Superconductivity in Bad Metals

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We introduce a model of the superconducting transition in a bad metal, and show that quantum and classical phase fluctuations prevent long-range order unless the resistivity $\rho(T)$ falls below a critical value. Application of these ideas to high temperature superconductors accounts for the variation of $\rho(T_c)$ in radiation-damaged films; gives an upper bound on $T_c \propto n_s$, where n_s is the zero-temperature superfluid density; and shows that, with screening, phase fluctuations give a linear temperature dependence of n_s at low temperatures.

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A number of the most interesting new materials discovered in the past few decades are "bad metals" in the sense that their resistance has a metallic (increasing) temperature dependence but, at sufficiently high temperatures, the mean free path l of a quasiparticle would be less than its de Broglie wavelength $\lambda_F = 2\pi/k_F$, were Boltzmann transport theory to apply. Among these materials are organic conductors, alkali-doped C_{60}, and high temperature superconductors. In this paper we show that, in a sufficiently bad metal, classical and quantum fluctuations of the phase of the superconducting order parameter depress the transition temperature T_c well below its mean-field value. Specifically, the conductivity at T_c must exceed a material-dependent critical value σ_c, otherwise number fluctuations are suppressed by the long-range Coulomb interaction, and long-range phase order cannot be established.

It is often assumed that the quasiparticle decay rate and the transport scattering rate $1/\tau$ in a metal must be small compared to the quasiparticle energy k_BT, as required by Fermi liquid theory. However, perfectly sensible metals, such as lead, fail to satisfy this condition at room temperature and yet their transport phenomena are well understood in terms of Boltzmann theory [1]. In fact, the concept of a propagating quasiparticle apparently does not break down entirely [2] until its mean free path is shorter than its de Broglie wavelength: $l < \lambda_F$. In normal metals with strong electron-phonon coupling, a symptom of this breakdown is resistivity saturation; for the $A15$'s, for example, the saturation value of the resistivity [1,3] ($\rho \approx 150\ \mu\Omega$ cm) corresponds to $l \approx \lambda_F$. (Thus the A15's are not "bad metals" as we have defined the term [3].)

Bad metals fail to exhibit resistivity saturation.—For example, in $La_{1.85}Sr_{0.15}CuO_{4-\delta}$, the resistivity in the a-b plane is a linearly increasing function of temperature from T_c up to 900 K, where its magnitude is about 0.7 mΩ cm; according to Boltzmann transport theory this implies $l/\lambda_F = 0.4$ at 900 K with no sign of saturation. The failure of bad metals to exhibit resistivity saturation strongly suggests that any theory based on conventional quasiparticles with more or less well-defined crystal momenta suffering occasional scattering events does not apply. *Since there is no crossover in the temperature dependence of the resistivity as the temperature is lowered, this conclusion applies by continuity even at lower temperatures where the putative mean free path deduced from the measured values of the resistivity would not, of itself, rule out the possibility of quasiparticle transport.* In other words, a bad metal behaves as if it is a quasiparticle insulator which is rendered metallic by collective fluctuations [4].

Given that the normal state of a bad metal is anomalous, the physics of the superconducting transition must be reconsidered. In the BCS-Eliashberg mean-field theory, which is an extremely good approximation for conventional superconductors, electron pairing and long-range phase coherence occur at the same temperature T_c^{MF}. However, in bad metals, especially the high temperature superconductors, the value of the bare superfluid density n_{s0} is quite low at zero temperature [5], so the *classical* phase ordering temperature, which is proportional to n_{s0}/m^*, can be substantially lower than T_c^{MF} [6]. Moreover, the phase ordering temperature is depressed further because the poorly screened Coulomb interactions suppress the local charge density fluctuations which are associated with phase order. Together, these effects determine the superconducting transition temperature T_c in a bad metal. They also imply that local superconducting fluctuations are important for a much larger range of temperatures above T_c than in good metals.

