Washboard Frequency of the Moving Vortex Lattice in $YBa_2Cu_3O_{6.93}$ Detected by ac-dc Interference

J. M. Harris,[1] N. P. Ong,[1] R. Gagnon,[2] and L. Taillefer[2]
[1]*Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544*
[2]*Department of Physics, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8*
(Received 31 October 1994)

The depinned vortex lattice in a type II superconductor represents a periodic structure moving in a random distribution of pinning centers. Such a system displays a "washboard" frequency given by $\omega/2\pi = v/a$ where v is the velocity and a the lattice period. By mixing a dc and an ac current (of frequency $\omega_{\rm ext}$), we demonstrate the existence of the washboard effect in the vortex state in an untwinned crystal of $YBa_2Cu_3O_{6.93}$. Jumps in the dc current-voltage characteristics are observed when ω and $\omega_{\rm ext}$ are harmonically related. We show that these jumps vanish when the line at the solid-to-liquid transition is crossed.

PACS numbers: 74.60.Ge, 74.40.+k, 74.72.–h

In the mixed state of a type II superconductor, the vortex lattice is pinned to the host lattice by defects in the host lattice. When depinned by the application of a strong current, the vortex lattice may display a variety of interesting phenomena such as hysteresis and low-frequency fluctuations in the flux-flow resistivity. Recent investigations in high-purity untwinned crystals of $YBa_2Cu_3O_{6.93}$ show that, in increasing field or temperature, the system undergoes a sharp transition to the resistive state [1,2]. Below the transition line, the depinning of the vortex lattice is accompanied by large resistance fluctuations and hysteretic behavior [1–3]. Distinct from these effects, there exists the interesting possibility of detecting a sharp frequency intrinsic to the moving lattice. Uniform motion of a periodic structure in a random distribution of pinning sites produces a periodic modulation of the pinning force. As a result, the velocity $\mathbf{v}$ acquires a weak ac component at a frequency (the washboard frequency) given by $\omega/2\pi = \langle v\rangle/a$, where a is the lattice period and $\langle v\rangle$ is the time-averaged (dc) velocity. In principle, if the dc driving current is augmented by a radio-frequency (rf) current $I_{\rm rf}e^{-i\omega_{\rm ext}t}$, interference between the intrinsic oscillations and $I_{\rm rf}$ occurs when ω and $\omega_{\rm ext}$ are harmonically related. In 1971 Fiory [4] performed such ac-dc interference experiments in superconducting thin-film aluminum, and observed steps in the ac impedance. Subsequently, in a different physical context, voltage oscillations at the washboard frequency were observed as "narrow band noise" in the sliding state of the charge-density wave (CDW) conductors [5], as well as in the spin-density wave (SDW) compounds [6]. The oscillations in CDW conductors also lead to a variety of striking interference effects when coupled to an rf current [7]. However, unlike the situation in CDW and SDW systems, the evidence for washboard oscillations in *bulk* superconductors remains highly uncertain. No reports (beyond Fiory's) have appeared for low-T_c or high-T_c superconductors. (We contrast below the washboard effect with the observation of Shapiro steps in a periodic Josephson junction array [8].) We report the detection of the washboard oscillations in a $YBa_2Cu_3O_{6.93}$ crystal by observing their interference with an rf current.

We selected a detwinned crystal with a very low defect density, as measured by the in-plane resistivity (along the chain axis $\mathbf{b}$, $\rho_b = 32\ \mu\Omega\,$cm at 100 K) and the high critical temperature ($T_c = 93.5$ K with a transition width of 0.21 K). The dc current $I_{\rm dc}$ and the rf current $I_{\rm rf}e^{-i\omega_{\rm ext}t}$ are applied simultaneously along the b axis, and the voltage measured in standard four-probe configuration (no periodic structure is fabricated on the crystal). The ac amplitude and frequency are held constant while the washboard frequency ω is scanned by slowly varying $I_{\rm dc}$. To detect the steps, we monitor the differential resistance dV/dI by adding a weak 30 Hz ac component to $I_{\rm dc}$ and using lock-in phase detection to measure the sample voltage (the steps appear as negative peaks in dV/dI).

