Isospin Breaking in Low-Energy Pion-Nucleon Scattering

W. R. Gibbs and Li Ai
Department of Physics, New Mexico State University, Las Cruces, New Mexico 88003

W. B. Kaufmann
Department of Physics and Astronomy, Arizona State University, Tempe, Arizona 85287
(Received 10 August 1994)

We have analyzed low-energy pion-nucleon data for isospin invariance by comparing charge-exchange amplitudes derived from charge-exchange data with those predicted from recent $\pi^{\pm}p$ elastic data through the application of isospin invariance. A discrepancy of the order of 7% is observed beyond the contributions of the $\pi^{\pm}p$ Coulomb interaction and the hadronic mass differences.

PACS numbers: 13.75.Gx, 24.80.Dc, 25.80.Gn

Of the eight reactions of the form $\pi N \rightarrow \pi' N'$ the only three directly accessible experimentally are those with charged pion beams and proton targets: $\pi^{\pm}p$ elastic scattering, with amplitudes $f_{\pm}$, and $\pi^- p \rightarrow \pi^0 n$ charge-exchange (CEX) scattering, with amplitude $f_{\rm CEX}$. Isospin conservation gives a relationship among these three amplitudes, the *triangle identity*

$$f_{\rm CEX} = \frac{1}{\sqrt{2}}(f_+ - f_-). \quad (1)$$

Since isospin symmetry is only approximate, the discrepancy in this relation, $D \equiv f_{\rm CEX} - (f_+ - f_-)/\sqrt{2}$, is expected to be nonzero. Interesting physics lies in the violation of this relation beyond effects due to the hadronic mass differences and the Coulomb interaction.

It is the purpose of this Letter to demonstrate that recent experimental data at low energies require a residual contribution to D, of the order of -0.012 *fm in the s wave.* This number is to be compared with the low-energy s-wave CEX amplitude of approximately -0.19 fm. There is a similar, but less well-determined, breaking in the p wave spin-flip amplitude.

The triangle identity implies a bound on the elastic and charge-exchange cross sections given by

$$\frac{1}{2}(\sqrt{\sigma_+} + \sqrt{\sigma_-})^2 \geq \sigma_{\rm CEX} \geq \frac{1}{2}(\sqrt{\sigma_+} - \sqrt{\sigma_-})^2. \quad (2)$$

In tests of this inequality [1] no evidence for isospin breaking was found. The difficulty with this test is that even a large breaking in isospin does not guarantee its violation. A better test of isospin invariance can be made by an analysis of the amplitude itself.

The study of isospin-breaking effects is favored at low energies for the following reasons. (1) Recent measurements of pion-nucleon elastic [2] and charge-exchange [3] scattering for $T_\pi \leq 50$ MeV have yielded data of exceptionally high quality. A new pionic atom measurement has also been recently obtained [4]. (2) There is a minimum in each of the amplitudes in this energy range. A small value of any one of the amplitudes is useful since Eq. (1) implies that if any of $(f_+, f_-, \sqrt{2} f_{\rm CEX})$ is zero the other two must have equal magnitudes. (3) At these low energies the imaginary part of the amplitudes is very small. (4) The s- and p-wave amplitudes suffice to describe scattering and have a smooth *and gentle* energy dependence.

The existence of a minimum in the charge-exchange cross section at 0° is very important, since its position in energy can be determined much more precisely experimentally than the absolute magnitude of the cross section. Fitzgerald *et al.* [3] determined the energy of the minimum to be $T_\pi = 45.1 \pm 0.5$ MeV.

While one could attempt to apply Eq. (1) directly, it seems more appropriate to correct for the known isospin breaking effects. To this end we fitted the scattering amplitudes with coupled-channel potential models [5]. These dynamical equations serve as an interpolating mechanism in energy, enforce unitarity and s-channel analyticity, and include the mass splitting and Coulomb interaction explicitly. We emphasize that this particular approach for the Coulomb corrections is expected to have very little effect on the result. One could use, for example, K-matrix approaches as well.

To estimate the model dependence in this technique we have repeated the fits with several forms for the strong-interaction potentials: a dipole separable form, three local potentials (sums of two Yukawa, exponential, or Gaussian functions), and the nonlocal form used in Ref. [5]. While the potentials derived from the fits are not unique, the scattering amplitudes produced show very little model dependence.

