Length Quantization in Levitation of Magnetic Microparticles by a Mesoscopic Superconducting Ring

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The equilibrium height of a magnetic microparticle levitated by a superconducting micronet ring of radius b is calculated. In the mesoscopic domain, when b is the same order as the Ginzburg-Landau coherence length ξ, the interacting particle-superconducting ring system exhibits a small set of measurably distinct, quantized, temperature dependent, levitation and suspension states. As the ratio ξ/b decreases, the number of states increases dramatically, manifesting the transition from a mesoscopic to a macroscopic system.

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Since its introduction [1], the superconducting (SC) micronet has been a subject of considerable theoretical, and recently experimental, investigations, e.g., [2–4]. Mesoscopic loops not only exhibit flux quantization, but there is recent evidence of anomalous Little-Parks oscillations [5], distinct resistance fluctuation rates depending on whether a current or a voltage source is applied [6], and the mesoscopic microladder has a first order critical transport current phase transition [7]. Technologically, an attractive feature of the SC micronet is the designability of the critical magnetic field $H_c(T)$ and current density $j_c(T)$ based on network topology, in lieu of material properties.

In this paper it is shown that fluxoid quantization by a mesoscopic SC ring with a micronet cross section can be employed to quantize a length: the equilibrium height of levitated and suspended submicron sized magnetic particles. As depicted in Fig. 1, a magnetic particle is modeled by a uniformly magnetized sphere of radius a, magnetic moment $\mathbf{M} = M_0\hat{\mathbf{z}}$, and weight W. The SC ring of radius b and the wire cross section $s_0 = wt$ carries an induced current I, and has self-inductance L. The SC ring material is characterized by the magnetic field penetration depth $\lambda(\tau)$ and the Ginzburg-Landau coherence length $\xi(\tau) = \xi_0/\sqrt{1-\tau^2}$, with $\tau = T/T_C$. In a levitation state, the height of the magnet above the SC loop is $h > 0$ and the SC current $I > 0$, as depicted. In a suspension state, $h < 0$ and $I < 0$.

The height h is determined self-consistently by minimizing the total free energy of the particle-SC ring system subject to fluxoid quantization and mechanical equilibrium constraints. Since the magnetic field at the SC ring is not a controllable external variable, the Helmholtz free energy F is the appropriate functional to be minimized. The difference between the SC state and the normal state free energy is

$$\Delta F = \frac{\Lambda(\tau)}{V_s}\int dv\left\{-N + \frac{1}{2}N^2 + \frac{1}{N}\left[J_\psi^2 + \frac{1}{4}(\xi\nabla N)^2\right]\right\} + \phi_a I + \frac{1}{2}LI^2 + Wh\,, \quad (1)$$

where $\Lambda(\tau) = \mu_0 V_s H_c^2(\tau)$, with V_s the volume of the SC ring, and H_c is the thermodynamic critical magnetic field. The contributions to ΔF are gravitational potential Wh; magnetic field energy $\phi_a I + 0.5LI^2$, with ϕ_a the "applied" magnetic flux in the SC ring due to M_0; in the integrand, the normalized condensation energy density $-N + 0.5N^2$ plus the kinetic energy density $\xi^2|\mathbf{p}\psi|^2$, where $N = |\psi|^2$, with $\psi = \Psi/\Psi_{\text{bulk}} = \sqrt{N}\exp(i\theta)$ the normalized pair wave function, $\mathbf{p} = -i\nabla + (2\pi/\phi_0)\mathbf{A}$, with $\mathbf{A}$ the magnetic vector potential, and $\phi_0 = h/(2|e|)$ the fluxoid quantum. The normalized quantum current density is defined by

$$\mathbf{J}_\psi = \xi Re(\psi^* \mathbf{p}\psi) = N\mathbf{Q}\,, \quad (2)$$

where $\mathbf{Q} = \xi[\nabla\theta + (2\pi/\phi_0)\mathbf{A}]$ is the normalized superfluid velocity. A contour integral of $\mathbf{Q}$ around the SC ring gives the fluxoid quantization constraint

$$\oint d\mathbf{l}\cdot\mathbf{Q} = \frac{2\pi\xi}{\phi_0}(\phi_a - LI + n\phi_0)\,. \quad (3)$$

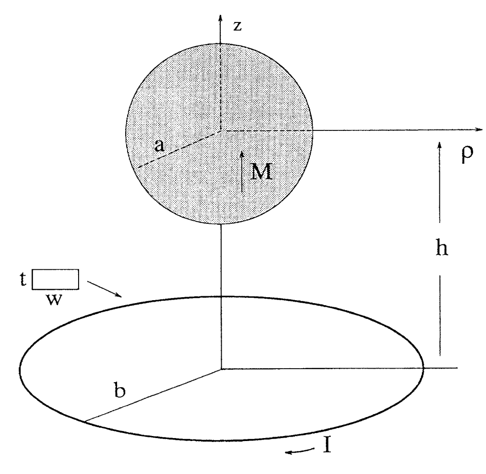

FIG. 1. A magnetic source is modeled by a uniformly magnetized sphere of radius a, and moment $\mathbf{M} = M_0\hat{\mathbf{z}}$. The SC ring of radius b and wire cross section $s_0 = wt$ carries an induced current I. The levitation height is h.

