Measurement of Atomic Motion in a Standing Light Field by Homodyne Detection

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It is shown that the photocurrent measured using homodyne detection of the phase quadrature on the output field of a cavity gives information about the transverse motion of an atom passing through the standing light field inside the cavity. Monte Carlo simulation techniques are used which result in quantum trajectories of the system conditioned on the measured photocurrent, i.e., the photocurrent defines the backaction of the continuous measurement on the atomic motion.

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A number of quantum measurements involving the interaction of an atom with a cavity light field have been proposed. The photon statistics of the cavity field may be probed by measuring the atomic phase shift [1] or the atomic deflection [2–4]. Alternatively the position at which an atom passes through the standing wave may be determined by measuring the phase shift imparted to the light field [5,6]. This yields position resolution better than the wavelength of the light. An alternative method to determine the position of the light has been demonstrated by utilizing a spatially varying level shift which enables one to correlate the atomic position with its resonance frequency [7]. This technique does not, however, give an entangled atom-field state. In this paper we show that it is possible to monitor the transverse motion of an atom in a standing wave light field by a continuous homodyne measurement of the phase of the light. We consider an atom passing through a standing wave light field in a cavity. The interaction of the atom will impart a phase to the light field, depending on which position in the standing wave the atom passes through. This phase can be detected by a homodyne measurement of the output field, giving a measure of the transverse motion of the atom. This system was analyzed by Storey *et al.* [5] for an atom passing quickly through the standing wave so that the transverse motion of the atom while in the standing wave can be ignored. A single measurement of the quadrature phase of the output light was sufficient to determine the position of the atom.

We wish to consider the situation where the transverse motion of the atom in the standing wave field cannot be neglected. In this case we can monitor the transverse motion of the atom by making continuous measurements on the quadrature phase of the output light. The continuous measurement is modeled using Monte Carlo simulation techniques described by Wiseman and Milburn [8] and by Carmichael [9].

We consider a two level atom described by the pseudospin σ_+, σ_-, and σ_z interacting with a quantized cavity field mode a with dipole constant g and detuning $\Delta = \omega_A - \omega_{\mathrm{cav}}$. The cavity decay rate is γ_{cav} and the spontaneous emission rate of the atoms is γ_A. The cavity is driven with a coherent driving field with amplitude β. A homodyne measurement is performed on the phase quadrature $X_{\pi/2} = i(a - a^\dagger)$ of the output field from the cavity.

Using the method of quantum trajectories the evolution of this system is described by the unnormalized stochastic Schrödinger equation:

$$d|\psi_c\rangle = \left\{\left[-\frac{i}{\hbar}H_{\mathrm{eff}} - i\sqrt{\gamma_{\mathrm{cav}}}\left(\sqrt{\gamma_{\mathrm{cav}}}\langle X_{\pi/2}\rangle + \zeta\right)a\right]dt + \sqrt{\gamma_A}\int_{-1}^{1} dk_r\, \tfrac{3}{4}(1 - k_r^2)e^{ik_r x}\sigma_-\, dN_{k_r}\right\}|\psi_c\rangle, \quad (1)$$

where

$$H_{\mathrm{eff}} = \frac{p^2}{2m} + \hbar\Delta\sigma_z + i\hbar g\cos(kx + \phi)(a^\dagger\sigma_- - a\sigma_+) - i\hbar\frac{\gamma_{\mathrm{cav}}}{2}a^\dagger a + i\hbar\sqrt{\gamma_{\mathrm{cav}}}\beta(a^\dagger - a) - i\hbar\frac{\gamma_A}{2}\sigma_+\sigma_- \quad (2)$$

is a non-Hermitian effective Hamiltonian. ζ represents Gaussian white noise, and the term $\sqrt{\gamma_{\mathrm{cav}}}(\sqrt{\gamma_{\mathrm{cav}}}\langle X_{\pi/2}\rangle + \zeta)$ is proportional to the measured photocurrent and is conditioning the system on the measurement. The noise increment $dN_{k_r}(t) = dN_{k_r}(t)^2$ represents spontaneous emission into the vacuum modes of the bath. In order to compensate losses and maintain a stable intracavity intensity the cavity is driven by a coherent driving field with amplitude β:

$$\beta = \left(\frac{1}{2}\sqrt{\gamma_{\mathrm{cav}}} + \frac{1}{4\sqrt{\gamma_{\mathrm{cav}}}}\frac{\gamma_A}{\Delta}\frac{g^2}{\Delta}\right)\alpha, \quad (3)$$

where α is the amplitude of the original coherent state in the cavity. The first term in (3) compensates for cavity losses and the second term for spontaneous emission. This can be derived by adiabatically eliminating the internal atomic degree of freedom in the large detuning limit $\gamma_A/\Delta \ll 1$, $(g/\Delta)|\alpha| \ll 1$. The net spontaneous emission loss is the emission rate γ_A times the population of the upper level, $(g^2/\Delta^2)\alpha\cos^2(kx + \phi)$, with $\cos^2(kx + \phi)$ averaged to $\frac{1}{2}$. We do not adjust the driving intensity to compensate for the phase shift due the atom, since this is precisely the effect we wish to measure.

The stochastic Schrödinger equation (1) describes the time evolution of the quantum trajectory wave function $|\psi_c(t)\rangle$. The subscript c indicates the fact that we are dealing with a conditioned wave function. It describes the state of the open system (atom plus cavity field) at time t, conditioned on the particular history of measurement records recorded at the detectors monitoring the system prior to t. In this particular case two different measurements are involved, which are modeled by two independent stochastic processes. There is a quantum diffusion process which gives the conditioning of the system due to the homodyne detection of the output field of the cavity. In homodyne detection the output field is added coherently at a beam splitter to a strong local oscillator field. This field is detected by a photoelectron detector. The measurement record is a stochastic photocurrent:

$$I(t) \propto \sqrt{\gamma_{\rm cav}}\,[\sqrt{\gamma_{\rm cav}}\,\langle X_{\pi/2}\rangle(t) + \zeta(t)]. \qquad (4)$$

The conditioning of the wave function on the measured photocurrent is given by a diffusion term of the form [8] $-i\sqrt{\gamma_{\rm cav}}\,(\sqrt{\gamma_{\rm cav}}\,\langle X_{\pi/2}\rangle + \zeta)a$. The second stochastic process describes the detection of a spontaneously emitted photon, and is given by a quantum jump process where the occurrence of a jump corresponds to the count of an emitted photon by the detector. This process is described by the stochastic term $\sqrt{\gamma_A}\,e^{ik_r x}\sigma_- dN_{k_r}$. There are periods of evolution of the system unaffected by this stochastic term interrupted at random times by the wave-function collapse $|\psi_c\rangle \longrightarrow \sqrt{\gamma_A}\,e^{ik_r x}\sigma_-|\psi_c\rangle$. The random collapse times are determined in a Monte Carlo fashion from the rate $\delta = \gamma_A\langle\tilde{\psi}_c(t)|\sigma_+\sigma_-|\tilde{\psi}_c(t)\rangle$, where $|\tilde{\psi}_c\rangle$ denotes the normalized wave function. The translation operator in momentum space $e^{ik_r x}$ defines the recoil of the emitted photon, where the momentum $\hbar k_r$ is given by the projection of the wave vector $\vec{k}$ of the emitted photon onto the transverse direction. The probability for spontaneous emission into a solid angle $d\Omega$ is $P = (3/8\pi)\sin^2\theta\,d\Omega$, where θ denotes the angle between the polarization direction of the linearly polarized incident field and the wave vector $\vec{k}$.

The connection between a single quantum trajectory $|\tilde{\psi}_c(t)\rangle$ and the solution of the corresponding master equation $\rho(t)$ is given by $\rho(t) = \langle|\tilde{\psi}_c(t)\rangle\langle\tilde{\psi}_c(t)|\rangle$, where $\langle\cdots\rangle$ denotes the ensemble average over realizations of the two stochastic processes. Our objective is to show the relation between the measured photocurrent and the transverse motion of an atom in the cavity field. This implies that we are more interested in the properties of a single quantum trajectory than in the solution of the master equation because $\rho(t)$ does not describe a particular quantum measurement scheme, that is, different measurements (for example, the measurement of the amplitude quadrature instead the phase quadrature) result in the same master equation.

