Electron-Electron Collisions and a New Hydrodynamic Effect in Two-Dimensional Electron Gas

R. N. Gurzhi, A. N. Kalinenko, and A. I. Kopeliovich
B. Verkin Institute for Low Temperature Physics and Engineering, National Ukrainian Academy of Sciences, 47, Lenin Avenue, Kharkov 310164, Ukraine
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Some characteristic features of momentum relaxation related to electron-electron collisions in the two-dimensional degenerate electron gas have been analyzed. It has been shown that, along with the Poiseuille flow, a new effect of a hydrodynamic type due to the one-dimensional diffusion of carriers exists in two-dimensional high-mobility wires. It is this effect that seems to be revealed in the experiments reported by Molenkamp and de Jong [Phys. Rev. B **49**, 5038 (1994)]. The sensitivity of the new effect to weak magnetic fields may facilitate its experimental identification.

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Recently, Molenkamp and de Jong [1] have reported that they found a hydrodynamic momentum transport mechanism in the electron gas, the mechanism having been predicted earlier by one of the authors of this paper [2]. The temperature dependence of the electrical resistivity of a two-dimensional high-mobility wire had a minimum. This result was discussed by the authors of the experiment using the theory of Ref. [2]. The latter predicts the appearance of a viscous Poiseuille flow in the three-dimensional electron gas because of frequent normal collisions, under the conditions

$$l_{ee} \ll d^2/l_{ee} \ll l_V, \tag{1}$$

where l_{ee} is the mean free path (MFP) for normal collisions, d is the sample thickness, and l_V is the electron MFP for bulk collisions without quasimomentum conservation. The aim of the present communication is to show that in the degenerate two-dimensional electron gas (2DEG) hydrodynamic phenomena of principally another type are possible, and these phenomena, most probably, were observed in the experiment of Ref. [1].

We shall start with discussing some features of electron-electron relaxation in the 2DEG that, as we believe, are essential for understanding a wide variety of effects studied experimentally. Afterwards, we shall discuss hydrodynamic phenomena.

Electron-electron collisions in the 2DEG are, generally speaking, small-angle processes. A characteristic scattering angle with a random choice of a partner, as follows from the energy and momentum conservation laws and the Pauli principle, has the order T/ε_F, where T is the temperature and ε_F is the Fermi energy. The exception is the collision of electrons with opposite momenta, i.e., $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{0}$, whose contribution to the 2DEG is not small [3]. In this case the pair of final states with zero total momentum is rotated in an arbitrary manner, so the scattering angle is arbitrary. Such collisions, however, by virtue of the symmetry of initial states, result in the relaxation of even-momentum distributions only. Thus it becomes clear that in the 2DEG there exist two substantially different relaxation MFP's: l_s and l_a for even- and odd-momentum distributions, respectively, with $l_s \ll l_a$. The MFP $l_s \simeq l_{ee} \sim (\varepsilon_F/T)^2$ is determined by collisions of electrons with almost opposite momenta $|\mathbf{p}_1 + \mathbf{p}_2| \lesssim (T/\varepsilon_F)p_F$. These collisions, as long as the momenta of the colliding electrons differ from being opposite, affect the odd relaxation. It appears that their contribution here is of the same order as that for small-angle processes with $|\mathbf{p}_1 + \mathbf{p}_2| \simeq p_F$. That is why the corresponding transport MFP $l_a \simeq l_{ee}(\varepsilon_F/T)^2$ [4,5]. The above concerns smooth angle distributions that noticeably vary at angles of the order of unity. The difference between the symmetric and antisymmetric relaxation decreases for a sharp distribution [6,7].

In order to investigate these questions we shall resort to the linearized electron-electron collision operator $J\{\chi\}$. The integral over energy of this quantity describes the angular relaxation. The result can be represented as

$$\int J\{\chi_{a,s}(\theta)\}d\varepsilon_1 \simeq \int_0^{2\pi}\int_0^{2\pi} V(\psi,\varphi)\,(\hat{D}_\psi^{\pm}\hat{D}_\varphi^{-})^2 \times \chi_{a,s}(\theta)d\psi\,d\varphi\,, \tag{2}$$

where

$$V(\psi,\varphi) = \int W_{1,2;3,4}n_1n_2(1-n_3)(1-n_4) \times \delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4),$$

$$\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3\,d\varepsilon_4\,,$$

$$\hat{D}_\gamma^{\pm}\{f(\theta)\} = \tfrac{1}{2}[f(\theta+\gamma/2) \pm f(\theta-\gamma/2)].$$