First, the elastic data (including the atomic point) were fitted by adjusting the parameters of the strong-interaction potential in the coupled-channel system, producing partial-wave amplitudes (s, p nonflip, and p flip) $f_{\pm}$. The resulting values of χ^2, evaluated with various potential models from the set of elastic data alone, are given in the second column of Table I. The elastic data sets, with $\chi^2_{\pm}$ of about 252 for 227 data points, are rather consistent. We have included 13 data sets taken by three different experimental groups at three different laboratories, so consistency of the data is quite significant. Charge-exchange amplitudes $f^P_{\rm CEX}$ are produced at the same time

 0031-9007/95/74(19)/3740(4)$06.00

TABLE I. Values of χ^2. The subscript R indicates the residual value after subtracting the elastic pion $\chi^2_\pm$, taken from column 1. If isospin were conserved, χ^2_R would be expected to be of the order of $\chi^2_{\rm CEX}$, from column 2. All χ^2 include contributions from the normalization uncertainties.

Model	$\chi^2_\pm$ 227 pts	$\chi^2_{\rm CEX}$ 37 pts	$\chi^2_R(P)$	$\chi^2_R(F)$
Local				
Yukawa	251	37	736	279
Gaussian	253	34	786	258
Exponential	252	33	725	245
Separable				
Ref. [5]	248	32	553	254
Dipole	253	34	596	251

by the solution of the coupled-channel system. The P on $f^P_{\rm CEX}$ indicates that this amplitude is a prediction, based solely on the elastic data and the assumption of isospin invariance of the strong potentials. The elastic data have a precision of the order of 2%, implying an error in the predicted CEX amplitude of the order of ± 0.003 fm.

Next, the CEX data alone were fitted, producing the CEX partial-wave amplitudes $f^F_{\rm CEX}$. As is seen from the second column of Table I, the CEX data set is very consistent, with a typical χ^2 of about 34 for 37 data points.

We briefly mention data sets which we have omitted or have included only in some of our analyses [6]. We have not included the data of Bertin *et al.* which have a normalization inconsistent with the later, more extensive, data from the meson factories; the normalization uncertainty is unknown. With the exception of the largest angle point, the elastic data of Auld *et al.* are consistent with our fits, while the six data points of Blecher *et al.* lie 8%–20% above them. We have not used the integral elastic data of Friedman *et al.* and of Kriss *et al.*, which disagree with each other in our energy range; the present fit is consistent with the latter. It is also consistent with the three charge-exchange data points of Ullman *et al.* The preliminary charge-exchange data of Pocanic in the SAID database have not been included; we look forward to the final analysis of these data.

The relevant question is *how close are $f^P_{\rm CEX}$, predicted from the elastic data with the assumption of isospin-invariance of the strong interactions, and $f^F_{\rm CEX}$, obtained directly from the charge-exchange data?* If isospin invariance were perfect (except for Coulomb and mass-splitting effects), we would expect these amplitudes to be equal. Our results, which show a clear discrepancy, are given in Fig. 1 for s-wave amplitudes and in Fig. 2 for the p-wave non-spin-flip and spin-flip amplitudes.

The excess χ^2 resulting from a fit to the combined data sets over the sum of the components provides a quantitative measure of isospin breaking. The χ^2 from the CEX data set alone calculated from the parameters determined by a fit to the elastic data is shown in column 3 of Table I. The *smallest* of these numbers (553 for 37 data points or a total χ^2 of 801 for 268 data points) leads

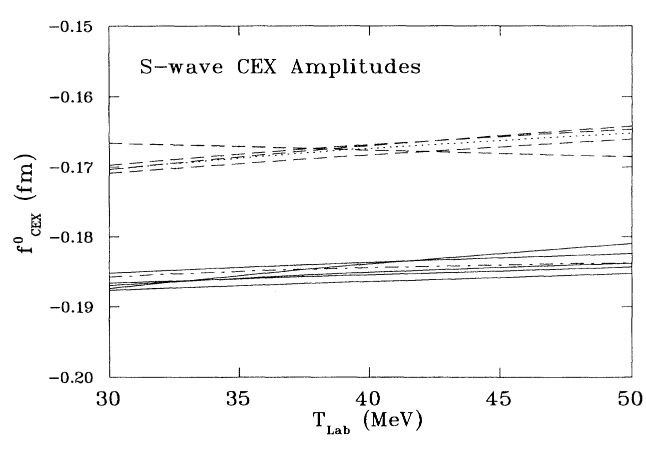

FIG. 1. s-wave π-nucleon charge-exchange amplitudes. The solid and dash-dotted curves correspond to fits to the CEX data and the dash-only or dot-only curves are predictions of the various models based on elastic data only. The dash-dotted and dotted curves are from fits with the Yukawa potential using only data published in refereed journals.