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The phase winding number n is zero or integer. The gravitational force and the interaction with the SC ring produce a net force on the magnet given by

$$\mathbf{F} = \mu_0 \int dv\, j(\hat{\boldsymbol{\rho}} H_z - \hat{\mathbf{z}} H_\rho) - \hat{\mathbf{z}} W, \tag{4}$$

where j is the SC current density, and H_z and H_ρ are the components of the field due to the magnet, evaluated at the SC ring. For the uniformly magnetized sphere, depicted in Fig. 1, the flux ϕ_a and field components are

$$\phi_a = \frac{\mu_0 M_0}{2b} \frac{1}{(1 + x^2)^{3/2}},$$

$$H_\rho = -\frac{3M_0}{4\pi b^3} \frac{x}{(1 + x^2)^{5/2}}, \qquad H_z = \frac{M_0}{4\pi b^3} \frac{2x^2 - 1}{(1 + x^2)^{5/2}}, \tag{5}$$

with normalized height $x = h/b$. At equilibrium, not only is the force $\mathbf{F} = 0$, but also $\partial F_z/\partial h = 0$. The z component of Eq. (4) is a mechanical constraint relating W, j, and H_ρ, and the derivatives of j and H_ρ with respect to height variation. Since the net horizontal force component must also be zero, the magnet must remain on the axis of symmetry of the SC ring. Stability is discussed below. For the moment assume the magnet remains as depicted in Fig. 1.

The wire cross section s_0 is assumed small enough that the transverse variation of N and H_ρ in the SC ring are negligible. In this approximation, since the system has cylindrical symmetry, the integrations in Eqs. (1), (3), and (4) are trivial. Minimizing $\Delta F(N, Q, I, h)$ subject to constraints (3) and (4) with $\mathbf{F} = 0$, and $\partial F_z/\partial h = 0$, leads to the relations

$$N = 1 - Q^2, \tag{6}$$

$$J(1 - \Omega) = NQ,$$

$$\Omega = H_\rho \frac{\partial}{\partial h}\left(\frac{\phi_a}{H_\rho}\right)\left[\frac{\partial}{\partial h}\left(\phi_a + \frac{WL'}{2\pi H_\rho}\right)\right]^{-1}, \tag{7}$$

where $L' = L/\mu_0 b \approx \ln(16b/w) - 1.75$. The normalized SC current density is $J = \lambda I/\sqrt{2}\, s_0 H_c$. The variables N, Q, and J in Eq. (7) are interpreted as mean values taken over the volume of the SC ring. Equation (6) is the usual relation between the superfluid velocity and the pair density for a uniform ring. However, Eq. (7) shows that J is not equal to the quantum mechanical expression $J_\psi = NQ$. Neglecting Ω is equivalent to neglecting the mutual inductance term $\phi_a I$ in Eq. (1). Combining Eqs. (6) and (7) gives the equilibrium height function y, which is the main result of this paper. It is

$$y(x) = \frac{J}{J_c}(1 - \Omega) + \cos\left[3\left(\frac{Q}{3J_c}\right)\right] = 0, \tag{8}$$

with $J_c = 2/\sqrt{27}$ the normalized critical current density of the ring. The variables Q and J are given by the constraint equations

$$Q = \frac{\xi}{b\phi_0}\left(\phi_a + \frac{WL'}{2\pi H_\rho} + n\phi_0\right), \tag{9}$$

$$J = -\frac{\xi^3 \kappa^2 W}{\phi_0 s_0 b} \frac{1}{H_\rho}, \tag{10}$$

with $\kappa = \lambda/\xi$. The function y in Eq. (8), with constraints (9) and (10), is a self-consistent equation for the normalized height $x(\tau, n) = h(\tau, n)/b$ of the levitated (or suspended) magnet. The function $y(x)$ has a root x_0 that corresponds to the minimum value of ΔF. For a given set of parameters, there is a minimum and a maximum value of n for which the root x_0 exists, i.e., $n_{\min} \le n \le n_{\max}$. For each n, as the temperature τ increases, x_0 disappears at $\tau = \tau_{\text{cutoff}} \le \tau_c$. Equation (8) also has a discrete set of temperature independent levitation and suspension solutions arising from exact flux quantization $Q = 0$, with $\Omega = 1$ [8]. Equation (8) is valid for any constant magnetic source with cylindrical or spherical symmetry. Levitation of one SC ring by another, involving two order parameters, is more complicated [9].

