Interedge Interaction in the Quantum Hall Effect

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We consider the effects of the interaction between electrons drifting along the opposite sides of a narrow sample under the conditions of the quantum Hall effect. A spatial variation of this interaction leads to backward scattering of collective excitations propagating along the edges. Experiments on propagation of the edge modes in samples with constrictions may give information about the strength of the interedge electron interaction in the quantum Hall regime.

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A potential, which confines a two-dimensional electron gas inside a sample, leads to the formation of the so-called edge states near the boundaries of the sample in the presence of a magnetic field [1]. When the chemical potential lies in a gap of the bulk levels the role of the edge states becomes dominant and many experiments in the quantum Hall regime may be interpreted by transport of the edge electrons [2,3] or by propagation of collective edge modes [4–6]. The collective edge excitations are analogous to the edge magnetoplasmon modes [7], but under the conditions of the quantum Hall effect the specifics of the energy spectrum of the electrons near the edges makes these modes rather peculiar [8].

In this work we discuss effects of the interaction between electrons drifting along opposite sides of the sample. These effects may be observed at time-resolved transport phenomena in samples with a nonconstant effective width, e.g., in a strip with constrictions. We show that the variation of the interedge interaction due to the constrictions leads to backward scattering of the collective excitations propagating along the opposite edges. The scattering of waves from one boundary of the sample to the other one is not related with the direct hopping of electrons between the edges. Because of a long-distance electron-electron interaction this scattering may happen when the electron hopping from one side of the sample to the opposite one is completely forbidden. For a particular sample with an extended constriction, which acts as a semitransparent cork, one may observe oscillations in the transparency of the sample as a function of the frequency of the edge wave. The magnitude of the effect gives information about the strength of the interedge interaction, while the period of the transparency oscillations provides one, for a given geometry of the sample, with the value of the velocity of the collective edge modes. In samples with rough boundaries the backward scattering of waves on random inhomogeneities of the boundaries opens a channel for relaxation of the electrons at the edges. This mechanism will be discussed in the final part of the paper.

The strength of the interedge interaction depends on the particular electrostatics of the sample. We intend to present here only a general idea of the study of the interedge electron-electron interaction. For this purpose we shall take as a base a simple picture of an abrupt potential near the edges when the transition between the filled and empty states is sharp. We believe that in such systems the phenomena related with the interedge electron coupling will be more pronounced compared with the systems where gate-confined electron density slowly varies in the lateral direction. For simplicity, we restrict the present discussion to situations when there is only one pair of edge states, i.e., when the filling factor of the lowest Landau level $\nu = 1/n$ where n is odd [9]. Below, we concentrate on the case $\nu = 1$ and we will comment upon the fractional filling in the concluding part.

To begin with, we consider the effect of the electron-electron interaction on the Hall conductance. It will be shown that despite the fact that the diamagnetic current along the boundaries is affected by the interaction, the static limit of the Hall conductance is still quantized when backward hopping of electrons from one edge to the opposite one is absent. This point was already discussed in Ref. [10], but now it will be proven in a rather general way.

We concentrate below on excitations with energy much smaller than the cyclotron frequency. Therefore, the transversal motion of electrons can be excluded by means of the adiabatic approximation. Then, only the longitudinal motion along the strip remains essential, and, finally, one comes to the picture of effectively one-dimensional fermions. The momentum quantization along a conducting strip of a quantum Hall device $k_n = 2\pi n/L_x$ ($n = 0, \pm 1, \pm 2, \ldots$; L_x is the length of the strip along the drift direction) leads to quantization of the center of the orbit of magnetized electrons according to $y_n = l_c^2 k_n$, where $l_c = \sqrt{\hbar c/eB}$ is the magnetic length. The momenta k_u and k_l of the last occupied states at the "upper" and "lower" edges correspond to the $\pm k_F$ of the 1D electron gas, while the drift velocity of the edge electrons is the analog of the bare Fermi velocity. The main difference of the effective one-dimensional theory at hand from the conventional 1D electron gas is the absence of time inversion symmetry: Particles which are moving in opposite directions are spatially separated and may have

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different velocities. For free electrons the quantization of the conductance can be easily obtained in this picture [11].