In order to develop the theory, we consider the effective action obtained by formally integrating out all the "microscopic" degrees of freedom other than the phase $\theta(\mathbf{r}, t)$ of the local superconducting order parameter. The result is a form of nonlinear sigma model, which must be regularized by introducing a short-distance cutoff. Consequently

we define the phase variable on a cubic (or, later, tetragonal) lattice of regions of size a labeled by positions $\mathbf{R}_j$. The phase and the *total* electron number in a region are conjugate variables, such that [7]

$$\hbar\dot{\theta}(\mathbf{R}_j, t) = 2e\phi(\mathbf{R}_j, t), \quad (1)$$

where ϕ is the local electrochemical potential. Then the contribution of the *screened* Coulomb interaction to the effective action, which is given by the integral of $\mathbf{E}\cdot\mathbf{D}/8\pi$, with

$$E_b(R_j) = -\nabla_b\phi(R_j) \equiv [\phi(\mathbf{R}_j) - \phi(\mathbf{R}_j + a\hat{\mathbf{e}}_b)]/a, \quad (2)$$

and $b = x, y, z$, can be expressed as a Fourier transform:

$$S_o = \frac{\beta\hbar^3 a}{32\pi e^2}\sum_{\mathbf{k},n}\Delta(\mathbf{k}a)\tilde{\epsilon}(\mathbf{k},\omega_n)\omega_n^2|\theta_{\mathbf{k},\omega_n}|^2. \quad (3)$$

Here $\beta = 1/k_BT$, $\omega_n = 2\pi n/\hbar\beta$ are the Matsubara frequencies, $\Delta(\mathbf{k}a) = 2\sum_{b=x,y,z}[1 - \cos(k_ba)]$, and ϵ is the dielectric function. (The tilde on ϵ and other functions indicates analytic continuation to imaginary frequency.) ϵ is related to the conductivity of the microscopic degrees of freedom in the usual manner:

$$\epsilon(\mathbf{k},\omega) = \epsilon_\infty + 4\pi\sigma(\mathbf{k},\omega)/i\omega. \quad (4)$$

The second contribution to the effective action describes classical phase ordering:

$$S_V = \frac{1}{\hbar\beta}\int_0^{\hbar\beta} d\tau\, d\tau' \sum_{\langle i,j\rangle}\{\mathcal{V}(\tau - \tau') \times [1 - \cos(\theta(\mathbf{R}_i,\tau) - \theta(\mathbf{R}_j,\tau'))]\}. \quad (5)$$

The Fourier transform of $\mathcal{V}(\tau)$ will be denoted by $V(\omega_n)$, and $V(\omega_n = 0) = \hbar^2 n_{s0}a/4m^*$, where m^* is the effective mass of an electron. It is remarkable that the details of the normal state conductivity completely determine the effective action [5]. For S_o, this reflects the quantum-mechanical nature of the problem, in which dynamics and thermodynamics are intimately linked. The frequency-dependent conductivity is the response function of all the degrees of freedom which have been integrated out, and is the measure of the efficiency with which the Coulomb potential is screened. As we shall see, the physics is determined by $\sigma(\mathbf{k},\omega)$ over a wide range of frequencies.

In the classical limit, $\sigma(\omega) \to \infty$, S_V describes phase ordering in the universality class of the X-Y model, with a critical temperature $T_\theta^{\max} = AV(0)/k_B$, where A is a dimensionless number of order 1, which depends on the details of the short-distance physics. For the X-Y model on a tetragonal lattice, A varies from 0.9 in the extreme two-dimensional limit to 2.2 for cubic symmetry [6].

In order to complete the specification of the model we set the area of the unit cell $a^2 = \pi\xi^2$ where ξ is the coherence length (so that there is one-half flux quantum per plaquette at the upper critical field), in recognition of the fact that variations of the amplitude of the order parameter are important at shorter lengths. For granular materials, the physical ξ typically is equal to the size of the grains. In general, this choice of a also ensures that the energy gap Δ of the superconductor exceeds the level spacing in the microscopic regions, i.e., $a^3 \gg \Delta/N(E_F)$, where $N(E_F)$ is the density of states at the Fermi level. Quasi-two-dimensional materials, such as the oxide superconductors, consist of weakly coupled planes, and the phase variables are defined on a tetragonal (or orthorhombic) lattice. The in-plane lattice constant does not enter any of our results, so the only relevant short-distance cutoff is the larger of the average spacing between the layers a_c and $\sqrt{\pi}\,\xi_\perp$, where $\xi_\perp$ is the coherence perpendicular to the layers. Finally, since we are interested in long wavelength physics, we will largely ignore the $\mathbf{k}$ dependence of the conductivity. Whenever this is not a good approximation (for example, in relatively good metals) there can be significant departures from the present results.

