Smooth Termination of Collective Rotational Bands

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(Received 15 November 1994)

Rotational cascades observed in neutron-deficient nuclei with $Z \approx 51$ are described as the first examples of smoothly terminating bands in heavy nuclei, since they show a continuous transition from high collectivity to a noncollective state. Excellent agreement between experiment and calculations in the Nilsson-Strutinsky cranking model is obtained for the ${}^{109}_{51}\mathrm{Sb}_{58}$ nucleus.

PACS numbers: 21.10.Re, 21.60.Ev, 23.20.Lv, 27.60.+j

A recurring theme in nuclear structure physics is the interplay between collective and noncollective degrees of freedom. Generally speaking, noncollective spectra are found at low energy in closed-shell nuclei and in nuclei with a few particles outside closed shells, since most states are excitations of the few valence particles in spherical j shells. When more valence particles are added, spectra become more collective, exhibiting rotational bands whose angular momentum is generated by small contributions from many particles. For nuclei with mass numbers 100–150, a minimum of ~10–15 valence particles (or, more generally, valence particles + valence holes) are needed before rotational bands are formed at low energy (e.g., [1]). With so few valence particles and with the core particles in filled j shells, it is evident that the angular momentum (spin) in the intrinsic configuration is limited [2]. Particles in the orbitals near the Fermi surface of this mass range can each contribute a spin of $3-4\hbar$, on average, and the limiting total spin is typically $40-50\hbar$ [2].

A very interesting situation arises when a particular configuration evolves continuously from high collectivity at low spin to where it becomes the state of highest spin available for the configuration. We define this as a *terminating band.* In this Letter we give evidence that the rotational bands recently identified in ^{109}Sb [3,4] are the first examples of collective bands in heavy nuclei which gradually exhaust the spin available in their intrinsic configurations. Previously, high-spin terminating *states* of maximal spin have been observed, e.g., in ^{158}Er [5], but their valence configurations are not the same as those of the collective bands which dominate the spectroscopy at lower spins. As illustrated in the upper panels of Fig. 1, the terminating configurations in this case lie well above the yrast states at low to medium spins, and are thus difficult to observe below the terminating spin. (The *yrast states* are defined as the set of states having the lowest excitation energy for a given spin.) Furthermore, the states above the yrast line interact with other states, so the configurations are not pure.

A detailed discussion of terminating states in heavy nuclei was given in Ref. [2], where calculations with the cranking model were presented for ${}^{158}_{68}\mathrm{Er}_{90}$. This nucleus can be viewed as a ${}^{146}_{64}\mathrm{Gd}_{82}$ doubly-closed-shell core + 12 valence particles. The terminating configurations in ^{158}Er are yrast only over a short spin range (see upper panel of Fig. 1), and therefore to follow them in the calculations over a longer spin range, we must somehow label them so that they can be distinguished from states

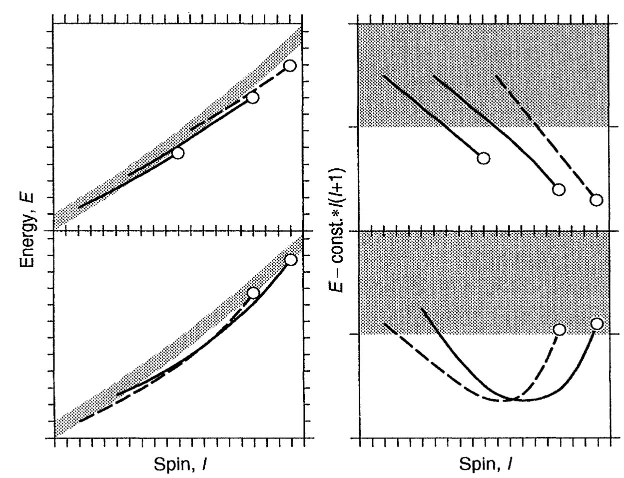

FIG. 1. Schematic illustration of rotational band termination for two different cases. For each case, excitation energy is plotted vs spin on the left, while an average rigid-rotor energy has been subtracted from the excitation energy (and the scale has been magnified) on the right. Terminating states are marked by circles. The units on the horizontal and vertical axes correspond approximately to $1\hbar$ and 1 MeV, respectively. Shaded areas represent states in the region above a smooth average yrast line. For terminations such as occur in ^{158}Er (top), the states below the terminating spins enter the shaded region and interact with other states. However, for terminations such as occur in ^{109}Sb and nearby nuclei (bottom), the rotational bands are below the shaded region over a large spin range and terminate smoothly.