Here the nonequilibrium correction to the distribution function is represented as $(-\partial n/\partial\varepsilon)\chi$, the subscripts s and a denote the symmetric and antisymmetric parts of the function χ (in the operator $\hat{D}_\gamma^{\pm}$ the superscripts + and − correspond to s and a), $n(\varepsilon)$ is the equilibrium Fermi distribution function, and W is the squared matrix element of the electron-electron interaction. We have assumed that the function χ depends only on the angle θ, which characterizes the point position on the Fermi surface [taking into account the energy dependence of χ, as estimates show, leads only to the appearance of a logarithmic factor of order $\ln(\varepsilon_F/T)$, as well as in

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Chaplic's formula [9] for energy relaxation]. The angle φ between the momenta of the initial $\mathbf{p}_1$ and the final $\mathbf{p}_3$ states is the measure of the momentum transferred on a collision, $\sin\varphi/2 = |\mathbf{p}_1 - \mathbf{p}_3|/2p_F$. The angle ψ between $-\mathbf{p}_1$ and $\mathbf{p}_2$ characterizes the total momentum of the pair of colliding electrons, $\sin\psi/2 = |\mathbf{p}_1 + \mathbf{p}_2|/2p_F$.

It is easy to see that $V(\varphi, \psi) \sim \exp(-\psi\varphi\varepsilon_F/T)$ for $\varphi\psi \gg T/\varepsilon_F$. This means that the kernel V is exponentially small outside the region $\varphi\psi \leq T/\varepsilon_F$. This property of the kernel V suffices for further calculations. For the even function in (2) the quantity $\hat{D}_\psi^+\chi_s$ always has the order of χ_s, and the finite difference value $\hat{D}_\varphi^-\chi_s$ is not small only for $\varphi \geq \theta_0$, where θ_0 is a characteristic angle scale of the $\chi(\theta)$ variation. Thus the region of the two-dimensional integration for an even function is bounded by the two conditions $\varphi \geq \theta_0$ and $\varphi\psi \leq T/\varepsilon_F$. For the odd function χ_a in (2) one also should take account of the additional restriction imposed by the factor $\hat{D}_\psi^-$. As a result, we obtain

$$l_s(\theta_0) \simeq l_{ee}, \quad l_a(\theta_0) \simeq l_{ee}[1 + (\theta_0/\theta_T)^4], \quad \theta_T = (T/\varepsilon_F)^{1/2}. \tag{3}$$

Thus, for the typical angles $\theta_0 \gg \theta_T$, the antisymmetric distributions relax slower than symmetric ones.

In what follows, it will be important to understand how the above features of relaxation processes affect the structure of current states in the coordinate space. If one neglects inefficient collisions with the scattering angle $\varphi \simeq T/\varepsilon_F$, then the process of current propagation is determined by colliding electrons with opposite momenta. As a result of the collision of a nonequilibrium electron $\mathbf{p}$ with an equilibrium one $-\mathbf{p}$, there appear (a) a "hole" with momentum $-\mathbf{p}$, which moves in the direction contrary to the initial electron and, therefore, carrying the same electric current; (b) a pair of electrons with opposite momenta, rotated through an arbitrary angle and, obviously, giving no contribution to the current. (By a hole we mean the absence of an electron in the appropriate state.) Further collisions turn the hole into an electron moving in the same direction as the initial one. Thus such collisions do not change the current, whereas the charge transport process is a one-dimensional diffusion of carriers with the step l_s. Collisions of electrons whose momenta are directed not too much opposite do affect the movement direction of such an electron-hole carrier, but a noticeable change (by the angle $\simeq\theta$) requires a time of order $\tau_a(\theta) = v_F^{-1} l_a(\theta)$, where v_F is the Fermi velocity.

It follows from what has been said above that in a 2DEG wire of width d the character of charge transfer depends on the ratio of the diffusion trajectory length before the collision with the boundary d^2/l_s to the antisymmetric relaxation MFP l_a.