Let us now discuss the quantization of σ_{xy} in the presence of electron-electron interactions. Because of the mutual coupling of electrons drifting along the edges, their velocity is renormalized. A convenient and economical way to describe the velocity renormalization is to derive the current operators using the continuity equation. When hopping of electrons from one edge to the other is absent the species of electrons on each boundary are well defined, and we can apply the continuity equation for each edge current separately,

$$J_{u,l}(p) = \frac{i}{p}\frac{d}{dt} e\rho_{u,l}(p) = \frac{e}{\hbar p}[H, \rho_{u,l}(p)], \quad (1)$$

where H is the Hamiltonian of the system, and the operators $\rho_{u,l}(p) = \frac{1}{\sqrt{L_x}} \sum_{k \approx k_{u,l}} a^{\dagger}_{k+p} a_k$ describe the creation of charge density excitations on the edges. For the part of the Hamiltonian that describes excitations with energy less than the cyclotron frequency the bosonized representation [8,12] will be exploited:

$$H = \pi \sum_p v_u \rho_u(p)\rho_u(-p) + \pi \sum_p v_l \rho_l(-p)\rho_l(p) + \frac{1}{2}\sum_p W_u(p)\rho_u(p)\rho_u(-p)$$
$$+ \frac{1}{2}\sum_p W_l(p)\rho_l(-p)\rho_l(p) + \sum_p U(p)\rho_l(p)\rho_u(-p) + \text{anharmonic terms}. \quad (2)$$

Here the terms with v_u and v_l represent the energy spectrum of magnetized electrons linearized near the edges, while the nonlinear part of this spectrum can be written in the bosonization technique in the form of an anharmonic interaction [13]. The amplitudes $W_{u,l}$ describe the intraedge Coulomb interaction which is responsible for the enhancement [4,7] of the velocity of the edge modes. The U term describes the interedge electron-electron interactions. In fact, our consideration holds for any Hamiltonian $H\{\rho\}$ describing the edge-state physics by means of a functional of the ρ operators. The $\rho_{u,l}$ operators in Eq. (2) have the standard 1D commutation relations [8,14],

$$[\rho_u(-p), \rho_u(p')] = [\rho_l(p), \rho_l(-p')] = \frac{p}{2\pi}\delta_{p,p'},$$
$$[\rho_l(p), \rho_u(-p')] = 0. \quad (3)$$

For operators commuting like that, performing commutation is equivalent to differentiation, i.e., $[F\{\rho_{u,l}\}, \rho_{u,l}(q)] = \pm(q/2\pi)\,\partial F\{\rho_{u,l}\}/\partial\rho_{u,l}(-q)$. Therefore we can rewrite $J_{u,l}$ in Eq. (1) as

$$J_{u,l}(p) = \pm\frac{e}{2\pi\hbar}\frac{\partial H}{\partial\rho_{u,l}(-p)}. \quad (4)$$

On the other hand, by definition, the chemical potential of the edges is

$$\mu_{u,l}(p) = \frac{\partial H}{\partial\rho_{u,l}(-p)}. \quad (5)$$

Thus, for the total current $I = J_u + J_l$ we obtain

$$I(p) = \frac{e}{2\pi\hbar}[\mu_u(p) - \mu_l(p)]. \quad (6)$$

This gives the quantization of the static Hall conductance in units $e^2/2\pi\hbar$ (in fact, the quantization holds for any p, i.e., locally). It should be emphasized that the electron interactions affect both the currents $J_{u,l}$ and the chemical potentials $\mu_{u,l}$. However, the structure of these corrections is such that the total current is changed in the same way as the difference of the potentials. As a result, these corrections proved to be canceled in the ratio that determines the conductance. Provided that there is no electron hopping between the edges, this fact is obtained here relying on the representation of the Hamiltonian as a functional of the ρ operators. Such representation is not well defined, however, when the density of states at the Fermi energy is singular. For that reason direct application of the above consideration to a system with alternating strips of compressible and incompressible liquids [15] is not possible.

