Phase-Sensitive Measurements of Vortex Dynamics in the Terahertz Domain

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Phase-sensitive spectroscopy is used to characterize fully the complex resistivity tensor of $YBa_2Cu_3O_x$ films in the mixed state, in the frequency range 100 to 500 GHz and temperature range 10 K to T_c. We compare our results with the predictions of a vortex model containing three parameters: viscosity, pinning, and Magnus force. The viscosity and Magnus parameter we measure are in disagreement with theoretically predicted values. We show that the discrepancies can be explained by incorporating the effects of a highly anisotropic or d-wave gap.

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Direct measurements of impedance in the MHz range have been used to probe vortex freezing near the glass transition temperature T_g in cuprate superconductors [1]. For $T \ll T_g$, dissipation at MHz frequencies is unmeasurably small. Probing vortex dynamics in this temperature range requires frequencies near the vortex relaxation rate in the glassy state, which approaches a microscopic time scale related to the quasiparticle lifetime, $\sim 10^{-12}$ s.

In this Letter we report direct measurements of the terahertz impedance in $YBa_2Cu_3O_x$ thin films. At each temperature we measured both the diagonal and the off-diagonal elements of the complex resistivity tensor. When we compare our results with a phenomenological description of vortex motion, the parameters we infer are not consistent with the predictions of microscopic models. We find a low-temperature viscosity that is approximately 20 times smaller than the Bardeen-Stephen prediction [2] and a Magnus parameter that is much smaller than the predicted value [3]. We are able to account for these unexpected results by considering the impact of an anisotropic gap on vortex structure and dynamics.

We measure the transmission of nearly single-cycle picosecond pulses through the $YBa_2Cu_3O_x$ thin films. Fourier transformation of the time domain data yields the complex, frequency-dependent transmission coefficient, related to the resistivity through the equation

$$t(\omega) = \frac{1}{n + 1 + Z_0 d/\rho(\omega)} \frac{4ne^{i\Phi(\omega)}}{n + 1}, \qquad (1)$$

where Z_0 is the impedance of free space, d is the film thickness, n is the substrate refractive index, and $\Phi(\omega)$ is the phase shift acquired as the pulse propagates through the substrate. The real and imaginary parts of the resistivity $\rho(\omega)$ are obtained by inverting Eq. (1), with no need for a Kramers-Kronig transformation.

In the presence of a magnetic field, both t and ρ become second-rank tensors. Because microtwinning averages over the crystalline anisotropy, ρ can be completely determined by two independent measurements at each frequency; in our spectrometer these are the transmission coefficients with the sample placed between parallel polarizers t_{xx} and crossed polarizers t_{xy}. Given these measurements, we use a tensor version of Eq. (1) to determine both the real and imaginary parts of ρ_{xx} and ρ_{xy} in the frequency range from 100 to 500 GHz.

The measurements were performed on three c-axis $YBa_2Cu_3O_x$ films. The first is a 70 nm thick film grown by off-axis sputtering on a sapphire substrate with a CeO_2 buffer layer, with $T_c = 85$ K [4]. The second is a 40 nm film grown on $LaAlO_3$ by the same method, with $T_c = 88$ K [5]. The third, 70 nm thick, was grown by laser ablation on $LaAlO_3$, with $T_c = 88$ K [6].

In Fig. 1 we plot the real and imaginary parts of $\rho_{xx}(T)$ and $\rho_{xy}(T)$ at 150 GHz for the 40 nm film in a 6 T magnetic field. Above T_c, the data agree well with dc measurements. Below T_c the resistivity remains measurable down to our lowest temperature 10 K. Notice that the Hall resistivity $\rho_{xy}(T)$ is more than an order of magnitude smaller than the diagonal part of the resistivity $\rho_{xx}(T)$ over the entire temperature range.