(1) Poiseuille flow is established provided that $d^2/l_s \gg l_a$. During this process at distances of the order $(l_s l_a)^{1/2} \ll d$ an odd-momentum drift distribution function $\chi(\mathbf{p}) = \mathbf{u} \cdot \mathbf{p}$ becomes established, $\mathbf{u} = \mathbf{u}(z)$ being the velocity of the ordered propagation of quasiparticles, parallel to the wire boundary. The transport MFP is $l_{\rm tr} \simeq d^2/l_s$, while the corresponding electrical resistivity is $\rho \sim T^{-2}$. These conclusions can be easily reached when solving the kinetic equation by expanding over small spatial gradients (actually, this is the Chapman-Enskog method applied to the quasiparticle gas [2]). The electron gas viscosity is proportional to the MFP $l_s \simeq l_{ee}$, but the expansion is convergent if the inequality $d^2/l_s \gg l_a$ is fulfilled. Thus, because of the existence of the two relaxation MFP's l_s and l_a that describe the normal collision effect, the conditions for the existence of the viscous Poiseuille flow are changed [cf. condition (1)],

$$l_a \ll d^2/l_s \ll l_V. \tag{4}$$

(2) In the opposite limit case, where $d^2/l_s \ll l_a$, most electrons lose their momentum at the boundary before the antisymmetric relaxation processes are revealed. The electric current is determined by a small group of grazing carriers propagating at small angles (of order θ_0) to the 2DEG boundary, i.e., the nonequilibrium distribution function is highly antistrophic. In these conditions the kinetic equation can be replaced by the equation system for the even and odd parts of the distribution function, where instead of the collision operator $J\{\chi_{a,s}(\theta)\}$, the appropriate relaxation time approximation is used,

$$v \sin\theta \frac{\partial \chi_a}{\partial z} + \frac{\chi_s}{l_s} = 0, \tag{5}$$

$$v \sin\theta \frac{\partial \chi_s}{\partial z} + \frac{\chi_a}{l_a(\theta_0)} = -eEv\cos\theta. \tag{6}$$

The use of the modified τ approximation in Eqs. (5) and (6) is justified by the following argument. The replacement of $J\{\chi_s\}$ by χ_s/l_s in the first equation, as can be easily seen, does not violate the momentum preservation law. In the second equation it was taken into account that a carrier's emergence from the current-carrying region at angles $\simeq\theta_0$ leads to its destruction at the boundary. It is not difficult to get a rather cumbersome expression solving Eqs. (5) and (6) for the diffusive scattering at the boundaries. Qualitatively, the result is given by the following expression for the transport MFP averaged over z:

$$[l_{\rm tr}(\theta)]^{-1} \simeq (d^2/l_s\theta^2)^{-1} + [l_a(\theta_0)]^{-1}, \quad \theta \ll d/l_s, \tag{7}$$

$$\theta_0 \simeq d/[l_a(\theta_0)l_s]^{1/2}. \tag{8}$$

Here the first term on the right-hand side of (7) is the one-dimensional diffusion path to the boundary of the charge carrier propagating at the angle θ. The second term in (7) takes into account that the carriers propagating at angles $\theta \leq \theta_0$ diffuse for so long that the slow antisymmetric relaxation processes have enough time to essentially change the direction of their motion and bring them to the boundary. For $\theta \gg d/l_s$, a Knudsen regime arises. Equations (3) and (8) make a system of algebraic equations, from which the angle θ_0 is found. The resistivity of the wire is $\rho \simeq (p_F/e^2 n) l_{\rm tr}^{-1}$, where

$$l_{\rm tr} = \int l_{\rm tr}(\theta) d\theta \simeq d[l_a(\theta_0)/l_s]^{1/2} \simeq d^{5/3} l_a^{1/6} l_s^{-5/6} \sim T\,, \tag{9}$$

$$l_s T/\varepsilon_F \ll d^2/l_s \ll l_a\,. \tag{10}$$

Thus the existence region of the Poiseuille flow becomes narrower, which is seen by comparing (4) with (1), since $l_a \gg l_s \sim l_{ee}$. But on the whole, due to the new mechanism (9) and (10), the region of the inverse ρ vs T dependence becomes wider, as compared to the three-dimensional case (1). The reason is that, instead of the inequality $l_s \ll d$, a much easier restriction $l_s\theta_0 \ll d$ must be fulfilled in the case of grazing electrons. (The diffusion step across the wire must be small compared with the wire width.) As a result, a decrease in resistivity may be observed with increasing temperature even in narrow ($d \ll l_{ee}$) 2DEG wires.

Up to now we assumed that bulk collisions not conserving momentum (e.g., those with impurities or phonons) are rather infrequent: $l_V \gg l_a \gg l_s$. It can be easily seen, however, that in contrast to the condition $l_V \gg l_s$ the requirement $l_V \gg l_a$ is necessary only for the Poiseuille flows (4). Collisions with loss of the quasimomentum can be accounted for in Eqs. (5) and (6) by the replacement $l_a(\theta_0)^{-1} \rightarrow l_a(\theta_0)^{-1} + l_V^{-1}$. The least of the MFP's delimits the existence time for the one-dimensional current-carrying state and determines the angular size of the effective carrier group, θ_0, and, simultaneously, the temperature dependence $\rho(T)$. It should be noted that with impurity scattering the linearity of the temperature dependence is preserved, although the dependence on the other parameters in (9) will obviously change.