Thus, the Hall transport in the static limit does not provide us with any information on the electron interactions. With a purpose to reveal an effect of the interaction between electrons on different edges let us consider the propagation of the edge modes in an inhomogeneous system. Then, the general form of the bilinear part of the Hamiltonian (2) is

$$H = \frac{1}{2}\int dx\,dy[V_u(x,y)\rho_u(x)\rho_u(y) + V_l(x,y)\rho_l(x)\rho_l(y) + 2U(x,y)\rho_u(x)\rho_l(y)]. \quad (7)$$

In order to diagonalize the Hamiltonian (7) we will write $\rho_{u,l}$ operators as

$$\rho_u(x) = \sum_n \rho_{\mathrm{I}}(-n)\eta_{-n}(x) - \rho_{\mathrm{II}}(-n)\chi_n(x),$$
$$\rho_l(x) = \sum_n \rho_{\mathrm{II}}(n)\eta_{-n}(x) - \rho_{\mathrm{I}}(n)\chi_n(x), \quad (8)$$

where ρ_{I} and ρ_{II} are new operators still satisfying the commutation relations of Eq. (3) in which $\rho_u \to \rho_{\mathrm{I}}$ and $\rho_l \to \rho_{\mathrm{II}}$. The transformation (8) is an inhomogeneous variant of the Bogoliubov transformation similar to the one used in the theory of superconductivity [16]. This transformation diagonalizes the Hamiltonian (7) if the conditions $[H, \rho_{\mathrm{I}}(n)] = v_{\mathrm{I}}(n)n\rho_{\mathrm{I}}(n)$ and $[H, \rho_{\mathrm{II}}(-n)] = v_{\mathrm{II}}(n)n\rho_{\mathrm{II}}(-n)$ are fulfilled. To derive the equations for the eigenfunctions of the wave modes η and χ, we rewrite

commutation relations (3) in the coordinate form and calculate the commutators $[H, \rho_{u,l}(x)]$. Then, replacing $\rho_{u,l}$ by means of $\rho_{\mathrm{I,II}}$ we obtain

$$\omega_n \eta_n(x) = i \int dy[\partial_x V_u(x,y)\,\eta_n(y) - \partial_x U(x,y)\,\chi_n(y)],$$
$$\omega_n \chi_n(x) = i \int dy[\partial_x U(y,x)\,\eta_n(y) - \partial_x V_l(x,y)\,\chi_n(y)], \tag{9}$$

where $\omega_n = v_{\mathrm{I}}(n)\,n$, and a pair of equivalent equations with $\omega_{-n} = v_{\mathrm{II}}(n)\,n$. In the presence of the interedge interaction the eigenmodes are not localized anymore near one of the edges, but are combined from excitations which are located on both sides. These modes can still be classified as left and right movers. In the homogeneous case when the potentials $V_{u,l}(x,y)$ and $U(x,y)$ depend only on $x - y$ Eqs. (9) reproduce correctly the well-known solution of the Tomonaga-Luttinger model [14], which gives two modes propagating with the velocities

$$v_{\mathrm{I,II}} = \pm\frac{1}{4\pi}\Big\{V_u(k) - V_l(k) \pm \sqrt{[V_u(k) + V_l(k)]^2 - 4U(k)^2}\Big\},$$

where $V_{u,l}(k)$ and $U(k)$ are the Fourier transforms of the potentials. The amplitudes $V_{u,l}(k)$ may have a logarithmic dependence on k, if the Coulomb interaction is not efficiently screened.