In a sufficiently bad metal, the phase ordering temperature is depressed substantially below $T_c^{\rm MF}$ where the amplitude of the order parameter is established, and hence the temperature dependence of $V(\omega_n)$ may be ignored. Here, we shall usually assume that $V(\omega_n)$ is a constant V_o for $|\omega_n|$ less than a cutoff frequency Ω.

In a good metal, $4\pi\sigma \approx \omega_p^2/i\omega$ and hence, continuing to imaginary frequency, $\tilde{\epsilon} \approx \epsilon_\infty + \omega_p^2/\omega_n^2$ at all but the lowest frequencies. With this expression for $\tilde{\epsilon}$, S_o in Eq. (3) becomes the effective action for the plasmon. Note that, in contrast to Ref. [8], $V(\omega_n)$ vanishes at high frequency, and does not contribute to the plasma frequency ω_p, unless Ω is very large. Conversely, the plasmon has very little effect on the superconducting transition.

Phase fluctuations in a bad metal.—The action in Eqs. (3) and (5) is formally similar to a model of an array of resistively shunted Josephson junctions considered previously [9,10]. Thus, while its physical origin is quite different, the model may be analyzed by using the same methods, with much the same results. Details will be presented in a forthcoming paper [11]. The term proportional to σ on the right side of Eq. (4) is dominant at low frequencies, and therefore ϵ_∞ may be dropped. It is easy to see that S_o is the larger contribution to the action ($S_o/S_V \sim \hbar\sigma/V_o$). Therefore it is straightforward to generalize the procedure of Refs. [9] and [10] to derive energy shell renormalization group equations to first order in V, taking Ω as the upper cutoff, since higher-frequency modes produce a trivial renormalization of V_o:

$$d\overline{V}/d\mathcal{L} = \overline{V}\{1 - \sigma_Q/\tilde{\sigma}(\Omega)\}, \quad (6)$$

where $\overline{V} = V/\hbar\Omega$, $d\mathcal{L} \equiv d\Omega/\Omega$, and $\sigma_Q = \mathcal{F}(2e)^2/ha$. Here $\mathcal{F}$ is a constant of order unity, which depends on the short-distance details of the screening [11]; for a $\mathbf{k}$ independent σ and a cubic lattice $\mathcal{F} = \frac{1}{3}$.

Zero temperature and constant conductivity.—For simplicity we first consider $\tilde{\sigma}$ to be frequency independent. In that case, the general solution of the full renormalization group equation for $\overline{V}$ must be of the form $\overline{V} =$

$f(\Omega/\omega^*)$, where $\hbar\omega^*$ is the physical energy scale. For the first-order equation (6)

$$f(x) = (x)^{(1-\alpha)/\alpha}, \tag{7}$$

$$\omega^* = \Omega_o(V_o/\hbar\Omega_o)^{\alpha/(\alpha-1)}, \tag{8}$$

where the subscript o refers to the bare value of a parameter, and $\alpha = \tilde{\sigma}/\sigma_Q$. Equation (8) gives an *exact* expression for the energy scale for small $\overline{V}_o$. According to Eq. (7), if $\alpha < 1$, $\overline{V} \to 0$ as $\Omega \to 0$. On the other hand, if $\alpha > 1$, $\overline{V}$ increases upon rescaling and the approximate renormalization group equation is insufficient to solve the problem. However, since V is proportional to the superfluid density, it is clear on physical grounds that the ground state has long-range phase order if $\alpha > 1$, and in that case $\hbar\omega^*$ is the characteristic energy scale of the superconducting state. In particular, in the self-consistent phonon approximation [9], the renormalized value of V_o at $T = 0$ is given by $(2\pi/\alpha\mathcal{F})^{1/(\alpha-1)}\hbar\omega^*$; thus $n_s/n_{s0} = (2\pi V_o/\alpha\mathcal{F}\hbar\Omega_o)^{1/(\alpha-1)}$, and $n_s \to n_{s0}$ as $\alpha \to \infty$.

