Color-Octet Fragmentation and the ψ' Surplus at the Fermilab Tevatron

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The production rate of prompt ψ''s at large transverse momentum at the Fermilab Tevatron is larger than theoretical expectations by about a factor of 30. As a solution to this puzzle, we suggest that the dominant ψ' production mechanism is the fragmentation of a gluon into a $c\bar{c}$ pair in a pointlike color-octet S-wave state, which subsequently evolves nonperturbatively into a ψ' plus light hadrons. This contribution to the fragmentation function is enhanced by a short-distance factor of $1/\alpha_s^2$, which may compensate for its suppression by v^4, where v is the relative momentum of the charm quark in the ψ'.

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The production of heavy quarkonium in high energy processes provides severe tests of our understanding of QCD in a domain that should be accessible to theoretical analysis. At high energies, QCD mechanisms for the direct production of charmonium are embedded in a large background from the production of b quarks, followed by the decay of the resulting B hadrons into charmonium states. Vertex detectors can be used to separate the charmonium states coming from b quarks from those that are produced by QCD interactions, thus allowing the direct study of the prompt QCD production mechanisms. In the 1991–92 run of the Fermilab Tevatron, the Collider Detector at Fermilab (CDF) group used their new vertex detector to study prompt charmonium production [1]. The results have proved to be an embarrassment to theory, revealing orders-of-magnitude discrepancies with theoretical predictions.

Until recently, the conventional wisdom was that the dominant mechanisms for prompt charmonium production at large transverse momentum p_T come from the processes such as gluon-gluon fusion which are leading order in the QCD coupling constant α_s [2]. The resulting predictions for prompt J/ψ production at large p_T are smaller than the CDF data by more than an order of magnitude. However, as pointed out by Braaten and Yuan in 1993 [3], the dominant production mechanism at sufficiently large p_T is the production of a parton with large transverse momentum, followed by the fragmentation of the parton into charmonium states. Including this fragmentation mechanism brings the theoretical predictions for prompt J/ψ production to within a factor of 3 of the data [4,5], close enough that the remaining discrepancy can be attributed to theoretical uncertainties. Unfortunately, even after including the fragmentation contribution, the prediction for the ψ' production rate remains a factor of 30 below the data. This remarkably large discrepancy between theory and experiment suggests than an entirely new production mechanism must be dominating the production of ψ' at large p_T.

Previous proposals for solving the ψ' surplus problem have focused on the production of higher charmonium states which decay into ψ'. Among the possible states are higher P-wave states, higher D-wave states, and states whose dominant Fock component is $|c\bar{c}g\rangle$ [6]. For each of these possibilities, there is reason to question whether the production rate can be large enough and whether the branching fraction into ψ' can be large enough to account for the observed rate for prompt ψ' production. In this Letter, we propose a completely different solution to the ψ' surplus problem. We suggest that the dominant production mechanism for ψ' at large p_T is gluon fragmentation into a $c\bar{c}$ pair in a pointlike color-octet S-wave state, followed by the nonperturbative evolution of this state into a ψ' plus light hadrons. We calculate the corresponding contribution to the fragmentation function for $g \to \psi'$ in terms of a well-defined nonperturbative matrix element, and we determine the value of this matrix element that would be required to explain the production rate of ψ' at the Tevatron. We then discuss possible tests of this solution to the ψ' surplus problem.

Our proposal is based on a general factorization analysis of the annihilation and production of heavy quarkonium by Bodwin, Braaten, and Lepage [7]. The factorization formalism of Ref. [7] allows the fragmentation functions for charmonium production to be factored into short-distance coefficients, which describe the production rate of a $c\bar{c}$ pair within a region of size $1/m_c$, and long-distance factors that contain all the nonperturbative dynamics of the formation of a bound state containing the $c\bar{c}$ pair. The fragmentation function for a gluon to split into a charmonium state X with longitudinal momentum fraction z can be written

$$D_{g\to X}(z,\mu) = \sum_n d_n(z,\mu)\,\langle 0|\mathcal{O}_n^X|0\rangle\,, \tag{1}$$

where $\mathcal{O}_n^X$ are local four-fermion operators that were defined in Ref. [7] in terms of the fields of nonrelativistic

QCD [8]. The short-distance coefficients $d_n(z,\mu)$ are independent of the quarkonium state X. For a fragmentation scale μ of order m_c, they can be computed using perturbation theory in $\alpha_s(m_c)$. The dependence on the quarkonium state X appears in the long-distance factors $\langle \mathcal{O}_n^X \rangle$. The relative importance of the various matrix elements for a given state X can be estimated by how they scale with v, the typical relative velocity of the charm quark in charmonium [7].