Without losing generality it will be assumed below that all effects of inhomogeneity are only due to $U(x,y)$. When the wavelength of an eigenmode is much shorter than the characteristic length on which the potential U changes, the adiabatic approximation can be applied. In that case, the propagating modes adjust themselves to the local value of the interaction U and no reflection occurs. The opposite situation, for which the sudden approximation is valid, occurs when the wavelength of the eigenmode is larger than the region where the potential U alters. It can be realized either in a sample with a sudden narrowing of the conducting strip or in a sample, which is partially covered by a metallic gate or by a material with a different dielectric constant. We will model this situation by a potential $U(x,y)$ that vanishes at $x,y < 0$, while for $x,y > 0$ we take $U(x,y) = U\delta(x - y)$ assuming that the characteristic length of the action of the potential $U(x,y)$ is shorter than the wavelengths of the eigenmodes. Consider now a mode propagating along the upper edge to the right. When the incident wave reaches the region of the interedge interaction a backward wave is excited on the lower edge, since in the presence of interedge coupling the eigenmodes are built from waves which are located on both sides. By matching the wave solutions for the semi-infinite strips, we find the transmission coefficient of the incident wave $T = \{1 + r(1 + r)\}^{-1}$, where $r = [(V_u - 2\pi v_{\mathrm{I}})/U]^2$. For the case of symmetric boundaries, when $V_u = V_l = V$, the velocity $v_{\mathrm{I}} = \sqrt{V^2 - U^2}/2\pi$, and for $U \ll V$ we obtain

$$T = 1 - \tfrac{1}{2}(U/V)^2. \tag{10}$$

Another geometry that we consider is a sample in which interedge interaction acts inside a constriction of length L_{int} : $U(x,y) = U\delta(x - y)$ for $0 < x,y < L_{\mathrm{int}}$ and is equal to zero otherwise. In such a free-interacting-free (FIF) junction we find that, due to the multiple back and forth reflections, the transmission coefficient oscillates according to

$$T(\omega) = \frac{1}{1 + |U/V|^2 \sin^2\left(2\pi\omega L_{\mathrm{int}}/V\right)}. \tag{11}$$

Here it was assumed again that $V_u = V_l = V$ and $U \ll V$. These oscillations resemble the oscillations of the differential resistance of a superconducting junction, caused by the Andreev reflection and known as the Tomasch oscillations [17]. An experiment on a FIF junction may provide us with information about the magnitude of the interedge interaction.

Now consider the reflection of edge waves due to inhomogeneities of the boundaries. In resonance experiments on quantum wires or annulus samples this reflection mechanism can determine the width of the resonance. A random variation of the shape of the boundaries creates a random sequence of potentials, which are similar to the potential that was studied at the derivation of Eq. (11). Assuming that the typical length of the inhomogeneities, L_{int}, is smaller than the wavelength, we model this situation by the interedge potential

$$U(x,y) = \sum_i U_i(x - a_i)\,\delta\left(\frac{x - y}{L_{\mathrm{int}}}\right), \tag{12}$$

where a_i are the locations of the inhomogeneities and U_i are some short range potentials.

The propagators of the edge waves will be defined by $\mathcal{D}_{u(l)}(x - y, t - t') = -i\langle T(\rho_{u(l)}(x,t)\,\rho_{u(l)}(y,t'))\rangle$. In the q,ω representation the free propagators are given by

$$\mathcal{D}_u^0 = \frac{q}{\omega - v_{\mathrm{I}} q + i\delta\,\mathrm{sgn}(q)},$$
$$\mathcal{D}_l^0 = \frac{-q}{\omega + v_{\mathrm{II}} q - i\delta\,\mathrm{sgn}(q)}. \tag{13}$$

In the presence of the potential (12) the averaged propagators $\mathcal{D}_{u,l}$ can be found by averaging over a_i as in the case of electrons scattered by random impurities [18]. For the self-energy Σ of the Dyson equation we obtain in this way

$$\Sigma_{u(l)} = \frac{c}{2\pi}\int \overline{L_{\mathrm{int}}^2 \tilde{U}_i^2}\,\mathcal{D}_{l(u)}^0(q)\,dq = -ic\frac{\overline{L_{\mathrm{int}}^2 \tilde{U}_i^2}}{2v_{\mathrm{II}(\mathrm{I})}^2}|\omega|. \tag{14}$$