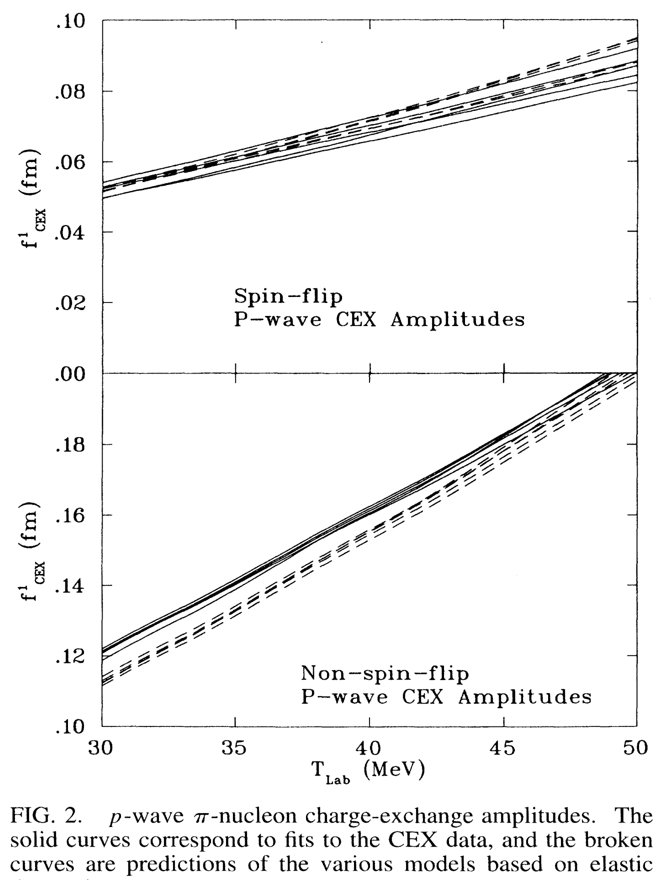

FIG. 2. p-wave π-nucleon charge-exchange amplitudes. The solid curves correspond to fits to the CEX data, and the broken curves are predictions of the various models based on elastic data only.

to a rejection of the hypothesis of isospin conservation at a very high level of confidence.

Of course, the strong-interaction parameters can be refitted to the entire data set with a consequent reduction in the residual χ_R^2 ($\equiv \chi_{\text{total}}^2 - \chi_\pm^2$) (column 4 in Table I). We have attempted to find acceptable fits to the global set of data (CEX and elastic together) in two different ways which differ according to the treatment of the data normalization errors.

First, the normalizations of the individual data sets were fixed at their values obtained in the separate fits to the elastic and CEX data. The best fits obtained with this procedure, assuming isospin conservation, have χ_R^2 of approximately 250 (column 4 in Table I), which is to be compared with about 34, the typical χ^2 for the CEX fit alone. The size of this number (χ_{total}^2 of around 500) shows a strong and statistically significant incompatibility (the probability that χ^2 would be this big or bigger is of the order of 10^{-14}) between the elastic and CEX data, assuming isospin invariance.

Second, we have allowed the normalizations of the data to be modified in the course of the global fit. In this case the χ_R^2 are in the range of 117 or greater. The result has a statistical confidence level of approximately 0.995 for the rejection of the conservation of isospin. We believe, however, that the case for isospin breaking is even stronger than this already significant number would suggest. The resulting changes in the normalization constants show strong correlations; a systematic rescaling of the charge-exchange differential cross section downward and the elastic cross sections upwards is seen.

We have included the sum $\chi_{\text{norm}}^2 = \sum_i [(N_i - 1)/e_i]^2$ to account for the uncertainty e_i in the normalization N_i of the ith data set. The N_i are not completely independent for the same experimental conditions (detectors, beam lines, etc.) except for different energies. To test the sensitivity of the calculated amplitudes to these correlations, we have experimented with the groupings of the data. For example, in the CEX data of Fitzgerald *et al.* [3], we have made separate fits with the two extreme cases: a single overall normalization ($\chi_{\text{norm}}^2 = [(N^{\text{Fitz}} - 1)/e^{\text{Fitz}}]^2$) and five independent norms, corresponding to each of the beam energies. Even though the best-fit values of N_i^{Fitz} differ somewhat, the resulting CEX amplitudes are nearly equal to those obtained with a single overall normalization constant. Similar comments apply to the Sadler *et al.* CEX data.

