

Isospin Dependence of the Spin-Orbit Force and Effective Nuclear Potentials

M. M. Sharma,¹ G. Lalazissis,² J. König,² and P. Ring²

¹Max Planck Institut für Astrophysik, Karl-Schwarzschildstrasse 1, D-85740 Garching, Germany

²Physik-Department der Technischen Universität München, D-85747 Garching, Germany

(Received 16 September 1994; revised manuscript received 4 April 1995)

The isospin dependence of the spin-orbit potential is investigated for an effective Skyrme-like energy functional suitable for density dependent Hartree-Fock calculations. The magnitude of the isospin dependence is obtained from a fit to experimental data on finite spherical nuclei. It is found to be close to that of relativistic Hartree models. Consequently, the anomalous kink in the isotope shifts of Pb nuclei is well reproduced.

PACS numbers: 21.30.+y, 21.10.Dr, 21.10.Ft, 21.60.Jz

The Hartree-Fock approach based upon phenomenological density dependent forces [1–3] has proved to be very successful in the microscopic description of ground state properties of nuclear matter and of finite nuclei over the entire periodic table. In all these calculations the spin-orbit potential has been assumed to be isospin independent. Its only parameter, the strength, is usually adjusted to the experimental spin-orbit splitting in spherical nuclei like ¹⁶O or ²⁰⁸Pb. The exchange term, however, causes for nuclei with neutron excess a strong isospin dependence of the corresponding single-particle spin-orbit field.

In recent years relativistic mean field (RMF) theory [4] with nonlinear self-interactions between the mesons has gained considerable interest for the investigations of low-energy phenomena in nuclear structure. With only a few phenomenological parameters such theories are able to give a quantitative description of ground state properties of spherical and deformed nuclei [5,6] at and away from the stability line. In addition, excellent agreement with experimental data has been found recently also for collective excitations such as giant resonances [7] and for twin bands in rotating superdeformed nuclei [8]. In many respects the relativistic mean-field theory is regarded as similar to the density-dependent Hartree-Fock theory of the Skyrme type [9,10]

Recently, however, detailed investigations of high precision data on nuclear charge radii in Pb isotopes [11,12] and of shell effects at the neutron drip line [13,14] have shown considerable differences between the Skyrme approach and the relativistic mean-field theory. In fact, density dependent Hartree-Fock calculations with Skyrme [11] or Gogny forces [15], which used, so far, antisymmetrized, isospin independent spin-orbit interactions, were not able to reproduce the kink in the isotope shifts of Pb nuclei (see Fig. 1). On the other hand, this kink is obtained in the RMF theory without any new adjustment of parameters [12]. Another considerable difference has been found in the theoretical investigation of shell effects in very exotic Zr isotopes near the neutron drip line: In conventional nonrelativistic Skyrme calculations the shell gap at isotope ¹²²Zr with the magic neutron configuration $N = 82$ is totally smeared out [13], whereas relativistic calculations us-

ing various parameter sets show at $N = 82$ a clear kink in the binding energy as a function of the neutron number [14]. This difference is caused by the different spin-orbit splitting of the single particle levels in these nuclei. Mass calculations within the finite range droplet model (FRDM) [16], which are based on an isospin dependent spin-orbit term carefully adjusted to experimental data, are in excellent agreement with the relativistic predictions.

This gives us a hint that one does not really need full relativistic calculations in order to understand these differences between conventional density dependent Hartree-Fock calculations and RMF theory. A spin-orbit term with a properly chosen isospin dependence might represent the essential part of a relativistic calculation. In this Letter, we therefore explore the isospin dependence of the spin-orbit term in nonrelativistic Skyrme calculations and analyze the consequences of this on the nuclear proper-