Since a narrow group of carriers with a long MFP is essential for the specifically two-dimensional hydrodynamic regime, this also causes the effect to be sensitive to a relatively weak transversal magnetic field. The field takes the carrier out of the angle range of order θ_0 during the time $r\theta_0/v_F$, where r is the Larmor radius. That is why the effect of a weak magnetic field can be qualitatively accounted for by adding $(\theta r)^{-1}$ to the expression for $[l_{\rm tr}(\theta)]^{-1}$. Thus a substantial positive magnetic resistance arises in this regime when $r\theta_0 \leq l_{\rm tr}(\theta_0)$ [values for $l_{\rm tr}(\theta_0)$ and θ_0 are taken when $H = 0$]. With increase of the field, the resistance grows as $r^{1/3}$ until either $r \gg l_s^2/d$ at $l_s \gg d$ or $r \gg d^2/l_s$ at $l_s \ll d$. In the first case, further increase of the field leads to the known result $l_{\rm tr} \simeq d\ln(r/d)$. In the second case, it results in a regime similar to the Poiseuille flow $l_{\rm tr} \simeq d^2/l_s$. For the sake of comparison, we recall that under the conditions of Poiseuille flow the magnetic field noticeably affects conductivity only at $r \ll l_s$, and, besides, the magnetic resistivity is negative here.

In the experiment of Ref. [1] the 2DEG wires were prepared in (Al,Ga)As heterostructures. The wires had the width $d \simeq 3.5$ μm, the electron-impurity MFP $l_i \simeq 12.4$–19.7 μm, and electron density about 2.2×10^{11} cm^{-2}. In the temperature range where the inverse dependence of electrical resistivity was observed, the condition $d^2/l_s \ll l_a$ was easily fulfilled. That is why the Poiseuille mechanism cannot be realized here. As estimates show, the parameter θ_0 varies from 0.3 to 0.8 in the inverse dependence range, and roughly speaking, $l_i \simeq l_a(\theta_0)$. Thus the results of the experiment of Ref. [1] can be qualitatively described by the above-suggested mechanism related to the one-dimensional diffusion of carriers. It could be experimentally confirmed by the appearance of positive magnetic resistivity in anomalously weak magnetic fields.

In conclusion, we have investigated the momentum relaxation mechanisms in 2DEG wires under the effect of frequent normal electron-electron collisions. We showed the existence of a principally new two-dimensional "hydrodynamic type" effect due to the one-dimensional diffusion of charge carriers. This effect is related to a narrow group of grazing carriers, and it can lead to the inverse temperature dependence of the resistivity of 2DEG wires even under the conditions when $l_{ee} \gg d$, i.e., opposite to the Poiseuille conditions. This effect appears to be very sensitive to the magnetic field. A qualitative explanation of the experiment by Molenkamp and de Jong has been given. It is noteworthy that in this experiment the full-size effect of the hydrodynamic type in the electron gas was observed for the first time.

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[3] Note that, in contrast to the 3D case, the quantity of processes with the opposite momenta of colliding electrons [or, rather, close to the opposite: $|\mathbf{p}_1 + \mathbf{p}_2| \leq (T/\varepsilon_F)p_F$, where p_F is the Fermi momentum] is of the same order as the quantity of collisions with arbitrary angle between the partners $\mathbf{p}_1$ and $\mathbf{p}_2$. Indeed, the total number of degrees of freedom is the same in both cases; it equals five: three degrees of freedom are fixed by the conservation laws. In the case $|\mathbf{p}_1 + \mathbf{p}_2| \simeq 0$, however, when $\mathbf{p}_1$ is fixed, then $\mathbf{p}_2$ is fixed as well (to within T/ε_F), and we have only one degree of freedom φ not restricted by the parameter T/ε_F. On the other hand, when $|\mathbf{p}_1 + \mathbf{p}_2| \simeq p_F$, then the choice of the partner $\mathbf{p}_2$ is arbitrary, while the freedom of the choice of finite states is restricted by the parameter T/ε_F, i.e., again one has only one "large" degree of freedom. For the 3D electron gas in the above cases we have two and three large degrees of freedom, respectively, so the contribution from the collisions with opposite momenta is small compared to the parameter T/ε_F.

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