0031-9007/95/74(20)/3935(4)$06.00

of lower energy. In Ref. [2] (see also [6]) rotating harmonic oscillator wave functions were used as basis states, and the configurations were labeled by the number of particles in different N shells (or more properly, N_{rot}). The terminating states predicted in ^{158}Er by this method have since been identified experimentally and a detailed understanding of the noncollective states in the $I = 40$–$50\hbar$ spin range has now been achieved [5].

Using essentially the same formalism, with the parameters of Refs. [7,8], we have performed theoretical calculations for $^{109}_{51}\mathrm{Sb}_{58}$, which can be viewed as a doubly-magic $^{100}_{50}\mathrm{Sn}_{50}$ core + 9 valence particles. In this nucleus, we obtain rotational bands of the type illustrated in the lower panels of Fig. 1. The results, which Fig. 2 shows in more detail, indicate that over a large spin range the bands with two $g_{9/2}$ holes are yrast relative to other states with the same parity and signature. From the configurations listed in the figure, we conclude that these bands have maximal spins of $I^{\pi} = 41.5^{-}$, 43.5^{+}, and 44.5^{+}, obtained by summing the maximal spins available from the constituents, taking into account the Pauli principle.

If we try to follow the terminating configurations of ^{109}Sb over a more extended spin range, say, down to $I = 20\hbar$, a new problem arises in the way we have labeled the configurations. The configurations are identified by a specific number of holes in the $g_{9/2}$ subshell and particles in other $N = 4$ subshells, $g_{7/2}$, $d_{5/2}$, etc., but these subshells all belong to the same principal shell of the rotating oscillator, so the distinction between configurations with 0, 1, or 2 holes in $g_{9/2}$ cannot be made as described above. However, at low deformation where the subshell of highest j (in this case $g_{9/2}$) is distinct from the other subshells, it is fairly straightforward to identify the $N = 4$ high-j orbitals by their single-particle wave functions. This can be done by picking out the orbitals which have the largest value of $\langle j^2 \rangle$ at a small rotational frequency and then following them continuously to high frequency.

In the three calculated configurations of ^{109}Sb shown in Fig. 2, the last spin units before the terminations are expensive to obtain in terms of energy, as seen from the upsloping E-vs-I curves in Fig. 1. Because of the very low energy of these configurations in the spin range 6–$10\hbar$ before termination, the high cost of building the terminating states can be absorbed while they remain close to yrast. Therefore over a large spin range these terminating configurations are below any other state with which they could interact. Consequently, their energy changes in a smooth way with increasing spin, i.e., the terminations are *band* terminations in which the spin available from the constituent valence nucleons is gradually exhausted.

The question now is whether such configurations can be identified with the observed rotational bands in ^{109}Sb [3,4]. Three bands were discovered in ^{109}Sb with the 8π spectrometer at Chalk River and were observed to very high rotational frequencies. Subsequently, the spectroscopy of ^{109}Sb was investigated at higher sensitivity with the early implementation version of the GAMMASPHERE detector array [4]; the three bands were confirmed, with the addition of a single high-energy transition to each, and two new band sequences were found. Furthermore, added knowledge of the decay scheme resulted in more information concerning the most probable spins of the band members.

The three bands most strongly populated in these experiments are drawn in Fig. 2 in a similar way as the calculated bands, with the spins proposed in Ref. [4]. With these spin values, it is striking how well theory