Unlike in microtwinned crystals, the onset of dissipation in this crystal is very abrupt. The differential resistance dV/dI is unresolvable from zero for a finite range of current less than the depinning current I_p. When I_p is exceeded, dV/dI rises steeply to approach its high-current value. In the inset in Fig. 1, we show dV/dI taken at 89 and 91 K with $I_{\rm rf}$ set to zero. (The zero-dissipation range is not apparent because we have plotted dV/dI against the dc electric field $\mathbf{E} = \mathbf{B} \times \langle\mathbf{v}\rangle$. The magnetic field $\mathbf{B}$ is applied along the c axis.) To detect the interference effect, we introduce an rf current with amplitude larger than the pinning current ($I_{\rm rf} > I_p$), so that the vortex lattice is depinned during a large fraction of the ac cycle. With a large $I_{\rm rf}$, dV/dI is close to its high-current value, except when E is close to zero (main panel of Fig. 1). The ac-dc interference appears as a comb of sharp minima in dV/dI. Comparison with the curves in the inset shows that the main peaks are at least 50 times larger than the background noise [9]. The fundamental (with index 1) typically extends to (5–8)% of the total resistance, and has a full width (at half maximum) $\Delta E_1 \sim \frac{1}{4}E_1$.

 0031-9007/95/74(18)/3684(4)$06.00 © 1995 The American Physical Society

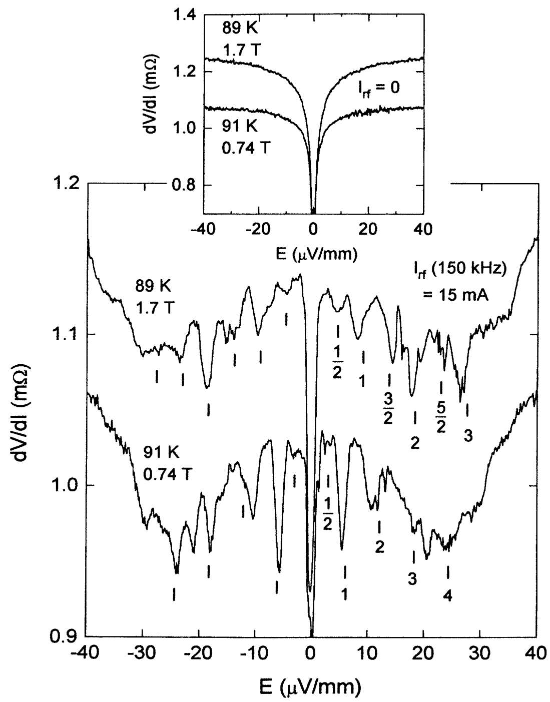

FIG. 1. The differential resistance dV/dI vs the dc electric field E in an untwinned $YBa_2Cu_3O_{6.93}$ crystal at 89 and 91 K in the presence of an rf current $I_{\rm rf}$ (crystal size is $1.5 \times 0.50 \times 0.055$ mm^3). Interference between $I_{\rm rf}$ and the washboard oscillations of the vortex lattice produces steps in the time-averaged velocity $\langle v \rangle$, which appear as negative peaks in dV/dI. The peaks occur when ω and $\omega_{\rm ext}$ are harmonically related. Vertical bars, indexed by p/q, indicate the position of the peaks predicted by $E'_{p/q} = (p/q)\omega_{\rm ext} aB/2\pi$, assuming that all vortices between the voltage leads are depinned ($f = 0$). At 91 K, two series of lines with slightly different washboard frequencies are apparent. The peaks vanish when $I_{\rm rf}$ is set to zero (see inset).