In previous calculations of fragmentation functions for charmonium production, only the matrix elements that dominate in the nonrelativistic limit $v \to 0$ have been considered. In the case of ψ', the only such matrix element in the notation of Ref. [7] is $\langle \mathcal{O}_1^{\psi'}(^3S_1)\rangle$. In this vacuum matrix element, the local operator $\mathcal{O}_1^{\psi'}(^3S_1)$ creates a $c\bar{c}$ pair in a color-singlet $^3S_1$ state, projects onto the subspace of states which in the asymptotic future contain a ψ', and then annihilates the $c\bar{c}$ at the creation point. Up to corrections of relative order v^4, this matrix element can be related to the wave function $\overline{R_{\psi'}}/\sqrt{4\pi}$ of the ψ' at the origin,

$$\langle 0|\mathcal{O}_1^{\psi'}(^3S_1)|0\rangle \approx \frac{3}{2\pi}\,|\overline{R_{\psi'}}|^2\,. \qquad (2)$$

In the case of S waves, the factorization approach reduces in the nonrelativistic limit to the color-singlet model [9], where the ψ' is treated as a $c\bar{c}$ pair in a color-singlet $^3S_1$ state with vanishing relative momentum.

In the case of P waves like the χ_{cJ} state, there are two matrix elements that survive in the nonrelativistic limit. In addition to $\langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J)\rangle$, which is related to the derivative of the wave function at the origin, there is also the color-octet matrix element $\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle$. The local operator $\mathcal{O}_8^{\chi_{cJ}}(^3S_1)$ creates a $c\bar{c}$ pair in a color-octet $^3S_1$ state, projects onto the subspace of states which in the asymptotic future contain a χ_{cJ}, and then annihilates the $c\bar{c}$ at the creation point. In the color-singlet model, only the matrix element $\langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J)\rangle$ is taken into account. This model is perturbatively inconsistent because of infrared divergences in the radiative corrections to P-wave production rates. In the factorization formalism of Ref. [7], this problem is solved by factoring the infrared divergences into the color-octet matrix element $\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle$ [10].

The restriction to the leading matrix elements in the $v \to 0$ limit is a great simplification, but it is not necessarily correct. The typical value of v^2 in the case of charmonium is only about 1/3, so factors of v^2 do not provide a very large suppression. In the case of (1), dropping contributions that are suppressed by powers of v^2 might be reasonable if the short-distance factors $d_n(z,\mu)$ were all comparable in size. However, these factors are of different order in α_s for different matrix elements. Short-distance factors of $\alpha_s(m_c)/\pi$ probably provide much more effective suppression than factors of v^2. Thus, in estimating the importance of various terms in the fragmentation function, one should take into account not only the scaling of the matrix element with v^2, but also the order in α_s of the short-distance coefficient.

In the case of gluon fragmentation into ψ', the short-distance factor associated with the matrix element $\langle \mathcal{O}_1^{\psi'}(^3S_1)\rangle$ is of order α_s^3. It describes the process $g^* \to c\bar{c}gg$, with the $c\bar{c}$ pair in a color-singlet $^3S_1$ state. By the velocity-scaling rules of Ref. [7], the matrix element $\langle \mathcal{O}_1^{\psi'}(^3S_1)\rangle$ is of order $m_c^3 v^3$, so the contribution to the fragmentation function is of order $\alpha_s^3 v^3$. While all other matrix elements are suppressed by powers of v^2, there are some with fewer short-distance suppression factors of α_s. In particular, there are some matrix elements for which the leading-order short-distance process is $g^* \to c\bar{c}$ and the rate is of order α_s. Of these matrix elements, the leading one in the nonrelativistic limit is $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$, which scales like $m_c^3 v^7$. The suppression factor of v^4 relative to $\langle \mathcal{O}_1^{\psi'}(^3S_1)\rangle$ reflects the fact that the $c\bar{c}$ pair in the ψ' can make a double $E1$ transition with amplitude of order v^2 from the dominant $|c\bar{c}\rangle$ Fock state to a $|c\bar{c}gg\rangle$ state in which the $c\bar{c}$ pair is in a color-octet $^3S_1$ state.

The contribution to the fragmentation function for $g \to \psi'$ from the short-distance production of a $c\bar{c}$ pair in a color-octet $^3S_1$ state was calculated to leading order in α_s in Ref. [11]. The calculation was carried out for the specific case of $g \to \chi_{cJ}$, but since the short-distance coefficients in (1) are independent of the quarkonium state, the result applies equally well to ψ'. The fragmentation function is

$$D_{g\to\psi'}(z,\mu) = \frac{\pi\alpha_s(2m_c)}{24m_c^3}\,\delta(1-z)\,\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle\,. \quad (3)$$

In Ref. [11], the corresponding fragmentation function for $g \to \chi_{cJ}$ was expressed in terms of a quantity H_8' defined by

$$H_8' \equiv \frac{1}{(2J+1)m_c^2}\,\langle 0|\mathcal{O}_8^{\chi_{cJ}}(^3S_1)|0\rangle\,. \qquad (4)$$

The form (3) is preferred, since it corresponds to the factorization of the fragmentation function into a short-distance factor proportional to α_s/m_c^3 and a long-distance matrix element.

