Band Staggering in Some Superdeformed States and Intrinsic Vortical Motion

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The staggering recently observed in some superdeformed rotational bands could be explained by a collective model including two quantized quantities. One of the latter being the angular momentum I, we propose for the other the Kelvin circulation J associated with an intrinsic uniform vortical motion. This explanation is consistent with the observed strong $E2$ intraband transitions. Some preliminary qualitative assessment of the relevant mass parameters is made.

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Recently, an energy staggering, yielding a $\Delta I = 4\hbar$ quasiperiodic structure, has been found experimentally for some rotational bands in superdeformed nuclear states. These structures whose amplitudes correspond merely to less than 0.1% of the observed transition energies have been found in the currently available data only in ^{149}Gd for one band [1] and in ^{194}Hg for three bands [2]. However, a few other candidates are tentatively proposed in nuclei of the same superdeformation regions or in the $A \approx 130$ region [3]. The $\Delta I = 4\hbar$ character of this phenomenon has prompted some explanations involving the hexadecapole deformation, namely, the Y_{44} collective degree of freedom [1,4,5]. Such approaches generally imply a significant static deformation for that mode, which does not yet seem to have been clearly established for these nuclei [6].

We will present here an alternative explanation, first on rather general grounds. In a second step, we will provide a possible collective model realization of this, namely, by taking into account a quantized intrinsic vortical motion.

Let us first assume that the energy of the system depends quadratically on two quantities I and J:

$$E(I,J) = \lambda\left(\frac{A}{2}I^2 - BIJ + \frac{C}{2}J^2\right). \tag{1}$$

In Eq. (1), the factor λ has been introduced for reasons which will appear clear later. The above quadratic assumption is made here only for the sake of simplification. We introduce now a smooth function $\overline{J}_{\text{yrast}}(I)$ yielding the minimum of J for a continuous energy function as given by Eq. (1). The resulting function $\overline{E}_{\text{yrast}}(I) = E(I,\overline{J}_{\text{yrast}}(I))$ could be named as a "smooth yrast energy." We thus have

$$\overline{J}_{\text{yrast}}(I) = \frac{B}{C}I, \qquad \overline{E}_{\text{yrast}}(I) = (AC - B^2)\lambda\frac{I^2}{2C}. \tag{2}$$

Let us assume now that I is the angular momentum and that J is the moment conjugated to a cyclic variable θ varying within the finite interval defined by $0 \le \theta \le 2\pi/p$, where p is an integer. Then both I and J are quantized. For instance, we suppose that they both take the values $2p\hbar$. Moreover, we assume that I^2 and J^2 are replaced in the 3-dimensional quantal version of the energy in Eq. (1) by $I(I+\hbar)$ and $J(J+\hbar)$, respectively, whereas the product of I by J stands unchanged (as it would be the case, e.g., for a scalar product of aligned vectors). Upon Taylor expanding the quantal energy around $J = \overline{J}_{\text{yrast}}(I)$, one obtains the following for the yrast energy corresponding to the quantal yrast value $J = J_{\text{yrast}}(I)$:

$$E_{\text{yrast}}(I) = \overline{E}_{\text{yrast}}(I) + \tfrac{1}{2}\lambda\hbar(A+B)I + \delta_{\text{yrast}}(I),$$
$$\delta_{\text{yrast}}(I) = \frac{\lambda C}{2}\left[J_{\text{yrast}}(I) - \frac{B}{C}I\right]^2. \tag{3}$$

Let us determine now $J_{\text{yrast}}(I)$. If J would be a continuous variable, one would have in lieu of the first Eq. (2):

$$J_{\text{yrast}}(I) = \frac{B}{C}I - \frac{\hbar}{2}. \tag{4}$$

In fact the quantized $J_{\text{yrast}}(I)$ is equal to either $J_{\langle} = 2\hbar[(B/C)p - \frac{1}{4}]$, where $[x]$ stands for the integer part of x, or $J_{\rangle} = 2\hbar[(B/C)p + \frac{3}{4}]$, whichever is closer to the continuous approximation of Eq. (4). The yrast band is made of pieces of parabola, each one corresponding to a different quantized value of J. These quantal leaps in J_{yrast} as a function of I are the causes of the staggering under consideration. Note that the same arguments could have been applied for states other than the yrast ones.