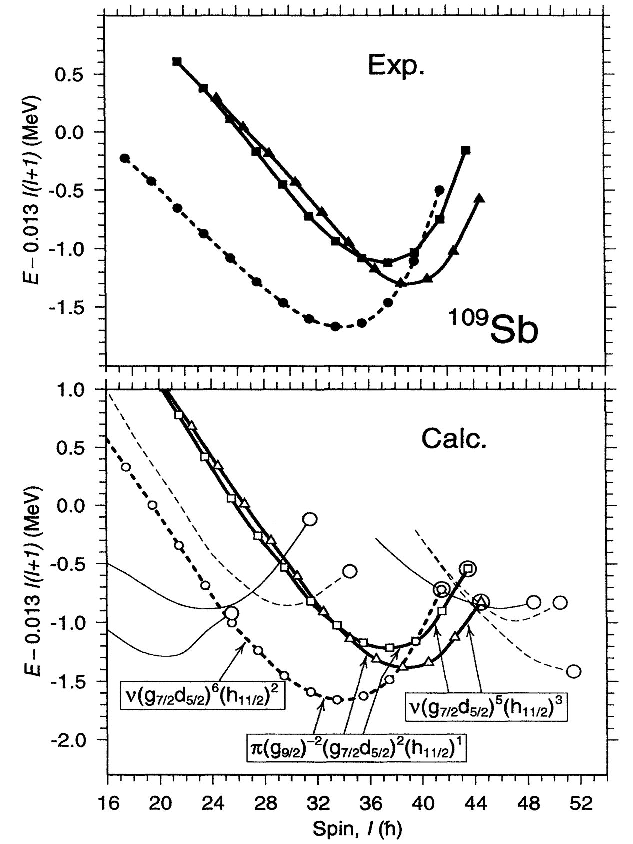

FIG. 2. Comparison between the three lowest calculated and observed rotational bands in ^{109}Sb. The same smooth reference has been subtracted in both cases, namely, $E_{\mathrm{ref}} = 0.007(158/A)^{5/3} I(I + 1)$ [5], representing rotation of a rigid body having a prolate deformation of $\varepsilon \approx 0.25$ ($r_0 = 1.2$ fm). Dashed (solid) lines indicate negative (positive) parity. The calculated band which we compare to experiment (thicker lines, with configurations indicated) are based on two-particle–two-hole (2p-2h) proton excitations across the $Z = 50$ gap (see text). The bands shown at higher spins are based on 3p-3h excitations and those at lower spins on 1p-1h and 0p-0h excitations. Terminations are indicated by large open rings. The spins of the observed bands have been assigned as explained in Ref. [4]; excitation energies have been chosen to match the calculated values, which lie within the range estimated from the experimental decay properties.

and experiment agree. The comparison suggests that all three of these bands have indeed been observed to their respective terminations. Band 1 is then identified with the negative-parity (dashed) band that terminates as $83/2^-$, and bands 2 and 3 with the positive-parity pair of signature-partner bands terminating at $87/2^+$ and $89/2^+$, respectively.

This identification is supported by evidence independent of spin assignments. The dynamical moment of inertia, $\mathcal{J}^{(2)} = (dE_\gamma/dI)^{-1}$, measures how the transition energies change with increasing spin, and is extracted directly from experiment. Obtained from the second derivative of the total energy with respect to spin, this quantity is a sensitive indicator of nuclear behavior. A potential hazard in the theory is that any spurious effects which ensue (due to interpolation inaccuracies, for example), even if small, are magnified when calculating the dynamical moment of inertia. It thus becomes very difficult to obtain a smooth behavior of $\mathcal{J}^{(2)}$ for the calculated bands. Note also that the calculations should be regarded as realistic only for high frequencies, $\hbar\omega \geq 0.7$–0.8 MeV (cf. Ref. [3]), since pairing correlations are neglected.

Experimental and theoretical $\mathcal{J}^{(2)}$ values are compared in Fig. 3; in one case, the theoretical values are taken directly from calculations, but in all other cases, the values are obtained from a polynomial in $I(I+1)$ fitted to the calculated energies. In this comparison, one should note that in all other mass regions where smooth rotational bands have been identified to such high spins, the $\mathcal{J}^{(2)}$ moments of inertia extracted from experimental transition energies lie within 30% of the rigid-body values. It is evident from Fig. 3 that ^{109}Sb (as well as some other nuclei nearby) is a special case, because the $\mathcal{J}^{(2)}$ value at the top of the band amounts only to $\sim 1/3$ of the rigid-body value. With this in mind, we find it remarkable that the experimental and smoothed calculated values agree so well, lying within $\simeq 1\hbar^2/\mathrm{MeV}$ of each other in the critical high-frequency region. This agreement is obtained with no fitting of parameters; the only parameters which enter are standard liquid-drop [7] and single-particle parameters [8]. Indeed, the fact that the $\mathcal{J}^{(2)}$ values become so small indicates that significant structural changes are occurring; it signifies that the total spin in the configurations is being exhausted.

It is interesting how the experimental and the calculated transition energies cluster in Fig. 3. In the calculation, we note that the bands behave according to the spin remaining to be aligned in the configuration, $I_{\max} - I$. This gives definite suggestions about the spins for the observed bands. These values agree with those used in Fig. 2.