In the fragmentation function for $g \to \chi_{cJ}$, the matrix element $\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle$ can be determined phenomenologically from data on charmonium production in B-meson decays. From Eq. (24) of Ref. [10], we have

$$\frac{\Gamma(B \to \chi_{c2} + X)}{\Gamma(B \to e\bar{\nu}_e + X)} \approx 14.8\,\frac{\langle 0|\mathcal{O}_8^{\chi_{c2}}(^3S_1)|0\rangle}{m_c^2 m_b}\,. \qquad (5)$$

The branching fraction for $B \to \chi_{c2} + X$ has recently been measured by the CLEO collaboration to be $(0.25 \pm 0.11)\%$ [12]. Assuming $m_c \approx 1.5$ GeV and $m_b \approx 4.5$ GeV, the expression (5) gives

$$\langle 0|\mathcal{O}_8^{\chi_{c2}}(^3S_1)|0\rangle = (0.016 \pm 0.007)\ \mathrm{GeV}^3\,, \qquad (6)$$

which corresponds to $H_8' \approx 1.4 \pm 0.6$ MeV. This value is significantly smaller than the estimate $H_8' \approx 3$ MeV obtained in Ref. [10] and used in Refs. [4] and [5]. It is just large enough to be compatible with the theoretical lower bound $H_8' \geq 1.4$ MeV derived in [11] from the condition that the fragmentation probability for $g \to \chi_{c0}$ be non-negative. With the new value (6), the contribution to J/ψ production from gluon fragmentation into χ_{cJ} that was calculated in Refs. [4] and [5] should be decreased by a factor of 2. The theoretical prediction for J/ψ production then falls below the data by about a factor of 5.

In the case of ψ', we can determine the matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ by fitting the CDF data on the production rate of prompt ψ' at large p_T. The production rate is given by convoluting hard scattering cross sections for the production of gluons with large transverse momentum with parton distributions for the colliding proton and antiproton and with the fragmentation function $D_{g\to\psi'}(z,\mu)$. The fragmentation function given by (3) is evolved up to the scale $\mu = p_T/z$ of the transverse momentum of the fragmenting gluon by using the Altarelli-Parisi equations. The production rate measured by CDF [1] in the 1991–92 run is shown in Fig. 1, along with three theoretical curves. The dotted line is the production rate from leading-order production mechanisms, such as $gg \to \psi' g$, while the dashed line is the color-singlet fragmentation contribution calculated in Ref. [4], which is dominated by charm quark fragmentation into ψ'. The solid curve is the color-octet fragmentation contribution, with the matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ adjusted to make the normalization agree with the data. The resulting value of the matrix element is

$$\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle = 0.0042\ \mathrm{GeV}^3\,. \qquad (7)$$

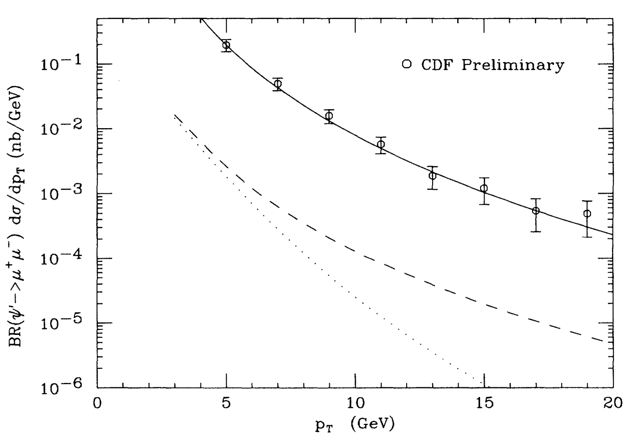

FIG. 1. CDF data for prompt ψ' production compared with theoretical calculations of the total leading-order contributions (dotted curves), the total color-singlet fragmentation contributions (dashed curves), and the color-octet fragmentation contribution (solid curves). The normalization of the color-octet fragmentation contribution has been adjusted to fit the data.

Note that the shape of the color-octet fragmentation contribution fits the data perfectly.

There is a very simple consistency check that the color-octet fragmentation mechanism must satisfy in order to provide a viable explanation for the ψ' surplus. The value of the matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ must be small enough relative to the matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ to be consistent with suppression by a factor of v^4. The color-singlet matrix element can be determined from the leptonic decay rate of the ψ' to be 0.11 GeV^3. The color-octet matrix element (7) is smaller by a factor of 25, consistent with suppression by v^4.