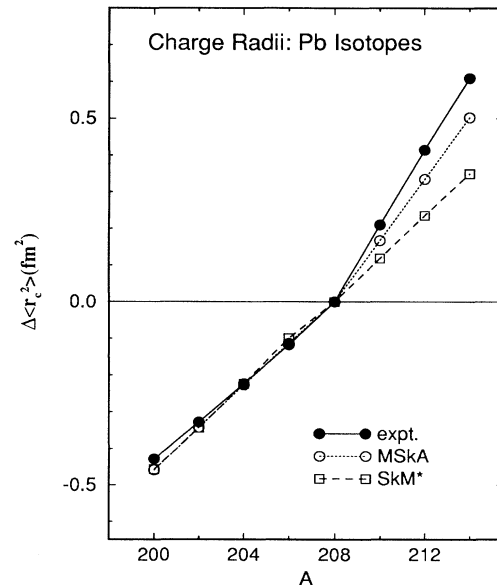


FIG. 1. Isotopic shifts in the charge radii of Pb isotopes normalized to the nucleus ²⁰⁸Pb as a function of the mass number A in the modified Skyrme ansatz (MSkA) as compared to the Skyrme force SkM* and the empirical data [19].

ties. We start from the Skyrme type force

$$\begin{aligned}
V(1,2) = & t_0(1 + x_0 P^\sigma) \delta(\mathbf{r}_1 - \mathbf{r}_2) \\
& + t_1(1 + x_1 P^\sigma) [\delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k}^2 + \text{H.c.}] \\
& + t_2(1 + x_2 P^\sigma) \mathbf{k} \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k} \\
& + \frac{1}{6} t_3(1 + x_3 P^\sigma) \delta(\mathbf{r}_1 - \mathbf{r}_2) \rho^\alpha \\
& + W_0(1 + x_w P^\tau) (\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}) \mathbf{k} \times \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k},
\end{aligned} \tag{1}$$

with $\mathbf{k} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$. In contrast to the conventional Skyrme ansatz, where the energy functional contains a Hartree and a Fock contribution, we neglect here the exchange (Fock) term for the spin-orbit potential in the last line of Eq. (1). Otherwise, the operator P^τ would be equivalent to +1 for spin saturated systems. For the rest of the potential the Fock terms are included. The associated 11 parameters t_i , x_i ($i = 0 \dots 3$), W_0 , x_w , and α of the modified Skyrme ansatz (MSkA) in Eq. (1) are determined by a fit to experimental data of finite spherical nuclei. The nuclear properties taken into consideration are the empirical binding energies and charge radii of the closed-shell nuclei ^{16}O , ^{40}Ca , ^{90}Zr , and ^{208}Pb . In order to take into account the variation in isospin we have also included Sn isotopes ^{116}Sn , ^{124}Sn , and the doubly closed nucleus ^{132}Sn , as well as one of the lead isotopes ^{214}Pb . The resulting force and its parameters are presented in Table I.

The spin-orbit term in the single-particle field derived from this force has the form $\mathbf{W}_\tau(\mathbf{r})(\mathbf{p} \times \boldsymbol{\sigma})$ with

$$\mathbf{W}_\tau(\mathbf{r}) = W_1 \nabla \rho_{\tau^+}, W_2 \nabla \rho_{\tau' \neq \tau}, \tag{2}$$

where ρ_τ is the density for neutrons or protons ($\tau = n$ or p) and $W_1 = W_0(1 + x_w)/2$, $W_2 = W_0/2$. Conventional Skyrme calculations use a spin-orbit potential without isospin dependence and include the Fock term. This leads to $x_w = 1$ and the relationship $W_1/W_2 = 2$, which is nearly a factor 2 different from the value 1.0005 obtained within the MSkA (see Table I). It is interesting to note that the fit leads to a value of $x_w \approx 0.1$ close to zero, which corresponds to the one without the Fock term.

In order to study the spin-orbit term in the RMF theory we start from the standard Lagrangian density [5]

TABLE I. Parameters of the interaction in the modified Skyrme ansatz (MSkA).

| | |
|--|-----------------|
| $t_0 = -1218.828$ (MeV fm ³) | $x_0 = 0.1190$ |
| $t_1 = 370.265$ (MeV fm ⁵) | $x_1 = 0.0157$ |
| $t_2 = -106.883$ (MeV fm ⁵) | $x_2 = 0.0335$ |
| $t_3 = 10834.819$ (MeV fm ^{3+3\alpha}) | $x_3 = -0.1990$ |
| $W_0 = 278.67$ (MeV fm ⁵) | $x_w = 0.1032$ |
| $\alpha = 0.7179$ | |

$$\begin{aligned}
\mathcal{L} = & \bar{\psi}[\gamma(i\partial - g_\omega \omega - g_\rho \vec{\rho} \cdot \vec{\tau} - eA) - m - g_\sigma \sigma] \psi \\
& + \frac{1}{2}(\partial\sigma)^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 \\
& - \frac{1}{4}\vec{\mathbf{R}}_{\mu\nu} \cdot \vec{\mathbf{R}}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}
\end{aligned} \tag{3}$$

which contains nucleons ψ with mass m , σ , ω , and ρ mesons, the electromagnetic field, and nonlinear self-interactions $U(\sigma)$ of the σ field,