The resistivity we measure is a combination of the response of the superfluid, thermally excited quasiparticles, and vortices [7]. At low temperatures where the quasiparticle contribution to the resistivity is negligible, the vortex and superfluid resistivities simply add $\rho = \rho_v + \rho_s$ [8]. To determine ρ_v we assume initially that ρ_s does not change appreciably when the field is applied because Cooper pairs are broken only inside the vortex cores. Since the fractional area of the cores is $H/H_{c2}(T)$ and H_{c2} is greater than 100 T, the change in ρ_s is expected to be very small for fields up to our maximum field of 6 T.

We compare ρ_v obtained using the above assumption with the well-known phenomenological description of vortex dynamics. This model assumes that the vortices form a rigid, massless lattice with a linear restoring force coefficient κ, a viscous damping coefficient η, and a Magnus parameter α [9]:

$$\eta \mathbf{v}_L + (\kappa/i\omega)\mathbf{v}_L = [(n_s h/2)\mathbf{v}_s - \alpha \mathbf{v}_L] \times \hat{\mathbf{z}}. \qquad (2)$$

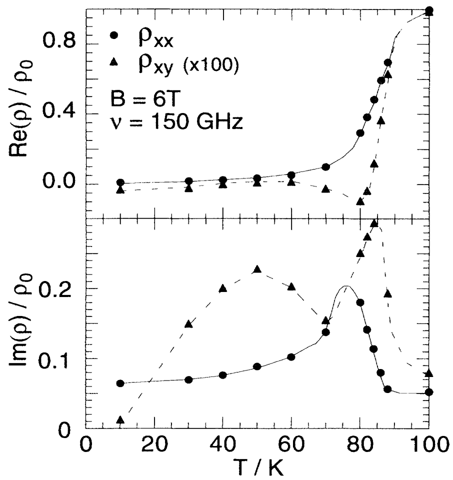

FIG. 1. Real and imaginary parts of ρ_{xx} and ρ_{xy}. Despite scaling by 10^2, the sign changes in ρ_{xy} from negative to positive near 70 K and back to negative near 20 K are barely discernable. All resistivities are normalized to ρ_0, the dc resistivity at 100 K. The data shown correspond to the 40 nm film on $LaAlO_3$. The lines are guides to the eye.

Equation (2) yields the vortex velocity $\mathbf{v}_L$ arising in response to a superfluid velocity $\mathbf{v}_s$, with $\mathbf{J}_s = n_s e \mathbf{v}_s$. $\hat{\mathbf{z}}$ is the direction of the magnetic field.

A vortex with velocity $\mathbf{v}_L$ generates an electric field $\mathbf{E}_v = \mathbf{B} \times \mathbf{v}_L$. With $\mathbf{v}_L$ obtained from Eq. (2) and the definition $\mathbf{E}_v \equiv \rho_v \mathbf{J}_s$, we find the vortex resistivity tensor

$$\rho_v^{xx} = iX_v \frac{(1 + i\omega/\Gamma)}{(1 + i\omega/\Gamma)^2 - (\alpha\omega/\kappa)^2}, \tag{3a}$$

$$\rho_v^{xy} = X_v \frac{\alpha\omega/\kappa}{(1 + i\omega/\Gamma)^2 - (\alpha\omega/\kappa)^2}, \tag{3b}$$

where $X_v = \omega\phi_0 B/\kappa$. The resistivity has two distinct regimes separated by the characteristic relaxation rate of a displaced vortex $\Gamma \equiv \kappa/\eta$. At low frequencies such that $\omega \ll \Gamma$, ρ_v^{xx} approaches iX_v, and the response is purely inductive. At high frequencies ρ_v^{xx} is purely dissipative and in the limit that $\alpha \rightarrow 0$ it approaches $B\phi_0/\eta$.

Equation (3) describes the longitudinal and transverse vortex response in terms of only three parameters: α, η, and κ. Because we measure four quantities at each temperature and frequency, the model is overconstrained. To compare the model with our experiment we first determine η and κ by using the fact that the magnitude of the Hall angle $|\rho_{xy}/\rho_{xx}|$ is less than 0.1 over the entire temperature and frequency range. This allows us to neglect the term $(\alpha\omega/\kappa)^2$ in the denominator of Eq. (3a), leaving η and κ uniquely determined by the measured ρ_v^{xx}.