There are several experimental consistency checks of our proposed explanation of the ψ' surplus. If color-octet fragmentation is indeed the dominant production mechanism, then most of the ψ''s produced at the Fermilab Tevatron must be accompanied by light hadrons. The leading order mechanisms, such as $gg \to \psi' g$, produce isolated ψ''s with no accompanying hadrons. Color-singlet fragmentation produces ψ''s that are associated with a charm quark jet, but with transverse momentum of order m_c relative to the jet. Color-octet fragmentation also produces ψ''s that are associated with a jet of light hadrons, but the transverse momentum relative to the jet is of order $m_c v$ or smaller. Thus measurements of the hadronic energy distribution as a function of the distance from the ψ' momentum in azimuthal angle and in rapidity could provide support for this new production mechanism. However, other proposed solutions to the ψ' surplus problem, such as fragmentation into charmonium states that lie above the $D\bar{D}$ threshold [6], could yield similar hadronic energy distributions. As recently pointed out by Cho and Wise [13], the spin alignment of the ψ''s, which is reflected in the angular distribution of their leptonic decays, provides a more distinctive signature for the color-octet fragmentation mechanism. This mechanism produces ψ''s that are 100% transversely polarized at leading order in α_s, while other mechanisms tend to produce unpolarized ψ''s.

If color-octet fragmentation is so important in the case of ψ' production, one might ask why it is not equally important for J/ψ production at large p_T. In fact, it might very well be. The main difference is that the J/ψ signal is fed by gluon fragmentation into χ_{c1} and χ_{c2} followed by their radiative decays into J/ψ, and this overwhelms the rate for color-singlet fragmentation directly into J/ψ. However, there is room for a substantial contribution from color-octet fragmentation into J/ψ, especially with the new determination (6) of the matrix element $\langle \mathcal{O}_8^{\chi_{c2}}(^3S_1)\rangle$, which leaves the predictions of Refs. [4] and [5] a factor of 5 below the CDF data. If the matrix element $\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle$ for J/ψ has a value comparable to that for ψ' given in (7), then the color-octet fragmentation mechanism for J/ψ production will be comparable in importance to gluon fragmentation into χ_{c1} and χ_{c2}.

If a color-octet contribution dominates the fragmentation function for $g \to \psi'$, one might worry that annihilation of a $c\bar{c}$ pair in a color-octet state should also dominate the decay rate of the ψ' into light hadrons. However, the relative importance of the color-singlet and color-octet contributions is different in the case of annihilation. The color-singlet contribution to the decay rate is of order $\alpha_s^3 m_c v^3$, while annihilation from a color-octet S-wave state contributes at order $\alpha_s^2 m_c v^7$. Thus the short-distance enhancement of the color-octet contribution is only one power of $1/\alpha_s$, rather than two powers as in the case of fragmentation. Furthermore, there is no apparent rigorous relation between the color-octet annihilation matrix elements and the color-octet production matrix elements beyond the fact that they have the same scaling in v [7].

Our proposal for the solution of the ψ' surplus problem has predictive power, because the matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ determined from ψ' production at large p_T can be used to predict ψ' production in other high energy processes. Unfortunately, the matrix elements $\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle$ and $\langle \mathcal{O}_8^{\psi'}(^3P_J)\rangle$ are of the same order in v^2 as $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$, and they must also be taken into account. Different production processes probe different linear combinations of these three color-octet matrix elements at leading order in α_s. Gluon fragmentation is rather unique in that $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ is the only color-octet matrix element that contributes at leading order in α_s and in v^2. In photoproduction of ψ', on the other hand, only the matrix elements $\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle$ and $\langle \mathcal{O}_8^{\psi'}(^3P_J)\rangle$ contribute at leading order in α_s and in v^2. While suppressed relative to the color-singlet contribution by a factor of v^4, they are enhanced by a short-distance factor of $1/\alpha_s$. Moreover, at leading order in α_s, the color-octet contribution is concentrated near $z = 1$, where z is the ratio of the energy of the ψ' to that of the photon. The cross section is known experimentally to peak strongly in the $z \to 1$ region, and charmonium states produced in this region are labeled "elastic." Thus a linear combination of the matrix elements $\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle$ and $\langle \mathcal{O}_8^{\psi'}(^3P_J)\rangle$ can be determined from the elastic peak in ψ' photoproduction [14].

In most of the work on the production of charmonium over the last decade, the dominant production mechanisms have been assumed to be ones in which the $c\bar{c}$ pair is produced at short distances in a color-singlet state. However, in the case of P-wave states, it is essential to include the color-octet mechanism to obtain consistent perturbative predictions. In the case of S waves, the color-octet mechanism is not essential for perturbative consistency, but it may be important nonetheless in cases where the color-singlet mechanism is suppressed by a short-distance coefficient. A global analysis of the data on ψ' production from all high energy processes could confirm the importance of color-octet mechanisms in the production of the S-wave states of heavy quarkonium.

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