The parameters A, B, and C appearing in Eq. (1) may depend on many yet unknown factors and vary from one nucleus to another. Then it may happen that in some cases that the ratio B/C is close to $\frac{1}{2}$. This particular case is well suited for the experimental phenomenon considered here. Indeed when $B/C = 1/2$, one finds

$$J_{\text{yrast}}(I) = 2\hbar\left(\frac{I}{4\hbar}\right) \tag{5}$$

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and

$$\delta_{\mathrm{yrast}}(I) = \frac{\lambda \hbar^2 C}{2}\left[2\left(\frac{I}{4\hbar}\right) - \frac{I}{2\hbar}\right]^2. \qquad (6)$$

The value of the latter is thus dependent on the divisibility of the even number $I/\hbar$ with respect to 4. It is vanishing whenever $I/\hbar$ is divisible by 4, and equal to $\lambda \hbar^2 C/2$, otherwise. If one defines the transition energies between two adjacent yrast states coupled by a collective electromagnetic field as

$$E_\gamma(I) = E(I + 2\hbar) - E(I), \qquad (7)$$

the absolute value of the associated staggering parameter $\Delta E_\gamma(I)$, according to the symmetrized quadratic interpolation of Ref. [2], is found to be I independent:

$$\Delta E_\gamma(I) = \frac{\lambda \hbar^2 C}{2}. \qquad (8)$$

If B/C would not have been exactly equal to $\frac{1}{2}$, but somewhat close to it, one would have observed a quasiperiodic pattern, namely, with alternating signs for $\Delta E_\gamma(I)$, except for some values of I where the positive or negative values could be observed twice.

Let us develop now a particular realization of the preceding collective model. We describe the dynamics of rapidly rotating nuclei by combining a global rotation with a uniform intrinsic vortical motion, further assuming the global angular velocity and the intrinsic vorticity to be aligned on the same axis (e.g., the z axis). The corresponding classical collective velocity field $\mathbf{u}(\mathbf{r})$ in the laboratory frame would have the following components on the principal axis of the nucleus:

$$u_x = -(\Omega + q\omega)y, \qquad u_y = \left(\Omega + \frac{\omega}{q}\right)x, \qquad u_z = 0, \qquad (9)$$

introducing the global Ω and vortical ω aligned angular velocities and defining q as the ratio of the characteristic lengths a_x and a_y in the x and y directions, respectively. This field is a particular solution of the well-known classical Dirichlet problem for the most general linear field bounded by an ellipsoidal surface as studied by Riemann and Chandrasekhar [7]. Using such a linear velocity field in nuclear physics had been already proposed by many authors, including Cusson [8], Rosensteel and Rowe [9–11], and one of the authors [12,13]. The special case where Ω and ω are collinear has been dubbed by Chandrasekhar as the Riemann S-type ellipsoid case. The case where these axes are not collinear is less suited to superdeformed bands and has been classically studied by Chandrasekhar (P-type ellipsoids). It may be relevant for the description of the so-called oblate bands. Recently, a semiquantal description of such S-type collective motions has been discussed [14]. It amounts to a generalization of the usual Routhian approach [15]. Upon solving such a generalized cranking problem, one gets the total laboratory energy E as a function of Ω and ω. Even though the study of the most general case is possible and has been sketched in Ref. [14], we will discuss here only the case where the energy is a quadratic function of Ω and ω:

$$E(\Omega, \omega) = \tfrac{1}{2}A\omega^2 + B\omega\Omega + \tfrac{1}{2}C\Omega^2. \qquad (10)$$