All figures are based on the following data: The magnetic particle is a YIG sphere of radius $a = 0.5\ \mu$m, magnetic moment $M_0 = 1.05 \times 10^{-13}$ A m^2, and weight $W = 2.67 \times 10^{-14}$ N. The SC ring has radius $b = 4.0\ \mu$m and wire cross section $w = 0.5\ \mu$m, $t = 0.02\ \mu$m. The SC is Al with $\lambda_0 = 0.05\ \mu$m and $\xi_0 = 1.6\ \mu$m. (A circular wire cross section of the same value gives very similar results, but the uncertainty in Q is reduced [8].)

Figure 2 shows the normalized, equilibrium levitation height $x_0(\tau, n)$ plotted as a function of the normalized temperature τ for the complete set $n_{\min} = -5$ to $n_{\max} = 3$. The quantization of x_0 is due to the integer n in the flux

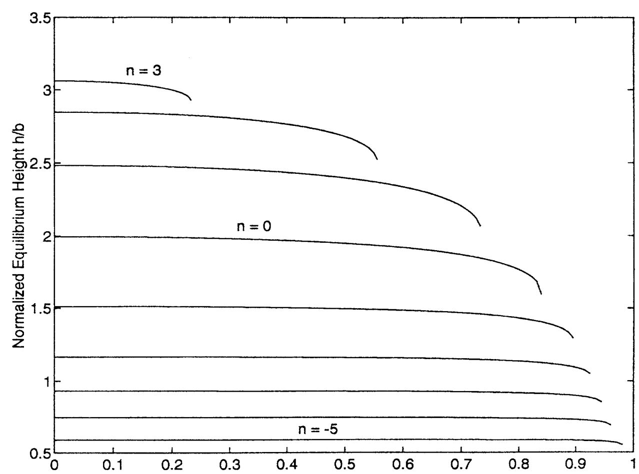

FIG. 2. The complete set of equilibrium levitation states $x_0 = h(\tau, n)/b$ is plotted as a function of $\tau = T/T_C$ for all values of the flux quantum number n in the range $n_{\min} = -5$ to $n_{\max} = 3$ for which levitation solutions exist. Each curve has a distinct cutoff temperature.

constraint, Eq. (9), and it is a function of temperature via the coherence length $\xi(\tau)$. The temperature dependence of κ is neglected. Figure 3 shows the complete set of suspension levels. Analysis of the horizontal force in Eq. (4) shows that the magnet is self-centering for $x < -1/\sqrt{2}$ and $0 < x < 1/\sqrt{2}$. Only the lowest levitation level in Fig. 2 is horizontally stable, whereas all of the suspension levels in Fig. 3 are stable. Figure 4 shows the normalized free energy $E = \Delta F/\Lambda$ plotted as a function of x for the suspension levels at $\tau = 0.5$. The value of x at the minimum values of E corresponds to the equilibrium heights $x_0(0.5, n)$ in Fig. 3. It is seen that for each n there is a local "potential well." If the magnetic particle is displaced to a point where the levels cross, it is expected that it would fall to a lower energy state. Figure 5 shows E plotted as a function of x for the four highest levitation levels at $\tau = 0$. The shallow potential well of the $n = 3$ curve is typical of all curves as τ approaches τ_{cutoff}, at which point the minimum vanishes. Since the energy for $n = 0$ crosses very close to the minimum of the higher levels, they are unstable with respect to height variation. The energy versus x plots for the lower levitation levels exhibit a structure similar to that shown in Fig. 4, which exhibits height variation stability. For any τ, as the quantum number n increases both the free energy and the levitation heights increase, but the SC pair density N decreases. Thus energy is extracted from the condensate to lift the sphere.

Replacing the aluminum SC ring with a lead ring of the same dimensions gives interesting comparative results. Lead has $\lambda_0 = 0.04\ \mu$m and $\xi_0 = 0.09\ \mu$m. The radius of the Pb ring is 44.4 times the zero temperature coherence length. The result is 86 closely spaced levitation levels with zero temperature range $0.6 \leq x_0 \leq 7.4$. The change from Al to Pb manifests an effective transition from a mesoscopic to a macroscopic system.