For a triangular lattice, the peak indexed by p/q (with $p, q = 1, 2, \ldots$) occurs when the time-averaged velocity is given by $\langle v \rangle = (p/q) a \omega_{\rm ext}/2\pi$, with $a = (2\phi_0/B\sqrt{3})^{1/2}$, where ϕ_0 is the flux quantum. With this estimate of $\langle v \rangle$, we may compute $E'_{p/q} = \langle v \rangle B$ and compare it with the electric field $E = V/\ell$ derived from the measured potential V ($\ell = 0.66$ mm is the distance between voltage leads). We have indicated $E'_{p/q}$ in Fig. 1 by short vertical bars. In all our traces, we find that $E'_{p/q}$ is fairly close to the observed peak positions. Because the pinning potential is not the same everywhere, we expect that the vortex lattice remains pinned in a fraction f of the space between the voltage leads. Since the chemical potential gradient is finite gradient across the depinned portion only, we should reduce $E'_{p/q}$ by a factor $1 - f$ before comparing it with E. The slight discrepancies between the observed peak positions and the tick marks then provide an estimate of f. The majority of our traces give values of $f < 0.1$, although f may occasionally be as large as 0.2.

As a further check on our results, we have taken traces at different frequencies. If the interference peaks arise from the washboard effect, their positions should increase linearly with $\omega_{\rm ext}$. In Fig. 2 (main panel), we display four traces taken at different $\omega_{\rm ext}$. The field and temperature are fixed at 3.3 T and 86 K, respectively. At 50 kHz, the peaks are small, but still resolvable up to $n = 2$. As $\omega_{\rm ext}$ increases, they shift to higher E (broken lines). In these traces, the slight difference between $E'_{p/q}$ (vertical bars) and the observed peaks at 1/2, 1, and 2 suggests that $f \sim 0.04$. In the inset, the applied rf frequency is plotted against the observed peak positions E (solid symbols). The close agreement between the data and the predicted positions (broken lines) provides strong evidence for the washboard effect.

We next turn to the effect of crossing the transition line. In Fig. 3 we display traces taken as the field is stepped through the transition field B_m (3.45 T at 86 K). At 3.2 and 3.3 T, the peaks at $n = 1$ and 2 are readily resolved. However, starting at 3.4 T, their amplitudes decrease rapidly, becoming barely resolved at 3.5 T. Schmid and Hauger (SH) [10] and Larkin and Ovchinnikov (LO) [11]

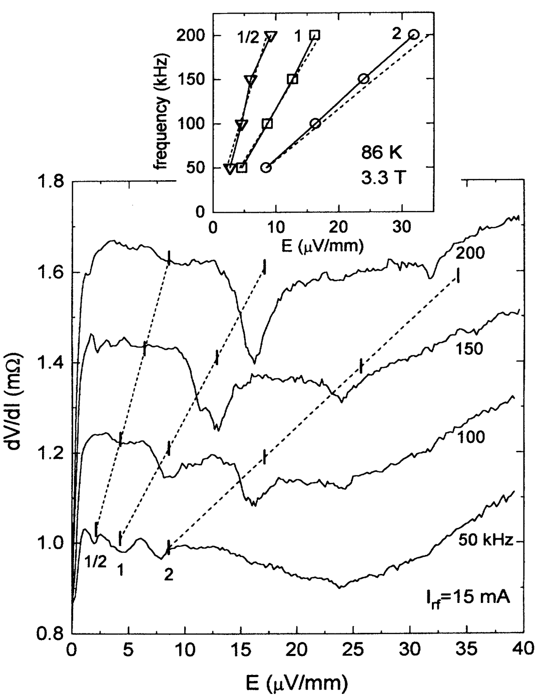

FIG. 2. Four traces of dV/dI vs E taken at 86 K in a 3.3 T field, at different rf frequencies $\omega_{\rm ext}$ (the curves are displaced vertically for clarity). As $\omega_{\rm ext}$ increases from 50 to 200 kHz, the peak positions shift to higher E fields, in agreement with the washboard effect. Vertical bars indicate the predicted positions $E'_{p/q}$ of the interference peaks with index p/q. The inset compares the observed positions of the peaks (open symbols) with the predicted positions $E'_{p/q}$ assuming the pinned fraction $f = 0$.