Here $\tilde{U}_i$ is the Fourier transform of the potentials U_i, the bar means averaging over the scatterers, and c denotes their concentration. Thus the averaged propagators are

$$\mathcal{D}_u(q,\omega) = \frac{q}{\omega - v_{\mathrm{I}} q + iv_{\mathrm{I}} q/|\omega|\tau_u}, \tag{15}$$

where $\tau_u(\omega)^{-1} = c(\overline{L_{\mathrm{int}}^2 \tilde{U}_i^2}/v_{\mathrm{II}}^2 v_{\mathrm{I}})\,\omega^2$, and a similar expression for $\mathcal{D}_l$. The result for τ^{-1} is consistent with

Eq. (11) in the long wavelength limit ($2\pi v/\omega \gg L_{\rm int}$). The obtained ω^2 dependence of the scattering rate is a consequence of the general properties of wave scattering in continuous mediums. When the space quantization is essential and the level spacing becomes larger than τ^{-1}, the integration in Eq. (14) should be substituted by summation over discrete momenta. In that case, the question of the symmetry between the boundaries becomes delicate. For symmetrical boundaries a self-consistent treatment of the resonance width gives $\tau^{-1} \propto |\omega|$.

The obtained result allows us to discuss the absorption of an electromagnetic field when the external electric field is parallel to the edges. Using the continuity equation we get from Eq. (15) the absorption coefficient,

$$\sigma = \frac{e^2}{\omega^2}\left(\frac{v_{\rm I}}{\tau_u(\omega)} + \frac{v_{\rm II}}{\tau_l(\omega)}\right). \qquad (16)$$

In the continuous case, when the level spacing is less than $\tau_{u,l}^{-1}$, the adsorption coefficient does not depend on the frequency of the ω^2 behavior of $\tau_{u,l}^{-1}$.

An interesting mechanism for electron relaxation arises as a consequence of the random spatial variation of the interedge interaction. For a homogeneous system the energy and momentum conservations forbid an emission of waves propagating opposite to a motion of the electron. Because of spatial inhomogeneities, the momentum conservation does not restrict the decay process anymore. After averaging over the inhomogeneities, the expression for $1/\tau^e(\varepsilon)$ determining the rate of emission of the edge waves can be written as

$$1/\tau^e_{u(l)}(\varepsilon) = \int \Sigma_{l(u)}(\omega)\, \mathcal{G}_{u(l)}(\varepsilon - \omega, q)\, dq\, d\omega\,. \qquad (17)$$

Here the integration with respect to the momentum variable q in the electron Green's function $\mathcal{G}$ should be performed independently of Σ. This yields $1/\tau^e(\varepsilon) \propto \varepsilon^2$ (or $\propto T^2$ at finite temperatures). The time τ^e determines the rate of the equilibration of electron states on the opposite edges.

In summary, we have discussed the effects related with the interedge electron interaction in the quantum Hall regime. We have shown that experiments on propagation of the collective excitations in samples with constrictions may give information about the amplitudes of the electron-electron interaction which are determined by the conditions of the screening. We expect that the interedge coupling effects will be considerably stronger in systems with a sharp transition between filled and empty states, which were under discussion here, compared with the systems with slowly varying electron density, where electron screening dominates [15]. Such effects may help to distinguish between these two types of systems.

The discussion above was related to the case when the filling factor $\nu = 1$. In the case of the fractional filling with $\nu = 1/n$ (n is odd) the physics of the two collective edge modes will be described, we believe, by a phenomenological Hamiltonian $H\{\rho\}$ of the type of Eq. (7). The specifics of the fractional state reveals in the commutation relations for ρ operators: on the right hand side of Eq. (3) the factor ν appears [8]. This modification does not alter the physics of the discussed phenomena. However, certain corrections should be performed, e.g., in Eq. (11) in the argument of the sinus a factor ν^{-1} should be introduced, because now $v_{\rm I,II} \propto \nu$.

This consideration is related to wavelengths which are shorter than the sample length. When the velocity of the edge mode is about 10^8 cm/sec as in the experiments of Refs. [5,6], for a sample with length of the order of 1 mm the frequency should be about 1 GHz. Presumably a geometry convenient for studying the effects of the interedge interaction is a pair of coupled rectangular mesas.

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