In our quantum trajectory treatment we have simulated the motion of a cesium atom. The active atomic transition is chosen to be the $6S_{1/2}, F = 4 \longrightarrow 6P_{3/2}, F = 5$ transition which has a wavelength $\lambda = 852$ nm. We choose the atom field coupling constant $g = 7.16$ MHz and the cavity lifetime $\tau_{\rm cav} = 0.18$ μs which are the experimental parameters of Kimble *et al.* [10]. Such a high atom field coupling and cavity lifetime is necessary for effective monitoring of the atomic motion. We choose for the detuning $\Delta = 26\gamma_A$, where γ_A for this transition is $\gamma_A = 32.3 \times 10^6$ rad/s. The choice of the detuning Δ is a compromise between a reasonably large effective atom-field interaction strength g^2/Δ, which gives the resolution of the atomic position stored in the cavity field, and suppressing the spontaneous emission, which disturbs the homodyne detection of the phase quadrature. The amplitude of the intial coherent cavity field state is choosen to be $\alpha = 3$.

Figure 1 shows the center of mass motion of an atom in the cavity light field. The abscissa gives the time in decay times of the cavity. The total time of the simulation is $320/\gamma_{\rm cav}$. The ordinate gives the expectation value $\langle x\rangle$ of the position operator in units of wavelength of the cavity field. The atom initially starts halfway between a node and antinode of the standing light field, where "0" in the figure indicates the location of an antinode. Furthermore, we assume that the atom is initially in the ground state. The initial state is a Gaussian wave function with standard deviation $\sigma = \lambda/20$, i.e., the atom starts well localized in the potential well. In the limit of large detuning one can think about the motion of the atom as if it were in the potential $-(g^2/\Delta)a^\dagger a\cos^2(kx)$. For this, one has to assume that the atom is for all times in the ground state. The figure shows an oscillatory motion of the atom in the potential well. This oscillation would be washed out by averaging over many trajectories since each would start with different values of $\langle x\rangle$. It should be mentioned here that there is a nonzero probability for the atom to go into one or both neighboring potential wells. In this case the atom is in general not trapped and travels through the

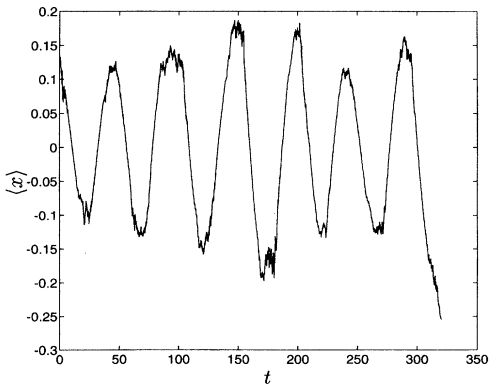

FIG. 1. The center of mass motion of the atom $\langle x\rangle$ (in units of the optical wavelength) is plotted versus the time in cavity lifetimes.

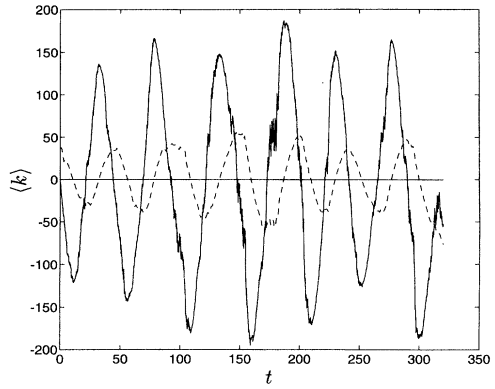

FIG. 2. The atomic velocity $\langle k \rangle$ (solid line) and the center of mass motion $\langle x \rangle$ (dashed line) versus the time in cavity lifetimes.

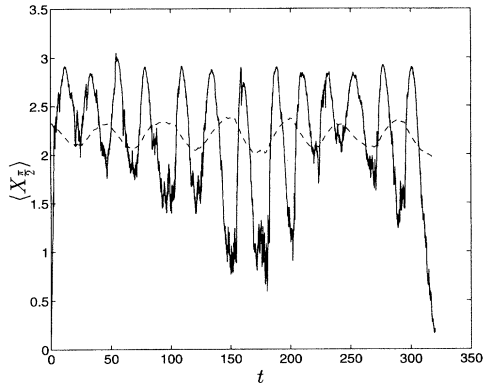

FIG. 3. The expectation value of the phase quadrature $\langle X_{\pi/2} \rangle$ (solid line) and the center of mass motion $\langle x \rangle$ (dashed line) versus the time in cavity lifetimes.

standing wave. We are more interested in those quantum trajectories where the atom is trapped for a sufficiently long time and moves in a single potential well. The main reason for this is that the measurement, as we will see, gives information about the atomic position relative to the node or antinode of the light field but not about which potential well or wells the atom is in.