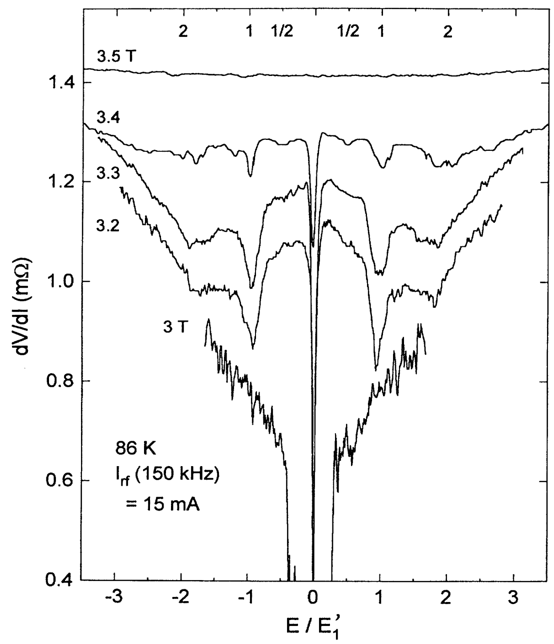

FIG. 3. Traces of dV/dI vs E taken at 86 K in fields B in the vicinity of the "melting transition" field $B_m = 3.45$ T (the E field is normalized to the predicted value E_1'). The peaks are prominent at 3.2 and 3.3 T, but rapidly decrease in amplitude between 3.4 and 3.5 T (where they are barely resolved). The vanishing of the peaks is consistent with the loss of lattice periodicity (see text). At 3 T (lowest curve), the peaks are not resolved. Instead, large low-frequency noise appears in the resistance. The vertical lines near $E/E_1' = \pm 0.3$ reflect the abrupt jumps and hysteretic behavior that appear at 3 T and below.

calculate that the peak amplitudes are determined mostly by contributions from modes close to the reciprocal lattice vectors **G** (see below). In the liquid state, these large contributions vanish. Thus, the observation of interference peaks may be interpreted as evidence for the crystalline phase of the vortex system, while the vanishing of these peaks represents the loss of shear rigidity in the liquid phase. The vanishing of the peaks at the transition has an obvious parallel with the vanishing of Bragg rings in an x-ray powder diffraction study of the melting of a crystalline material.

We have observed the interference peaks only in a restricted range of fields below B_m. For instance, in Fig. 3 the peaks are undetected in the 3.0 T curve (which shows an abrupt increase in noise). The appearance and disappearance of the peaks uncover an interesting vortex phase diagram in the presence of a driving current. Figure 4 displays the profiles of dV/dI versus field, for several values of I_{dc} between 0 and 20 mA (all with $I_{rf} = 0$). In the curve with $I_{dc} = 0$ the abrupt step increase in dissipation at B_m (3.45 T) corresponds to the melting transition reported in Ref. [2]. At finite I_{dc}, however, we distinguish three dissipative regimes. Below ~3.0 T, vortex motion is accompanied by significant low-frequency fluctuations and hysteresis in the resistance, consistent with Refs. [1–3]. The dc resistivity fluctuates by (5–10)% on time scales longer than 1 min (see the trace at 20 mA). Using a spectrum analyzer, we observe the noise to be broadband and featureless. Between 3 and 3.45 T, we observe the interference peaks displayed in Fig. 3. As is apparent in that figure, the low-frequency noise is strongly suppressed in this range. Finally, above 3.45 T, we enter the "liquid" state in which the I-V curves are linear (Ohmic) and the interference peaks vanish. The sharp dip at 2.95 T in the 10 mA curve in Fig. 4 is a precursor to the intermediate phase. The dip corresponds to a sharp cusp in the profile of the depinning current versus B, which is reminiscent of the "peak effect" well studied in low-T_c superconductors (see inset) [12].

The existence of interference steps is characteristic of mode locking in a nonlinear system subject to two or more driving frequencies. We consider a lattice moving with an average velocity $\mathbf{v} + \mathbf{a}\sin\omega_{\text{ext}}t$. In the limit treated by SH and LO (large velocity), the pinning potential $U(\mathbf{r})$ introduces a small dc perturbation $\Delta\mathbf{v}$ to the velocity. In the rest frame of the lattice, $U(\mathbf{r})$ generates an ac force at multiple frequencies $\omega_{\mathbf{k}} = \mathbf{v}\cdot\mathbf{k}$, as well as a mixing between $\omega_{\mathbf{k}}$ and harmonics of the applied frequency $n\omega_{\text{ext}}$. The result is that each lattice mode $\mathbf{x}_{\mathbf{k}}$ is driven at the shifted frequency $(\omega_{\mathbf{k}} - n\omega_{\text{ext}})$. Since, in a rigid lattice, the force matrix $D(\mathbf{k}) \to 0$ at $\mathbf{G}$, the response is