It has previously been demonstrated [2] that with a number of valence particles distributed over high-j shells, a terminating state having a relatively low energy is created, as in ^{158}Er. This state is noncollective, with the spin vectors of the valence particles quantized along the rotation axis. The valence particles can be visualized

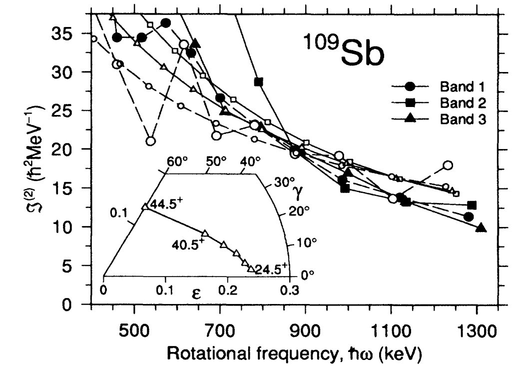

FIG. 3. Comparison between observed (filled symbols) and calculated $\mathcal{J}^{(2)}$ moments of inertia for the three bands shown in Fig. 2, using the same symbols and line types as in that figure. The calculated values are smoothed by fitting a four-parameter $I(I+1)$ expansion to the 13 highest calculated energies in the band. For one of the bands the unsmoothed values are also shown (larger symbols). Inset: plot of the calculated intrinsic shape of the ^{109}Sb nucleus as a function of spin, in steps of $4\hbar$, for the rotational band predicted to lie lowest in energy at the highest spins observed. The shape is defined by ε and γ coordinates, representing quadrupole stretching and nonaxial deformation, respectively. At intermediate spins the calculated shape is approximately prolate deformed with axial symmetry ($\varepsilon \approx 0.24$, $\gamma \approx 0°$), the rotation occurring about a nonsymmetry axis. With increasing spin it gradually moves away from being prolate, becoming oblate noncollective ($\gamma = 60°$) at termination. Results are very similar for the two other bands which we compare to experiment.

as rotating near the equator of the nucleus, making an oblate state. But unlike ^{158}Er, the collectivity of bands at moderate spin in ^{109}Sb involves both particles and *holes,* since it depends on particle-hole excitations across the $Z = 50$ gap. In this way the number of valence particle + valence holes becomes large enough to polarize the whole nucleus at moderate spin, resulting in collective rotation about an axis perpendicular to the symmetry axis, where the intrinsic shape is deformed prolate with approximate axial symmetry. At higher rotational frequencies, the individual spin vectors will become increasingly aligned along the rotation axis. Eventually the spins of the valence particles will be fully quantized along the axis, leading to a noncollective oblate state, with the "rotation axis" coinciding with the new symmetry axis, as illustrated in the inset to Fig. 3. Under these circumstances, however, alignment of the valence holes is unfavorable. Consequently, it occurs only at very high rotational frequencies and at a high cost in energy. This leads to increasingly large transition energies, implying small $\mathcal{J}^{(2)}$ moments of inertia close to the terminations.

Rotational bands showing smooth terminations are expected in the region of nuclei with valence particles outside ^{100}Sn. For nuclei with fewer valence particles than ^{109}Sb (e.g., ^{108}Sn [9] or ^{106}Sn [10]), lower terminating spins in the two-particle–two-hole (2p-2h) bands are expected; however, the collectivity might not be as well developed. Calculations predict that if several valence particles are added beyond $A = 109$, the terminating states in multiparticle-two-hole configurations lie higher above the yrast line and have much higher spins than in ^{109}Sb, and consequently are difficult to observe (cf. ^{115}I [11]). In ^{109}Sb, the number of valence particles is large enough to make the bands collective at low and intermediate spin, but small enough that the terminating states are experimentally reachable. Such a compromise seem necessary for the formation of smooth, terminating bands in heavy nuclei. Elsewhere, calculations suggest that nuclei with 10–12 valence particles outside a $^{90}_{40}\mathrm{Zr}_{50}$ core have bands which are yrast over a large spin range up to their termination. In principle, similar bands can occur in nuclei close to ^{154}Dy, with typically two holes and 10 particles outside a ^{146}Gd core, except that in these nuclei interactions between the $N = 4$ orbitals which lie above the $Z = 64$ gap make the terminations less well defined [12]. Beyond the region of nuclei which can be studied to high spins with current experimental techniques, nuclei with $Z \sim 58$ and $N \sim 84$ are promising candidates.

In summary, we have shown how the collective bands observed in ^{109}Sb and in neighboring nuclei can be understood as rotational bands which terminate in a smooth manner. They illustrate in a concrete way the interplay between collective and noncollective states, particularly the effect of finite particle number on the structure of nuclei at high spin. The termination features are different from those which have previously been observed in heavy nuclei. In further studies of these bands, it would be important to determine their spin values and to measure transition matrix elements near termination.

We wish to thank Ramon Wyss for input into Woods-Saxon cranking calculations which we performed, substantiating the modified-oscillator result reported here, and David Ward for valuable discussions. This work is supported by the Swedish Natural Science Research Council, AECL Research, the Canadian Natural Sciences and Engineering Research Council, the U.S. National Science Foundation, the U.K. Engineering and Physical Sciences Research Council, and NATO.

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