Figure 3 shows the real part of the 0° CEX amplitude which results from the fitted value of f_{CEX}^F and the "predicted" value of the charge-exchange amplitude f_{CEX}^P. These amplitudes would be equal if the only isospin symmetry breaking were due to mass splitting and Coulomb effects. *From this figure it is clear that the prediction from the elastic amplitudes gives values of the crossing energy which are consistently lower than the CEX fits (or the direct data). The amplitude difference is nearly constant with a value of* -0.010 ± 0.003 *fm. Note that the comparison with the predicted zero crossing is independent of the normalization of the CEX data.*

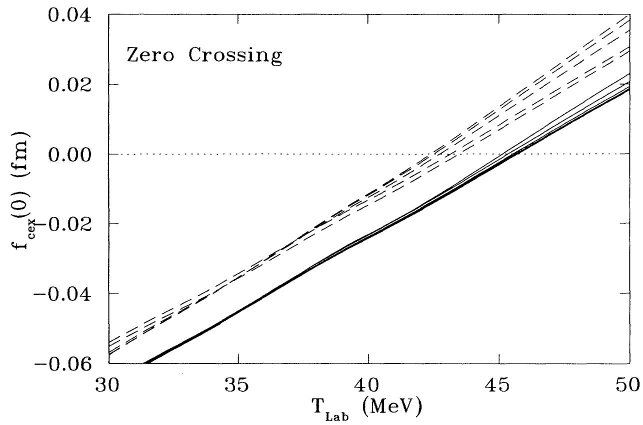

FIG. 3. Energy dependence of the predicted and fitted charge-exchange amplitudes at 0°, $f_{\text{CEX}}^P(0)$ and $f_{\text{CEX}}^P(0)$. The solid curves correspond to fits to the CEX data and the broken curves are predictions of the various models based on elastic data only.

It is instructive to compare the size of isospin breaking observed to that of the hadronic mass splitting and the Coulomb interaction which are reliably calculated in our coupled-channel model [5]. In the Gaussian potential model the contribution of the Coulomb interaction to the s-wave charge-exchange amplitude is approximately -0.005 fm at 10.4 MeV, and decreases to -0.002 fm at 51.8 MeV. In the same model the contribution of the hadronic mass splitting to the s-wave charge-exchange amplitude is -0.012 fm at 10.4 MeV and falls to -0.001 fm at 51.8 MeV. Note that, while these corrections are significant at very low energies, in the range just below 50 MeV, where our result is most relevant, the corrections are of the order of 10% of the difference seen in Fig. 1.

The radiative-capture channel has not been included in the present analysis, but previously it was estimated [5] that the effect on the amplitude is of the order of 0.5%.

One cannot distinguish in the present work where the breaking actually occurs; it could as well be in the charge exchange, in the elastic scattering amplitudes, or in a combination. The triangle discrepancy can be decomposed in terms of the amplitudes defined in Ref. [7],

$$f_{\text{CEX}} - \frac{1}{\sqrt{2}}(f_{\pi^+} - f_{\pi^-}) = -\frac{a_3}{\sqrt{2}} - \frac{\sqrt{3}\,a_6}{2} + \sqrt{\frac{3}{5}}\,\frac{a_7}{2} + \frac{a_8}{\sqrt{10}}. \quad (3)$$

Since a_8 is believed to be very small and a_6 is due largely to Coulomb effects which we have explicitly included, the test can be considered as measuring a combination of the amplitudes a_3 and a_7. Isospin breaking in the NN system is most often discussed in terms of $\rho\omega$ [8] and $\pi\eta$ [9] mixing. If they are valid hadronic theories their concepts must apply to pion-nucleon scattering as well. Within a meson-mixing model, a_3 may be attributed to $\rho\omega$ mixing, which affects elastic scattering only in leading order, while a_7 may be attributed to $\pi\eta$ mixing, which affects charge-exchange scattering only [7]. It is impossible to distinguish between these sources of isospin breaking solely by use of the triangle discrepancy. This separation of a_3 and a_7 requires a study of an additional reaction.

Low energy pion-deuteron elastic charge asymmetries are sensitive to a_3. From recent data [10] and published calculations of Coulomb breaking [11] one can estimate this amplitude. If we assume that all of the breaking in Eq. (3) is in the a_3 amplitude (the elastic scattering) then there is consistency in sign and approximate magnitude between the two experiments. If one assumes that the ρ coupling constant to the pion is the same as to the nucleon, then the calculation of the nucleon-nucleon isospin breaking through the $\rho\omega$ mixing [8] can be applied directly to the present case [12]. Such a calculation gives about the right magnitude but the opposite sign to the present (and deuteron) result. Recently, the derivation of the standard result has been called into question by treatments which calculate the mixing microscopically [13]. These approaches often find isospin-breaking potentials of the opposite sign to the standard calculations which could bring about agreement with the present work but would fail to explain the breaking in the *NN* interaction.