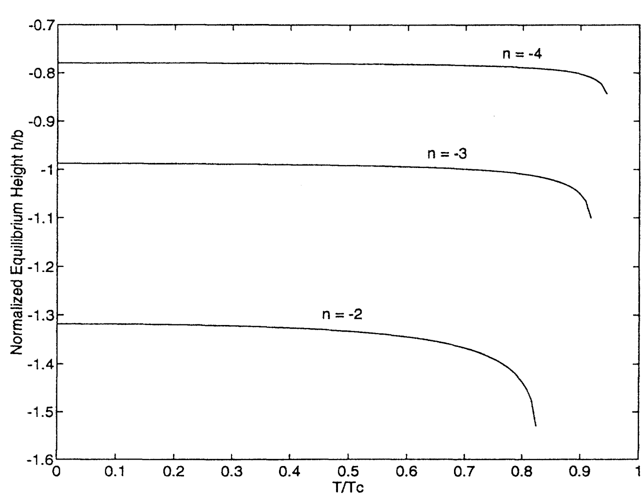

FIG. 3. The complete set of stable, equilibrium suspension states $x_0 = h(\tau, n)/b$ is plotted as a function of τ.

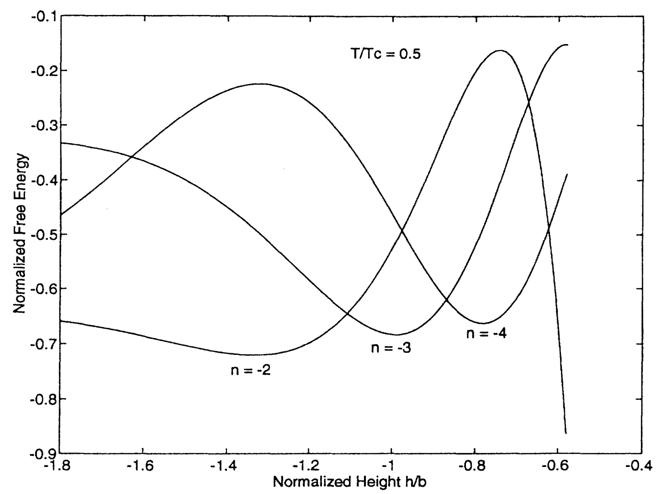

FIG. 4. The normalized total energy E of the suspension states is plotted as a function of normalized height h/b at temperature $T/T_C = 0.5$. The values of h/b at the energy minima correspond to the equilibrium values in Fig. 3. The relative position, the level crossings, indicates stability of the states with respect to height variation.

Can one experimentally observe the quantization effects predicted here? The suspension states and the lower lying levitation states are stable and should be observable in the mesoscopic domain. Recent advances in the development of optical "tweezers" using laser microbeams to trap and manipulate micron-sized biological particles may make it possible to position a magnetic microparticle [10]. To observe non-self-centering states, the magnet must be externally constrained to the symmetry axis of the SC ring. If the suggested levitation experiment is performed, the author believes it would be the first direct observation

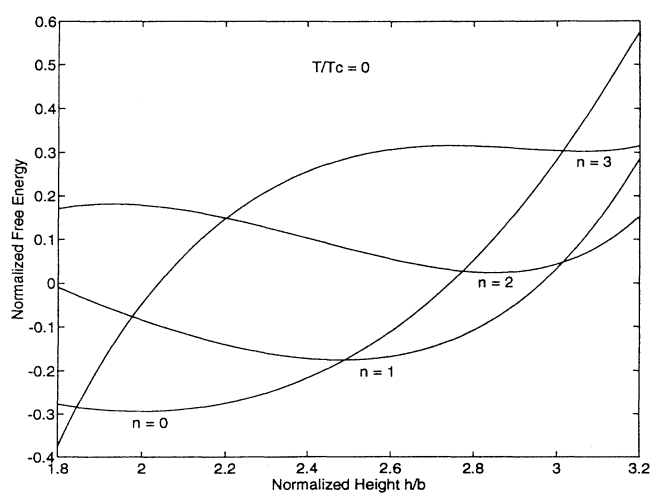

FIG. 5. The plot shows the energy of the highest four levitation states at $T/T_C = 0$. The position of the level crossings indicates instability.

of a quantized relative spatial coordinate of a corpuscular object.

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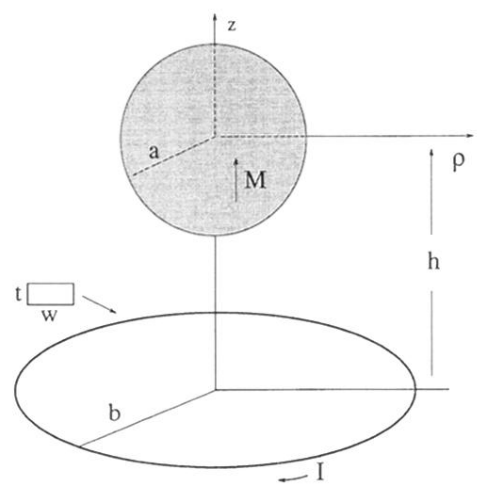

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