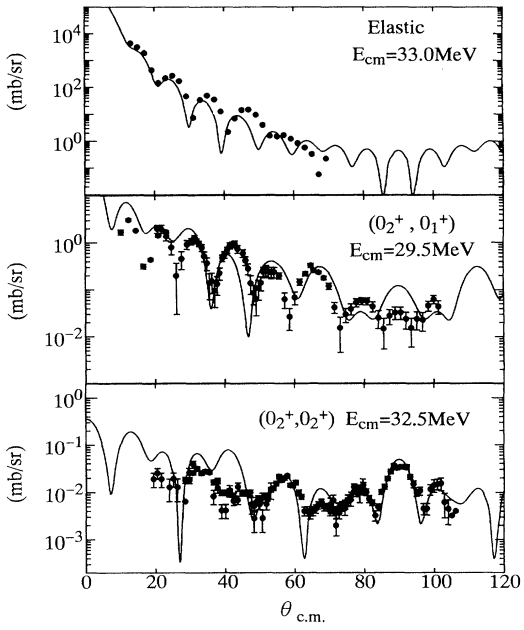

FIG. 3. Calculated angular distributions are compared with the available experimental data, the elastic scattering in the top panel, the single-excited inelastic scattering in the middle panel, and the mutual-excited inelastic scattering in the bottom panel. The parameters values used in the imaginary part of the optical potential are the same as those in the excitation function calculations. The real parts of the optical potential are calculated by the folding model without any adjustable parameter. No χ^2 fitting is made.

which the six-α-particle linear chain configuration is claimed. In the present calculation, the dominant partial waves at the resonance energy are grazing ones around $l = 18$. Of course, lower partial waves contribute appreciably, which are necessary for the reproduction of the enhancement at 90° as discussed in Ref. [3]. As is seen in Fig. 1(b) with open squares, a series of resonances appear to be similar to a rotational band whose slope might not be so different from that of the linear chain configuration of six α particles. But it should be noticed that the resonances in Fig. 1(b) are potential resonances in the 0_2^+-0_2^+ channel and thereby of a weak coupling structure with $l = J$, not of a strong coupling structure with the linear chain intrinsic configuration. It would be useful to remind one here that prominent resonances are expected to be observed around grazing partial waves in combinations of the stable light nuclei, considering the strongly absorptive nature of heavy ion interactions, which are discussed elsewhere by the number of open channels [13].

In summary, the resonance observed at $E_{\text{c.m.}} = 32.5$ MeV in the inelastic channel of $^{12}C + ^{12}C$ both in the 0_2^+ state is well understood by the BCM, i.e., by the crossing of the excited molecular band with the elastic one, which are calculated with DDM3Y nucleon-nucleon force and the realistic densities of ^{12}C nucleus. The structure of the resonance is predicted to be of weakly coupled two three-α-particle states, rather than the linear chain of six α particles. It is felicitous that a resonance is predicted in the single-excited channel, consistently with the resonance in the mutual-excited channel, and is in good agreement with the previously observed resonance at $E_{\text{c.m.}} = 29$ MeV. The structure of this resonance is now predicted to be of three α particles coupled weakly with the other compact ^{12}C nucleus. In addition, the present model predicts a resonance in the 0_2^+-3_1^- channel, which has a structure of weakly coupled three α particles traveling around the ^{12}C core in the 3^- excited state. Other resonances in the 0_2^+-2_2^+ and 2_2^+-2_2^+, etc. channels are also expected simply from a band crossing diagram, which are now under analysis. A systematic experimental study is strongly called for, which surely clarifies resonance mechanisms and enables us to comprehensively explore exotic nuclear structures such as multi-α-particle ones in high excitation of atomic nuclei.

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