Figure 2 shows the velocity as measured by the atomic wave number scaled by the optical wavelength $k = \lambda p/\hbar$. The velocity of the atom is as expected closely related to the center of mass motion: the velocity is greatest if the atom is in the minimum of the potential and least at the turning points of the motion. To show this, we have also plotted a graph of $\langle x \rangle$ (dashed line).

The trapped atom is, in general, better localized than the initial Gaussian state, i.e., $\mathrm{Var}(x)(t) < \mathrm{Var}(x)(t = 0)$. If the atomic center of mass becomes close to a node of the standing light field (a maximum of the potential), the atom becomes delocalized. This can cause $\mathrm{Var}(x)$ to be significantly larger than its intial value. Furthermore, it can be observed that the localization of the atom changes periodically, i.e., the atomic wave function breathes. The breathing of the wave function is not closely related to the atomic motion. The atom is always less localized in momentum space than the intial value $\mathrm{Var}(k)(t = 0)$. Unless the atom is close to a node of the field, the dynamical behavior of $\mathrm{Var}(k)$ is the inverse of that of $\mathrm{Var}(x)$: An improvement of the localization in position space causes a delocalization in momentum space, and vice versa. The uncertainty product $\mathrm{Var}(x) \times \mathrm{Var}(k)$ is always larger than the minimal value, so the atom is not in a Gaussian state. This uncertainty product has, roughly speaking, its best (smallest) value if the atom is close to an antinode of the field.

Figure 3 shows the relation between the expectation value of the phase quadrature $\langle X_{\pi/2} \rangle$ (solid line) and the center of mass motion of the atom $\langle x \rangle$ (dashed line). As mentioned above, the measured photocurrent is given by $\langle X_{\pi/2} \rangle$ plus white noise. It can be seen in Fig. 3 that the phase quadrature is oscillating with double the frequency of the center of mass motion. $\langle X_{\pi/2} \rangle(t)$ is greatest if the atom is located in the minimum of the potential, and least if the atomic position is closest to a node of the standing wave, i.e., at the turning points of the motion. The amplitude of the oscillation of $\langle X_{\pi/2} \rangle$ is related to the amplitude of $\langle x \rangle$, i.e., a larger amplitude of the atomic motion causes, in general, a larger amplitude of $\langle X_{\pi/2} \rangle$. This effect can be best observed if the atom reaches a node of the field. The amplitude of the atomic motion is random, i.e., it differs from trajectory to trajectory.

The measured photocurrent is given by $I(t) \propto \sqrt{\gamma_{\mathrm{cav}}}[\sqrt{\gamma_{\mathrm{cav}}}\langle X_{\pi/2} \rangle(t) + \zeta(t)]$. To get from this noisy photocurrent an interpretable signal one has to apply an "electronic" filter. We have applied a function which mimics a Butterworth filter of third order to $I(t)$. The result is shown in Fig. 4 (solid line) on which $\langle X_{\pi/2} \rangle$ has also been plotted (dashed line). The Butterworth filter phase shifts the signal and reproduces correctly the frequency of the oscillation which carries the main

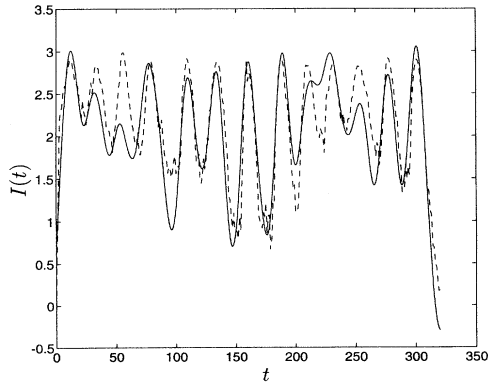

FIG. 4. The filtered photocurrent $I(t)$ (solid line) and the phase quadrature $\langle X_{\pi/2} \rangle$ (dashed line) versus the time in cavity lifetimes.