Finite temperature and constant conductivity.—The scaling must stop here when $\hbar\Omega \sim k_BT$. For $\alpha < 1$, Eq. (7) demonstrates that, at this point, $V \ll k_BT$, i.e., the system is thermally disordered, as expected, since the ground state is nonsuperconducting. For $\alpha > 1$, the superconducting transition temperature must be proportional to the characteristic energy scale of the renormalization group theory: $k_BT_c = A'\hbar\omega^*$, where A' is universal. From Eq. (8) $\hbar\omega^* \to V_o$ in the classical limit $\alpha \to \infty$. Thus A' is equal to the constant A introduced below Eq. (5). The expression (8) for ω^* may be inverted to obtain $\sigma(T_c) = \sigma_c$, the conductivity at T_c:

$$\sigma_c = \sigma_Q \frac{\ln[A\hbar\Omega_o/k_BT_c]}{\ln[AV_o/k_BT_c]}. \tag{9}$$

In the limit $T_c \to 0$, this reduces to $\sigma_c = \sigma_Q$, a constant [defined below Eq. (6)] which does not depend strongly on material properties. When the transition temperature is finite, σ_c is always larger than σ_Q.

The above general considerations must be modified in two ways for application to high temperature superconductors. First, the conductivity is strongly frequency dependent, and, in particular, $\tilde{\sigma}(\omega = 0)$ typically is much larger than $\tilde{\sigma}(\omega)$ for a wide range of frequencies. Second, the materials are quasi-two-dimensional so that, although the Coulomb interaction is still three dimensional, the conductivity and the bare superfluid density are highly anisotropic, with values in the a-b plane, $\sigma_\parallel$ and $V_\parallel$, which are much larger than their values in the c direction, $\sigma_\perp$ and $V_\perp$, respectively.

It is a simple matter to generalize the renormalization group procedure to the anisotropic case. $V_\parallel$ satisfies Eq. (6) with $\tilde{\sigma}(\Omega)$ replaced by $\tilde{\sigma}_\parallel(\Omega)$ and $\mathcal{F}$ by another [11] nonuniversal constant which is equal to $\frac{1}{2}$ for large anisotropy and **k** independent screening. The equation for $V_\perp$ shows that, under renormalization, $V_\perp/V_\parallel$ always decreases, and that consequently it will always be the growth of $V_\parallel$ which signals the approach to the superconducting transition [11]. Even with a frequency-dependent $\sigma_\parallel$, it is straightforward to integrate the equation for $V_\parallel$ as above. The result is the same as before provided that $\tilde{\sigma}$ is replaced by $\tilde{\sigma}_\parallel(0)$, and Ω_o is changed to Ω_o', which satisfies the inequality $\Omega_o' > \Omega_o$ if $\tilde{\sigma}_\parallel(\omega)$ is a decreasing function of ω. Thus the critical value of σ_{dc} is increased, reflecting the fact that the appropriately frequency-averaged conductivity is smaller than σ_{dc}. An explicit expression for Ω_o' is given in Ref. [11].

The idea that the superconducting transition is controlled by the value of the conductivity at T_c correctly predicts many trends in the transition temperature of high temperature superconductors. It has been considered as an explanation of the variation of T_c upon purposely reducing the conductivity of $YBa_2Cu_3O_{7-\delta}$ by radiation damage by Sun *et al.* [12]. As illustrated in Fig. 1, which shows $R_\square$, the resistance per square per copper oxide plane, their data are well explained by our theory. The solid line in the figure is obtained from Eq. (9) with σ_c/σ_Q replaced by $R_Q/R_\square$ and $R_Q = 6(h/4e^2) \approx 40\ \mathrm{k}\Omega$, $T_\theta^{\max} = AV_o/k_B = 100$ K, and $A\hbar\Omega_o' = 1,200$ K. Here, R_Q is determined by the resistivity of the films as $T_c \to 0$, while the assumed value of $T_\theta^{\max}$ is close to estimates obtained from $4\pi n_s = m^*[c/e\lambda(T)]^2$, where $\lambda(T)$ is the in-plane penetration depth at zero temperature [6]. It also has been assumed that the radiation damage does not induce large changes in n_{s0}. The fits are largely insensitive to changes in the assumed value of Ω_o' by factors of 2 or 3 so, for definiteness, we have set $A\hbar\Omega_o' \approx J$, the antiferromagnetic exchange energy in $YBa_2Cu_3O_{7-\delta}$.