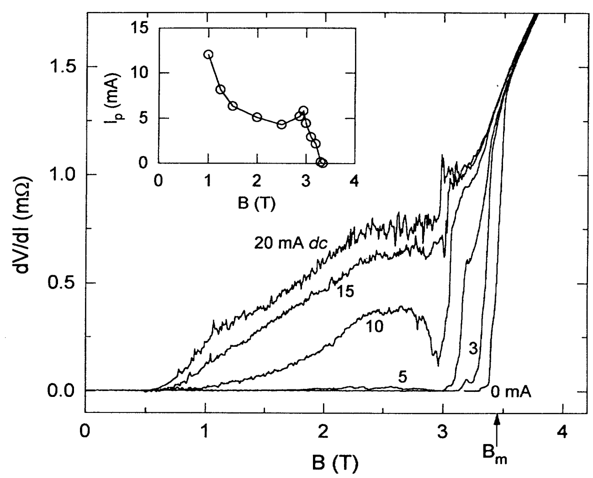

FIG. 4. The differential resistance dV/dI vs B taken at 86 K with the dc bias current I_{dc} fixed at values between 0 and 20 mA ($I_{rf} = 0$). (The resistance is 6.9 mΩ at 94 K at 14 T.) Three regimes of dissipation may be distinguished. In the low-field phase ($B < 3$ T), the resistance displays large-amplitude, low-frequency noise and hysteretic jumps. In the intermediate phase ($3 < B < 3.45$ T), interference peaks are observed in dV/dI in the presence of an rf current. The peaks vanish in the liquid phase above 3.45 T. The depinning current I_p attains a sharp peak just below the intermediate phase (inset).

largest for modes close to $\mathbf{G}$ (driven at the frequency $\omega - n\omega_{\text{ext}}$). When $\omega - n\omega_{\text{ext}}$ is tuned through zero (for some n), these modes become phase coherent. In particular, the simultaneous vanishing of $\text{Im}\,\mathbf{x}_{\mathbf{k}}$ at $\omega - n\omega_{\text{ext}} = 0$ produces a step in $\Delta\mathbf{v}$. The phase coherence leads to macroscopic effects despite the random nature of $U(\mathbf{r})$. This discussion is summarized by SH's expression $\Delta\mathbf{v} \sim \sum_{\mathbf{k},n} iU(\mathbf{k})U(-\mathbf{k})\,\mathbf{k}k^2J_n^2(\mathbf{a}\cdot\mathbf{k}/\omega_{\text{ext}})\,[D(\mathbf{k}) + i\eta(\mathbf{v}\cdot\mathbf{k} - n\omega_{\text{ext}})]^{-1}$, where J_n is a Bessel function and η the viscosity [10]. The contributions from $\mathbf{k}$ near $\mathbf{G}$ (where the summand is largest) lead to steps in $\Delta\mathbf{v}$ whenever $\mathbf{v}\cdot\mathbf{G} = n\omega_{\text{ext}}$. We note that if a gap appears in $D(\mathbf{k})$ at $\mathbf{G}$, these contributions vanish. Thus, the loss of shear rigidity in the moving lattice results in the disappearance of steps in $\Delta\mathbf{v}$.

We distinguish our experiment from giant Shapiro step experiments in a periodic array of $N \times N$ Josephson junctions [8]. In these array experiments, large interference effects are possible because coherence is imposed by the periodicity of the fabricated array (the steps are equally prominent in zero and finite field). In our experiment, by contrast, there is no periodic array of junctions or potential barriers; the phase coherence is established at large velocity by the periodicity of the vortex lattice itself.