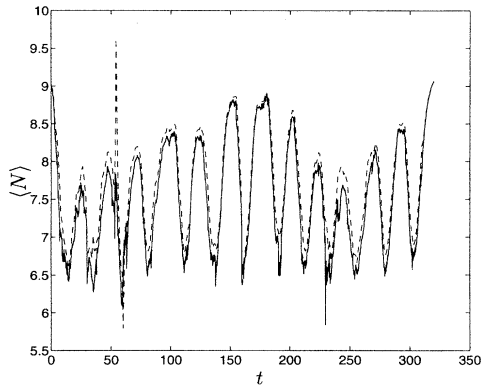

FIG. 5. The mean photon number $\langle N\rangle$ (solid line) and its variance $\mathrm{Var}(N)$ (dashed line) versus the time in cavity lifetimes.

information about the atomic motion. The phase shift is not shown in Fig. 4. To get a reasonably good signal-to-noise ratio the atomic motion should be slow on the time scale given by the cavity life time τ_{cav} and the amplitude of the oscillations of the phase quadrature $\langle X_{\pi/2}\rangle$ should be reasonably large.

In Fig. 5 the mean photon number $\langle N\rangle(t)$ (solid line) of the cavity field is shown, and the variance of N, $\mathrm{Var}(N)(t)$ (dashed line). The time dependence of $\langle N\rangle$ is closely related to that of $\langle X_{\pi/2}\rangle$, i.e., the oscillation of $\langle N\rangle$ is phase shifted by $\pi/2$ with respect to the oscillation of the phase quadrature $\langle X_{\pi/2}\rangle$. If the phase of the field becomes maximal through the interaction with the atom, the field will be maximally off resonant with the constant driving field, and this causes the greatest decrease in the field intensity. A decrease of the field phase causes by the same effect an increase in the field intensity. The variance $\mathrm{Var}(N)$ is usually a little bit greater than $\langle N\rangle$, but is sometimes a little smaller than $\langle N\rangle$ due to spontaneous emission, so the cavity field state is always close to a coherent state.

Figure 6 shows the inversion, $\langle\sigma_z\rangle$, of the two atomic levels. It turns out that due to the large detuning Δ, the

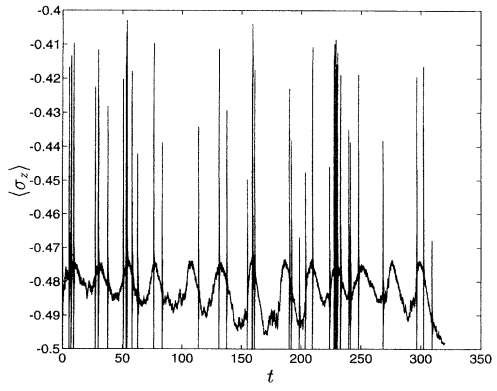

FIG. 6. The inversion $\langle\sigma_z\rangle$ versus the time in cavity lifetimes.

mean of σ_z differs by approximately 2% from the ground state. The spikes in Fig. 6 are the atom jumping into the ground state when a photon is spontaneously emitted, followed by damped Rabi oscillations that bring the atom approximately back to the value of $\langle\sigma_z\rangle$ before the jump. The Rabi oscillations occur on a time scale which is much shorter than the cavity lifetime. In Fig. 6 a further low frequency oscillation can also be seen. This oscillation is caused by $\langle\sigma_z\rangle$ following, dynamically, the motion of the atom, i.e., $\langle\sigma_z\rangle$ is greatest if the atom is located in the minimum of the potential. Therefore the time dependent behavior of $\langle\sigma_z\rangle$ is correlated to that of $\langle X_{\pi/2}\rangle$.

To summarize, we have shown that the oscillatory transverse motion of an atom in a standing light field inside a cavity yields an oscillation in the measured photocurrent. The photocurrent is maximal if the atom is close to an antinode of the cavity field, and minimal if the atom is closest to a node of the field. We have demonstrated that the atom can be tracked through a watchdog effect via the coupling to the measurement device, where the continous measurement of the phase quadrature on the output field of the cavity gives the backaction on the atom.

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