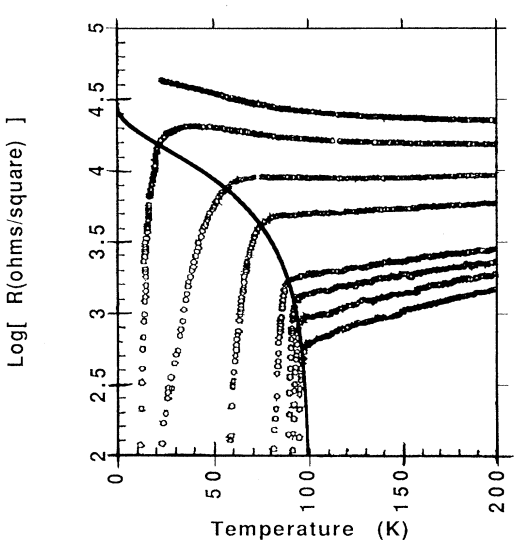

FIG. 1. The solid line is the logarithm of the resistance in ohms per square per CuO_2 plane vs T_c as derived from Eq. (10) for $R_Q = 40\ \mathrm{k}\Omega$, $AV_o = 100$ K, and $A\hbar\Omega_o = 1200$ K. The resistivity data for radiation-damaged films of $YBa_2Cu_3O_{7-\delta}$ are from Ref. [12].

It is clear from Eq. (9) that the classical phase ordering temperature $T_\theta^{\max} = AV_o$ is an *upper bound* on T_c, which is significant whenever $T_\theta^{\max} < T_c^{\mathrm{MF}}$. Since $V_o \propto n_{s0}$, this provides a new interpretation [6] of the empirical correlation [13] between T_c and $n_s(0)$ in undamaged materials. For underdoped materials, there is a considerable range of temperatures between T_c and T_c^{MF} in which the existence of a well-developed amplitude of the order parameter should be observable as a pseudogap [6]. It also is interesting to note that, according to Eq. (9), the reduction of T_c due to quantum phase fluctuations in pristine materials is about 6%, which is small but not insignificant. [Typically [14] $R_\square(T_\theta^{\max})$ is about 1 kΩ.]

Other implications of our model for high temperature superconductors include the existence of a substantial range of temperatures in the neighborhood of T_c in which critical phase fluctuations dominate the low frequency electromagnetic response, and a low temperature regime in which $n_s(T)$ will have an anomalous temperature dependence as a result of phase fluctuations: $n_s(T)/n_s(0) = 1 - \alpha \mathcal{F} k_B T/2V_o(\alpha - 1)$, to first order in T [11]. Here, it is assumed that the Coulomb interaction is screened by the finite residual far IR conductivity observed in $YBa_2Cu_3O_{7-\delta}$ [15]. If we use $\sigma(\omega) = 500\ (\Omega\ \mathrm{cm})^{-1}$ (which is roughly consistent with experiment [15]) together with $R_Q \approx 40\ \mathrm{k}\Omega$ ($\mathcal{F} \approx \frac{1}{6}$), as deduced from the data of Fig. 1, the linear temperature dependence of the superfluid density agrees quantitatively with the observation of Hardy *et al.* [16] in $YBa_2Cu_3O_{7-\delta}$ crystals [17].

In addition to its consequences for high temperature superconductors, a minor modification of our approach explains the fact that it is the value of the *normal state* resistivity that determines whether dirty metallic films ultimately become superconducting or not, and that the critical value of the resistivity, within a factor of 2, is independent not only of material, but also of morphology [18]. This is remarkable, given the other striking differences in the low temperature behavior of granular and homogeneous films. In our approach [11] both of these observations are explained by the fact that most of the screening of the Coulomb interactions occurs at frequencies larger than Δ_o, and so is not very sensitive to low energy details. One prediction of the theory is that a normal metal overlayer, separated from a disordered metallic film by an oxide layer thick enough to suppress tunneling, can still screen the Coulomb interaction and thus enhance T_c. This suggests that a strategy for making new high temperature superconductors is to fabricate structures consisting of alternating layers of a bad metal with strong pairing to induce local superconductivity, and a good metal to screen the Coulomb interaction.

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