The solid and open symbols in Fig. 2 show $\rho_v^{xx}(\omega)$ at two temperatures for the 70 nm film on $LaAlO_3$. The curves in Fig. 2 are fits generated using the modified form of Eq. (3a). Although η and κ are completely constrained at each ω, frequency-independent values for these parameters yield a satisfactory fit over our frequency range [10]. Similar fits at other temperatures allow us to find the temperature dependence of η and κ.

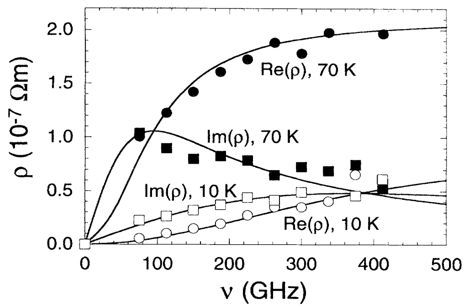

FIG. 2. The vortex contribution to the resistivity plotted vs frequency at 10 and 70 K for $B = 6$ T. The lines are fits to the data using Eq. (3a). The data shown were taken on the 70 nm film on CeO_2/Al_2O_3.

In Fig. 3 we plot the values of η and κ found at each temperature. The third parameter α can be found by fitting the real and imaginary parts of ρ_{xy}. We cannot plot α in Fig. 3 because to account for the behavior of ρ_{xy}, α must be frequency dependent and complex. In the following, we compare η, κ, and α determined from the resistivity tensor with the predictions of microscopic models.

We compare the low-temperature value of η with the Bardeen-Stephen prediction $\eta_{BS} = B_{c2}\phi_0\sigma_n$ [2], where σ_n is the normal-state conductivity. Since $YBa_2Cu_3O_x$ is a clean superconductor, nearly all the electrons condense at low temperature, and we can rewrite this equation as $\eta_{BS} = 4\pi\mu_0 H_c^2 \tau_{qp}$. With $H_c/\mu_0 \approx 1$ T, this leaves τ_{qp}, the quasiparticle scattering time, as the only unknown value needed to calculate η_{BS}.

Selecting a value for τ_{qp} poses a problem because the quasiparticle scattering rate in $YBa_2Cu_3O_x$ is expected to be different for the normal and superconducting states at the same temperature. It is not obvious which one should

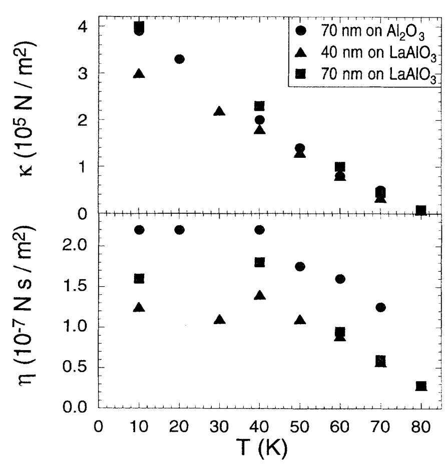

FIG. 3. The vortex pinning and damping parameters κ and η plotted as functions of temperature for all three samples.

be used to estimate η_{BS}, and we are not aware of any theoretical justification for using one value or the other. However, it turns out that because the low-temperature scattering rate of our films is limited by an extrinsic scattering rate, the two values are not as far apart as they would be for cleaner samples. The normal state quasiparticle scattering rate decreases linearly with temperature. For our samples, the scattering rate is about 3.5 THz at 100 K. Extrapolating this rate to 10 K yields a normal state quasiparticle lifetime $\sim$0.5 ps. We determined the superconducting state's quasiparticle lifetime from a two-fluid analysis of the complex conductivity in zero field [11]. For $T \leq 40$ K, we find that τ_{qp} reaches a value ~ 0.7 ps. This is a factor of 10 smaller than reported for single crystal samples of $YBa_2Cu_3O_{6.95}$ [11], and it is quite close to the extension of the normal-state scattering rate. Using either value in the Bardeen-Stephen relation for the viscosity, we obtain $\eta_{BS} \approx 5 \times 10^{-6}$ N s/m^2, which is approximately 20 times larger than our low-temperature experimental result.