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4. \tag{4}$$

In a nonrelativistic approximation of the corresponding Dirac equation [17], we obtain the following single particle spin-orbit term:

$$\begin{aligned}
\mathbf{W}_\tau(\mathbf{r}) = & \frac{1}{m^2 m^{*2}} (C_\sigma^2 + C_\omega^2 + C_\rho^2) \nabla \rho_\tau \\
& + \frac{1}{m^2 m^{*2}} (C_\sigma^2 + C_\omega^2 - C_\rho^2) \nabla \rho_{\tau' \neq \tau},
\end{aligned} \tag{5}$$

with $C_i^2 = (m g_i / m_i)^2$ for $i = \sigma, \omega, \rho$, which is similar in form to the spin-orbit field (2), but which contains \mathbf{r} -dependent parameters

$$W_1 = \frac{1}{m^2 m^{*2}} (C_\sigma^2 + C_\omega^2 + C_\rho^2), \tag{6}$$

$$W_2 = \frac{1}{m^2 m^{*2}} (C_\sigma^2 + C_\omega^2 - C_\rho^2), \tag{7}$$

with $m^*(\mathbf{r}) = m - g_\sigma \sigma(\mathbf{r})$. The r dependence drops out in the ratio W_1/W_2 which is 1.13 for the parameter set NL1 [5] and 1.10 for the parameter set NL-SH [6]. This is only slightly higher than the value 1.0005 obtained in the modified Skyrme ansatz by the fit to the empirical data (see Table I). The absolute size of the spin-orbit term turns out to be \mathbf{r} dependent, which stems from the \mathbf{r} dependence of the effective mass $m^*(\mathbf{r})$. It can be approximated by $m/m^* \approx 1 + C_\sigma^2/m^3 \rho(\mathbf{r})$. A more careful consideration would therefore require an explicitly density dependent spin-orbit term. It has not been included in the present investigation.

In Table II we show nuclear matter results obtained in the MSkA and compare it with the values from the conventional Skyrme force SkM* [18]. The saturation density is obtained as $\rho_0 = 0.1531 \text{ fm}^{-3}$ in MSkA. This is

TABLE II. Nuclear matter properties obtained in the modified Skyrme ansatz (MSkA).

| | MSkA | SkM* |
|-----------------|-------------------------|-------------------------|
| ρ_0 | 0.1532 fm ⁻³ | 0.1603 fm ⁻³ |
| $(E/A)_\infty$ | 15.995 MeV | 15.776 MeV |
| K | 313.4 MeV | 216.7 MeV |
| $\rho_0^2 e'''$ | 99.4 MeV | 57.9 MeV |
| J | 30.3 MeV | 30.0 MeV |
| m^*/m | 0.79 | 0.79 |

the same as the value obtained from an extensive fit of the FRDM mass formula [16]. The binding energy per particle E/A is 15.995 MeV, which is close to that of other Skyrme forces. The compression modulus $K = 313$ MeV is somewhat higher than that of the currently adopted Skyrme forces, but it lies within the error bars of the analysis based upon the breathing mode energies. The asymmetry energy J is close to the empirical value of 33 MeV. The effective mass m^* is in good agreement with that of SkM*.

In Table III we show binding energies and charge radii obtained in the MSkA for a number of spherical nuclei and compare them with those from SkM*. A comparison with the empirical values shows that the binding energies obtained with MSkA have improved over those of SkM*. The slightly reduced charge radii in MSkA seem to be connected with a slightly higher binding energy, which improves the results for the lighter Pb isotopes. In general, the charge radii are improved including that of ^{16}O .