In order to determine the amount of breaking due to a_7, the most direct measurement would be a comparison of the position of the CEX minimum from the reactions $\pi^- p \rightarrow \pi^0 n$ and $\pi^+ n \rightarrow \pi^0 p$. Since analog transitions are known to display the minimum clearly, a comparison of $\pi^- \, {}^3\mathrm{He} \rightarrow \pi^0 \, {}^3\mathrm{H}$ with $\pi^+ \, {}^3\mathrm{H} \rightarrow \pi^0 \, {}^3\mathrm{He}$ could be used to obtain the same information.

In conclusion, by analyzing recent high-quality pion-nucleon elastic and charge-exchange scattering data we have found clear indications of isospin breaking in the pion-nucleon interaction beyond the effects of Coulomb and hadronic mass splitting. The size of the breaking is represented by the triangle discrepancy of $D = -0.012 \pm 0.003$ fm for the s wave alone or $D = -0.011 \pm 0.03$ fm for the sum of the s and p waves at 40 MeV.

W. B. K. would like to thank the members of T-5 and LAMPF at the Los Alamos National Laboratory and New Mexico State University for support while a portion of this work was done. W. R. G. wishes to acknowledge very helpful conversations with M. E. Sadler, S. A. Coon, T. Goldman, and J. Stern. This work was supported by the U.S. Department of Energy.

[1] J. A. Wightman *et al.*, Phys. Rev. D **38**, 3365 (1988).

[2] J. S. Franks *et al.*, Phys. Rev. D **28**, 1569 (1983); J. T. Brack *et al.*, Phys. Rev. C **41**, 2202 (1990); **38**, 2427 (1988); **34**, 1771 (1986); U. Wieder *et al.*, Phys. Rev. Lett. **58**, 648 (1987); C. Joram, πN Newsletter No. 3 (1991); SAID interactive dial-in program.

[3] M. E. Sadler *et al.*, πN Newsletter, No. 5 1992; M. Salomon *et al.*, Nucl. Phys. **A414**, 493 (1984); J. Duclos *et al.*, Phys. Lett. **43B**, 245 (1973); D. H. Fitzgerald *et al.*, Phys. Rev. C **34**, 619 (1985); A. Bagheri, Phys. Rev. C **38**, 885 (1988); M. E. Sadler (private communication); SAID interactive dial-in program.

[4] W. Beer *et al.*, Phys. Lett. B **261**, 16 (1991).

[5] P. B. Siegel and W. R. Gibbs, Phys. Rev. C **33**, 1407 (1986).

[6] P. Y. Bertin *et al.*, Nucl. Phys. **B106**, 341 (1976); E. G. Auld *et al.*, Can. J. Phys. **57**, 73 (1979); M. Blecher *et al.*, Phys. Rev. C **20**, 1884 (1979); E. Friedman *et al.*, Nucl. Phys. **A514**, 601 (1990); B. J. Kriss *et al.*, in *Proceedings of the 5th International Symposium on Meson-Nucleon Physics and the Structure of the Nucleon, Boulder, 1993,* edited by G. Hoehler, W. Kluge, and B. M. K. Nefkens, Vol. 1, p. 17; J. L. Ullman *et al.*, Phys. Rev. C **33**, 2092 (1986).

[7] W. B. Kaufmann and W. R. Gibbs, Ann. Phys. **214**, 84 (1992).

[8] S. A. Coon and R. Barrett, Phys. Rev. C **36**, 2189 (1987).

[9] P. Langacker and D. A. Sparrow, Phys. Rev. C **25**, 1194 (1982); S. A. Coon and M. Scadron, Phys. Rev. C **26**, 562 (1982).

[10] M. D. Kohler *et al.*, Phys. Rev. C **48**, 1884 (1993).

[11] J. Fröhlich, B. Saghai, C. Fayard, and G. H. Lamot, Nucl. Phys. **A435**, 738 (1985).

[12] B. L. Birbrair and A. B. Gridnev, Gatchina Report No. TH-24-1994-1966 (to be published).

[13] T. Goldman, J. A. Henderson, and A. W. Thomas, Few Body Systems **12**, 123 (1992); G. Krein, A. W. Thomas, and A. G. Williams, Phys. Lett. B **317**, 293 (1993); J. Piekarewicz and A. G. Williams, Phys. Rev. C **47**, 2462 (1993); T. Hatsuda, E. M. Henley, T. Meissner, and G. Krein, Phys. Rev. C **49**, 452 (1994); K. Maltman, Phys. Lett. B **313**, 203 (1993).