We next examine $\kappa(T)$. κ is an extrinsic property that depends on the density and type of defects in the sample, so it can range from zero to a maximum value which will be estimated later. Given this possible range, it is remarkable how well different measurements of κ agree with each other. Wu and Sridhar [12] inferred the pinning strength in $YBa_2Cu_3O_x$ crystals by measuring the inductance at 6 MHz. They found $\kappa = 2.22 \times 10^5$ N/m^2 at low temperature with a temperature dependence similar to that which we observe. Our values are also quite close to those measured by Pambianchi *et al.* [13], which is surprising because their samples were mixed c-axis and a-axis oriented films.

The parameter α, which describes the "Magnus force," is often ignored in analyses of high-frequency vortex dynamics. α determines the size of the Hall effect by setting the angle between vortex motion and superfluid flow. In a perfectly ordered crystal the force on a vortex line is expected to be independent of the frame of reference and dependent only on the difference $\mathbf{v}_s - \mathbf{v}_L$. In this clean limit, $\alpha = n_s e \phi_0$. It has generally been assumed that the force on pinned vortices depends on $\mathbf{v}_s$ only and $\alpha = 0$. Recently this assumption has been questioned by Ao and Thouless who argue that $\alpha = n_s h/2$ regardless of disorder [3]. However, we find that α is very small (less than $0.07 n_s h/2$ at all frequencies and temperatures) and has an unexpected peak near 40 K.

To summarize the results so far, we found major discrepancies between the values of the vortex parameters predicted by microscopic models and those we extract from our data by following a conventional analysis. We found that in order to fit our data, the viscosity must be 20 times smaller than the microscopic prediction of Bardeen and Stephen, the pinning parameter must be nearly independent of sample quality, and the Magnus parameter must be complex, frequency and temperature dependent, and very small. To resolve these discrepancies, we reexamine the assumptions made in the conventional analysis in light of the strong possibility that $YBa_2Cu_3O_x$ has an anisotropic gap.

We stated earlier our assumption that the superfluid density is essentially unaffected by the application of a magnetic field. This assumption is valid for an isotropic s-wave superconductor, where the fractional change in superfluid density is H/H_{c2}. We are not aware of any treatments of vortex dynamics in $YBa_2Cu_3O_x$ that do not assume conventional s-wave behavior. In the following we discuss the implications of strong gap anisotropy on the electrodynamic response of the cuprates in the mixed state. The resulting modifications to the standard picture allow us to explain the anomalous behavior.

With gap anisotropy the reduction in superfluid density may be considerably larger in the cuprates than in an isotropic superconductor. The reason is related to the strong nonlinearity of the London constitutive relation in the Meissner state predicted by Yip and Sauls [14]. Because the gap vanishes in certain directions, the critical velocity for breaking Cooper pairs $\nu_c \approx \Delta/m\nu_F$ extends down to zero. As a consequence, the superflow around a vortex will lead to depairing at distances much greater than ξ from the center of the vortex.

Another way of expressing the increased pair breaking is through the effective density of states at the Fermi level in the mixed state $N_F(B)$. Recently, Volovik [15] calculated that for a superconductor with a line node in the gap function $N_F(B) \propto N_F \xi/R$, where R is the distance between vortices, and N_F is the density of states at the Fermi level in the normal state. This density of states is far greater (by a factor R/ξ) than in an isotropic superconductor. A straightforward extension of this theory would predict that in a highly anisotropic s-wave superconductor, R would be replaced by $\min\{R, \hbar\nu_F/\Delta_{\min}\}$.

Increased pair breaking affects the Bardeen-Stephen prediction for the viscosity, bringing it much closer to our measured value. The Bardeen-Stephen prediction can be written explicitly in terms of the density of states, $\eta_{BS} = B\phi_0\sigma_n N_F/N_F(B)$. Volovik's density of states implies that η is reduced by the factor ξ/R in a d-wave superconductor and that $\eta_{BS} = \phi_0(BB_{c2})^{1/2}\sigma_n$ replaces the conventional formula $\eta_{BS} = B_{c2}\phi_0\sigma_n$. This reduced Bardeen-Stephen viscosity for an anisotropic superconductor mitigates the factor of 20 discrepancy discussed earlier. At 6 T, $R = 183$ Å, corresponding to a reduction in viscosity $\sim \xi/R \sim 0.1$ for $YBa_2Cu_3O_x$.