Defining two functions I and J of Ω and ω, and their inverses, such that

$$\frac{\partial E}{\partial I} = \Omega, \qquad \frac{\partial E}{\partial J} = \omega, \qquad (11)$$

one then gets

$$I = B\omega + C\Omega, \qquad J = A\omega + B\Omega \qquad (12)$$

and the expression of Eq. (1) for the energy with the following overall factor λ:

$$\lambda = 1/(AC - B^2). \qquad (13)$$

Some questions remain, however, to be answered before proposing the model of a global rotation combined with an intrinsic vortical motion as a possible explanation for the $\Delta I = 4\hbar$ staggering. The first one concerns the quantification of J which obviously plays a major role in the proposed explanation. Its quantized character may be hinted, somewhat intuitively, by the fact that it is canonically conjugated with an angle θ defining the intrinsic rotation [12,14]. Indeed, the motion defined by Eq. (9) can be decomposed as the product of an inverse scaling to a sphere, further rotated by an angle θ, scaled back to its original deformed shape, and then rotated as a whole by an angle Θ. In order to obtain the interaction term $B\omega\Omega$ in Eq. (10), both rotation axes must be collinear. The matching conditions at the boundaries of the definition interval for the angular variable θ, together with the C_2 symmetry associated with the so-called left-right symmetry, accounts for the quantization of J by steps of $2\hbar$. A more rigorous proof of the latter has been provided by the group theoretical approaches of Refs. [9] and [13]. In Ref. [13], for instance, the components of $\mathbf{J}$ are shown to satisfy Poisson bracket relations (thence the commutation relations) similar to those of an angular momentum, leading thus to the integer (or half-integer in general) character of J. In Ref. [9], it is shown in particular that whenever the projections of both $\mathbf{I}$ and $\mathbf{J}$ on the quantization axis are vanishing, the integer numbers I and J (in units of $\hbar$) should have the same parity, implying thus the quantization of J by steps of $2\hbar$ as a consequence of the C_2 symmetry. Let us note finally that J is called, in Ref. [11], for instance, the Kelvin circulation.

Another point must be clarified now. Indeed the previous speculations are merely based on energies. Clearly more insight should be gained by assessing the electromagnetic properties of such states. In particular, does the fragmentation of the yrast line into several parabolic cusps prevent such a set of states to constitute a band whose members are linked by strong $E2$ transitions? Let us answer this question in the idealized case of the $\Delta I = 4\hbar$

energy structure given by Eq. (6). It is likely that the residual interaction will mix for a given value of I, the unperturbed yrast state (which is an eigenstate of $\mathbf{I}^2$ and $\mathbf{J}^2$) and its closest neighbor, as for instance

$$|I\rangle_{\text{yrast}} = \cos\varphi|I, J_0\rangle + \sin\varphi|I, J_0 + 2\rangle, \quad (14)$$

$$|I - 2\rangle_{\text{yrast}} = \cos\psi|I - 2, J_0\rangle + \sin\psi|I - 2, J_0 + 2\rangle, \quad (15)$$

with obvious notation. It may be proven [14] that the Poison bracket of J with an $E2$ perturbative electromagnetic field is vanishing. Therefore, in the quantal case, the J value is not affected by this supplementary field (see also Ref. [9]), so that between two yrast states with spins I and $I - 2$ the $E2$ transition probability will write

$$\langle I|O(E2)|I - 2\rangle = \cos(\varphi - \psi)X, \quad (16)$$

where X is some matrix element involving the intrinsic state common to both states. If one assumes I to be a spin value where parabola crossing occurs and that the mixing is maximum there (i.e., $\cos^2\varphi = 0.5$), and if one also estimates the mixing for the other spin to be reasonably weak (for instance, $\cos^2\psi = 0.9$), one gets a quenching factor for the relevant matrix element of ~ 0.9, which is within the experimental error bars from lifetime measurements in superdeformed states [16]. Our hypothesis for the staggering seems therefore not contradicted by the present status of electromagnetic properties measurements.

We will now provide some semiclassical estimates of the three inertia parameters A, B, and C using a truncated $\hbar$ expansion of the solution of Eq. (1) when the effective nucleon-nucleon interaction is a full-fledged Skyrme force. This approach, described in Ref. [14], is a generalization of what has been done in Ref. [17] for the Routhian case only. The density will be schematically described as being constant and bounded by a sharp-edged ellipsoid whose semiaxes are given by

$$a_x = a_0 q^{2/3}, \qquad a_y = a_z = a_0 q^{-1/3}, \quad (17)$$

where a_0 is given in terms of the total number of particles N and of the usual size parameter r_0 by $a_0 = r_0 N^{1/3}$, whereas q is, as before, the a_x/a_y ratio. We will also use below the following unit of moment of inertia:

$$\gamma = \frac{2}{5} m r_0^2 N^{5/3} q^{1/3}, \quad (18)$$

which is merely the rigid body moment of inertia for a sphere multiplied by the shape factor $q^{1/3}$. The various inertia parameters are given below to order $\hbar^2$ (the first terms being the Thomas-Fermi estimates):

$$A = \gamma R\left[1 - \left(\frac{D}{\gamma}\right)R\right], \qquad B = \gamma\left[1 - \left(\frac{D}{\gamma}\right)R\right], \qquad C = \gamma R\left[1 - \left(\frac{D}{\gamma R}\right)\right], \quad (19)$$

where R is a geometrical factor given by $R = \frac{1}{2}(q + 1/q)$. The semiclassical expansion factor D is approximately given in terms of the effective mass in nuclear matter $(m^*/m)_{\text{NM}}$ by

$$\frac{D}{\gamma} = 10(9\pi)^{-2/3}(m^*/m)_{\text{NM}}^{-1} N^{-2/3} q^{-1/3}. \quad (20)$$

(Note that its inverse surface dependence makes D/γ small with respect to 1—typically of the order of a few percent—in heavy nuclei.) The contact periodicity in I is given up to second order in D/γ by

$$\frac{2C}{B} = 2R\left[1 + \left(\frac{D}{\gamma R}\right)(R^2 - 1)\right]. \quad (21)$$

Disregarding the small second order correction, one gets exactly from Eq. (21) the $\Delta I = 4\hbar$ energy structure when $q = 2 \pm \sqrt{3}$. This is of course not the expected superdeformed value. This discussion shows merely that such a description of the observed structure by an intrinsic vortical motion is not completely out of range. However, one has to improve it in two directions: first release the ellipsoidal shape constraint and, more importantly, include shell and pairing effects. One may also point out that the spin degrees of freedom have not been included in our discussion. It appears, however, that they are contributing only to second order in the inertia tensor [14,17]. It is worth noting that the collective alignment appearing in Eq. (12) is found, even at second order in $\hbar$, to be equal to J/R, i.e., size independent.

The staggering amplitude is given from Eqs. (8), (13), and (19) by

$$\Delta E_\gamma = \frac{\hbar^2}{2\gamma(R - 1/R)}\left[1 + \left(\frac{D}{\gamma}\right)(R - 1/R)\right]. \quad (22)$$

In the Thomas-Fermi limit, one may express it in terms of the irrotational moment of inertia ($I_{\text{irrot}} = C - B^2/C$) as

$$\Delta E_\gamma = \frac{\hbar^2}{2I_{\text{irrot}}}. \quad (23)$$

For a heavy ($A \approx 200$) superdeformed ($q \approx 2$) nucleus, this amplitude takes a value of about 9 keV. This is indeed 1 order of magnitude higher than experimentally observed. However, it is clear that the mixing of collective band states due to the residual interactions is able to reduce this value substantially.

In this Letter, we have demonstrated that the energy staggering observed in some superdeformed bands may be explained by a generic Hamiltonian depending (e.g., quadratically) on two physical quantities which are quantized (e.g., as angular momenta). The collective Hamiltonian coupling a global rotation with a uniform vortical motion has been suggested here as a possible candidate for being such a Hamiltonian. Semiclassical estimates of the model parameters, if not exactly consistent with the values deduced from the experimental data within this model, are not, however, orders of magnitude away. Clearly further theoretical work is needed to assess the validity of the present suggestion, in particular, by taking into account the quantal shell and pairing effects. Nevertheless, it has been proven that the level of quenching of $E2$ transitions resulting from our modeling of the yrast line is not inconsistent with the data, considering currently available accuracy. A more fundamental question, common to most collective model descriptions, concerns the starting point of our approach: Is the simple description of rotations within a two mode coupling Hamiltonian scheme *à la* Riemann and Chandrasekhar, at all sufficient? In other words, do real states correspond to almost exactly conserved Kelvin circulation quantum numbers? If the answer to this question is rather positive, then whatever the fate of our present suggestion about the above discussed energy staggering (which may be invalidated by any fine tuning of the inertia tensor), one would be forced to consider more consistently now the intrinsic vortical motion as a relevant tool for deciphering nuclear collective excitations.

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