As described in Fig. 4, the interference peaks are observed only in a restricted region of the solid phase below the melting curve. A clean observation of the washboard frequency requires that the velocity correlation length ξ_v should be very long. However, if the pinning potential $\langle U\rangle$ is much larger than the energy for domain formation, the depinned lattice will display a broad distribution of velocities, and a short ξ_v. From the dramatic increase in low frequency noise below 3.0 T (Fig. 4), we infer that the flux-flow pattern fluctuates erratically. Both the large hysteresis and the jumps in dc voltage suggest that the depinned state switches randomly between many deep metastable configurations. These features preclude the observation of interference peaks. At higher fields in the intermediate regime (3 to 3.45 T), the appearance of well-resolved interference peaks suggests that $\langle U\rangle$ rapidly decreases while ξ_v increases to macroscopic lengths. The experiment provides direct evidence that the depinned lattice in this range of fields retains shear rigidity, i.e., the vortex system behaves as a *solid* when it is in motion. The peak effect appears to be a distinctive feature separating the strongly fluctuating state at low fields from the weakly pinned state. The existence of finite rigidity and a long ξ_v in a narrow range of fields *above* the peak effect may offer further insight into the origin of this interesting feature [12].

Finally, at the melting field B_m (3.4 T), the loss of shear rigidity and periodicity removes the largest contributions to the sum in $\Delta\mathbf{v}$ at $\mathbf{G}$, so that the interference effect vanishes in the liquid state. The observation of the interference peaks below B_m, but not above, provides essential evidence that the vortex system actually undergoes a transition from the solid to the liquid phase with a concomitant loss of shear rigidity. Without information on the rigidity, it seems possible to consider an alternate interpretation of the jump of the resistivity at B_m as a depinning process unrelated to a thermodynamic transition. However, since the rigidity is unaffected by depinning, we should expect interference peaks on both sides of B_m. Our results rule out this scenario.

J. M. H. and N. P. O. are supported by the U.S. Office of Naval Research. The work of R. G. and L. T. is funded by NSERC of Canada and FCAR of Quebec. L. T. acknowledges support from the Canadian Institute for Advanced Research and the A. P. Sloan Foundation.

[1] H. Safar, P.L. Gammel, D.A. Huse, and D.J. Bishop, Phys. Rev. Lett. **69**, 824 (1992).

[2] W.K. Kwok, J. Fendrich, S. Fleshler, U. Welp, J. Downey, and G.W. Crabtree, Phys. Rev. Lett. **72**, 1092 (1994).

[3] H. Safar (private communication); D.J. Bishop, in *Proceedings of the International Conference on Materials and Mechanisms of High Temperature Superconductivity, Grenoble, France, 1994* (North-Holland, Amsterdam, 1994).

[4] A.T. Fiory, Phys. Rev. Lett. **27**, 501 (1971); Phys. Rev. B **7**, 1881 (1973); **8**, 5039 (1973).

[5] R.M. Fleming and C.C. Grimes, Phys. Rev. Lett. **42**, 1423 (1979).

[6] E. Barthel, G. Kriza, G. Quirion, P. Wzietek, D. Jérome, J.B. Christensen, M. Jørgensen, and L. Bechgaard, Phys. Rev. Lett. **71**, 2825 (1993).

[7] P. Monceau, J. Richard, and M. Renard, Phys. Rev. Lett. **45**, 43 (1980); A. Zettl and G. Grüner, Solid State Commun. **46**, 29 (1983). For a review, see P. Monceau, in *Electronic Properties of Inorganic Quasi-One-Dimensional Materials II,* edited by P. Monceau (Reidel, Dordrecht, 1985), p. 139.

[8] S.P. Benz, M.S. Rzchowski, M. Tinkham, and C.J. Lobb, Phys. Rev. Lett. **64**, 693 (1990); see also P. Martinoli *et al.,* Phys. Rev. Lett. **36**, 382 (1976).

[9] The peaks are well resolved only if I_{rf} exceeds I_p by 50%. As I_{rf} is reduced the peaks rapidly broaden. A study of the variation with I_{rf} is reported elsewhere.

[10] Albert Schmid and Wolfgang Hauger, J. Low Temp. Phys. **11**, 667 (1973).

[11] A.I. Larkin and Yu.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **65**, 1704 (1973) [Sov. Phys. JETP **38**, 854 (1974)].

[12] The peak effect in YBCO is discussed in W.K. Kwok *et al.,* Phys. Rev. Lett. **73**, 2614 (1994).