In Fig. 1 we show the isotope shifts of Pb nuclei for the MSkA together with experimental data and results of conventional Skyrme calculations. For MSkA we observe a clear kink at the double magic nucleus ^{208}Pb , whereas the conventional Skyrme force SkM* with an isospin independent spin-orbit term gives an almost straight line. For the lighter isotopes both theories give excellent agreement; on the heavier side MSkA comes closer to the experimental isotope shifts. It may be recalled that the RMF theory [12] is successful in reproducing the full size of this kink. MSkA uses an isospin dependent, but density independent spin-orbit force. In contrast the spin-orbit term derived from the RMF theory [see Eq. (5)] is implicitly density dependent through the density dependence of the effective mass $m^*(r)$. A density dependence of the spin-orbit term in Skyrme theory might improve the charge radii of

the heavier Pb isotopes. It has also been observed that a density dependent pairing force can possibly improve the situation in this context [11]. This requires, however, further investigations.

In Fig. 2 we present binding energies of Zr nuclei about the neutron drip line as a function of the mass number. It is observed that in agreement with earlier investigations [13] within the conventional Skyrme theory, shell effects about the closed shell nucleus ^{122}Zr are weakened considerably. In contrast, RMF calculations exhibit strong shell effects in the drip line region. It has been surmised in Ref. [14] that this is caused by the differences in the spin-orbit terms in the two approaches. It is gratifying to see that the introduction of an isospin dependent (not antisymmetrized) spin-orbit term in Skyrme theory leads to stronger shell effects. This is also in agreement with the predictions of the FRDM [16], where also an isospin dependent spin-orbit term is used. A more quantitative analysis shows that the ratio W_1/W_2 in the FRDM is close to 1.06 for the Pb nuclei and 1.09 for ^{122}Zr .

Summarizing, we conclude that the isospin dependence of the spin-orbit term has an essential influence on the details of anomalous isotope shifts of Pb nuclei. A new modified Skyrme ansatz has been proposed, and the isospin dependence of the spin-orbit strength has been determined. The magnitude of this isospin dependence x_w is in agreement with the deductions from the relativistic mean-field theory. A reasonably good agreement with the experimental data on the binding energies and charge radii has been obtained. The kink in the isotope shifts of Pb nuclei has been obtained in the modified Skyrme ansatz. However, the agreement with the empirical isotope shifts for heavy Pb nuclei is not so good as in the RMF theory. This calls for further investigations including a density dependence of the spin-orbit term.

TABLE III. The binding energies and charge radii obtained with the modified Skyrme ansatz (MSkA) in the Hartree-Fock approximation as compared with the normal Skyrme force SkM* and the empirical values.

| Nuclei | Binding energies (MeV) | | | Charge radii (fm) | | |
|-------------------|------------------------|---------|---------|-------------------|-------|-------|
| | Expt. | MSkA | SkM* | Expt. | MSkA | SkM* |
| ^{16}O | -127.6 | -128.3 | -127.7 | 2.730 | 2.743 | 2.811 |
| ^{40}Ca | -342.1 | -343.4 | -341.1 | 3.450 | 3.467 | 3.518 |
| ^{48}Ca | -416.0 | -416.5 | -420.1 | 3.500 | 3.506 | 3.537 |
| ^{90}Zr | -783.9 | -781.3 | -783.0 | 4.270 | 4.274 | 4.296 |
| ^{116}Sn | -988.7 | -983.3 | -983.4 | 4.626 | 4.621 | 4.619 |
| ^{124}Sn | -1050.0 | -1047.2 | -1049.0 | 4.673 | 4.675 | 4.678 |
| ^{132}Sn | -1102.9 | -1104.7 | -1110.7 | ... | 4.727 | 4.727 |
| ^{200}Pb | -1576.4 | -1568.7 | -1568.4 | 5.464 | 5.465 | 5.468 |
| ^{202}Pb | -1592.2 | -1586.0 | -1586.0 | 5.473 | 5.476 | 5.478 |
| ^{204}Pb | -1607.5 | -1603.1 | -1603.4 | 5.483 | 5.486 | 5.489 |
| ^{206}Pb | -1622.3 | -1619.9 | -1620.3 | 5.492 | 5.496 | 5.501 |
| ^{208}Pb | -1636.5 | -1635.6 | -1636.4 | 5.503 | 5.506 | 5.510 |
| ^{210}Pb | -1645.6 | -1644.1 | -1645.6 | 5.522 | 5.522 | 5.520 |
| ^{212}Pb | -1654.5 | -1652.4 | -1654.5 | 5.540 | 5.537 | 5.531 |
| ^{214}Pb | -1663.3 | -1662.3 | -1663.1 | 5.558 | 5.553 | 5.541 |