The same physics suggests that the value of κ inferred from the conventional interpretation of $\mathrm{Im}\rho_{xx}$ may not be the actual value of the pinning parameter. We noted previously our assumption that $\Delta\rho(B)$ can be attributed to ρ_v, rather than a change in ρ_s with magnetic field. However, gap anisotropy leads to a greater number of quasiparticles associated with each vortex and to corresponding changes in the superfluid density and resistivity,

$\Delta\rho_s/\rho_s = \Delta n_s/n_s \approx \xi/R \approx \sqrt{B/B_{c2}}$. This change in ρ_s with B may dominate $\Delta(\mathrm{Im}\rho_{xx})$. If this increased inductance were incorrectly interpreted in terms of pinning, an apparent force constant $\kappa_{\mathrm{eff}} = \phi_0(BB_{c2})^{1/2}ne^2/m^*$ would be inferred in samples in which the vortices were more tightly bound. This may explain why different samples yield such similar results for κ.

The possibility that κ is much larger than the value obtained using the conventional model allows us to reconcile the very low values of ρ_{xy} we observe with the prediction that $\alpha = n_s h/2$. Note that the magnitude of ρ_{xy} depends on the ratio α/κ. Our measurement of ρ_{xy} determines α only if we assume that we know κ from ρ_{xx}. If κ is actually much larger (by more than 1 order of magnitude) then $\alpha = n_s h/2$ could be consistent with the magnitude of ρ_{xy} observed. The required value of κ is lower than the theoretical upper limit $\kappa_{\max}$, which can be estimated using a simple model of a stretched vortex line. If the vortex line is strongly pinned by point defects with a linear density of n_p, then the expression $\varepsilon = 4\pi\xi^2 U_c \ln\kappa_{\mathrm{GL}}$ for the line tension [16] leads to $\kappa \approx 8\pi U_c(n_p\xi)^2 \ln\kappa_{\mathrm{GL}}$ (U_c is the condensation energy and κ_{GL} is the Ginzburg-Landau parameter). This expression implies $\kappa_{\max} \approx 8\pi U_c \ln\kappa_{\mathrm{GL}}$ for point defects at the maximum density $n_p = 1/\xi$ (this is equivalent to a line defect). With $H_c/\mu_0 \approx 1$ T and $\kappa_{\mathrm{GL}} \approx 10^2$, we estimate $\kappa_{\max} \approx 5 \times 10^7$ N/m^2. Therefore, not only could κ be large enough that $\alpha = n_s e\phi_0$ is consistent with ρ_{xy}, but the quasiparticle Hall effect may dominate the vortex contribution to the Hall effect, as was previously argued [17].

To summarize, we have used phase-sensitive spectroscopy to characterize fully the linear response in the mixed state of three $YBa_2Cu_3O_x$ films, in the frequency range 100–500 GHz. We compared our results with the conventional low-frequency description of the vortex lattice as an overdamped harmonic oscillator. Because we measure two complex response functions, the three parameters of the model, viscosity, pinning force constant, and Magnus force, are overconstrained. We found that the Magnus parameter α is small, $\leq 0.07 n_s h/2$, in contrast to the clean limit prediction of unity. The low-temperature viscosity is a factor of approximately 20 smaller than the Bardeen-Stephen prediction for conventional superconductors and the pinning force constant is 2 orders of magnitude smaller than the maximum value consistent with the condensation energy. We reinterpreted these parameters by proposing modifications to mixed state electrodynamics due to gap anisotropy. We found that the viscosity, Magnus parameter, and the pinning force constant could be successfully accounted for by the increase in the number of quasiparticles associated with a vortex in a highly anisotropic superconductor.

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