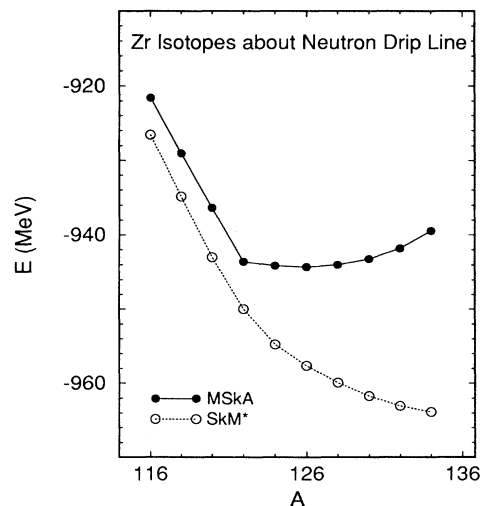


FIG. 2. Binding energy of Zr isotopes at neutron drip line in the MSkA and SkM* calculation.

Finally a remark about the isospin dependence of the spin-orbit term: Our investigations lead to the interesting result that the parameter x_w in the Eq. (1) is very small. This leads practically to an isospin independent single-particle spin-orbit field in Eq. (2). This is in agreement with relativistic calculations, where the entire isospin dependence of the spin-orbit field is caused by the parameter C_ρ in Eq. (5), which is, in fact, rather small. In contrast the two-body spin-orbit potential in conventional Skyrme theory is isospin independent. The exchange term, however, causes a strong isospin dependence in the corresponding single-particle spin-orbit field. On the other hand, in RMF theory the spin-orbit field has its origin in Lorentz covariance. There is no contribution from a two-body spin-orbit potential, and an exchange term is therefore excluded.

One of the authors (G.A.L.) acknowledges support from the E.U., HCM program, Contract EG/ERB CHBICT-930651. This work is also supported in part by the Bundesministerium für Forschung und Technologie under Project 06 TM 743.

-
- [1] D. Vautherin and D.M. Brink, Phys. Rev. C **5**, 626 (1972).
[2] P. Quentin and H. Flocard, Ann. Rev. Nucl. Sci. **28**, 523 (1978).

- [3] J. Decharge and D. Gogny, Phys. Rev. C **21**, 1568 (1980).
[4] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
[5] Y. K. Gambhir, P. Ring, and A. Thimet, Ann. Phys. (N.Y.) **198**, 132 (1990).
[6] M.M. Sharma, M.A. Nagarajan, and P. Ring, Phys. Lett. B **312**, 377 (1993).
[7] D. Vretenar, H. Berghammer, and P. Ring, Nucl. Phys. **A581**, 679 (1995).
[8] J. König and P. Ring, Phys. Rev. Lett. **71**, 3079 (1993).
[9] M. Thies, Phys. Lett. B **166**, 23 (1986).
[10] P.-G. Reinhard, Rep. Prog. Phys. **52**, 439 (1989).
[11] N. Tajima, P. Bonche, H. Flocard, P.-H. Heenen, and M.S. Weiss, Nucl. Phys. **A551**, 434 (1993).
[12] M.M. Sharma, G.A. Lalazissis, and P. Ring, Phys. Lett. B **317**, 9 (1993).
[13] J. Dobaczewski *et al.*, Phys. Rev. Lett. **72**, 981 (1994).
[14] M.M. Sharma, G.A. Lalazissis, W. Hillebrandt, and P. Ring, Phys. Rev. Lett. **72**, 1431 (1994).
[15] J.L. Egido (private communication).
[16] P. Möller, J.R. Nix, W.D. Myers, and W.J. Swiatecki, At. Data Nucl. Data Tables (to be published).
[17] J. König (to be published).
[18] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Håkansson, Nucl. Phys. **A386**, 79 (1982).
[19] E.W. Otten, in *Treatise on Heavy-Ion Science*, edited by D.A. Bromley (Plenum, New York, 1989), Vol. 7, p. 517.