# Experimental Observation of Stochastic Resonancelike Behavior of Autonomous Motion in Weakly Ionized rf Magnetoplasmas

Lin I and Jeng-Mei Liu
*Department of Physics, National Central University, Chungli, Taiwan 32054, Republic of China*
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The stochastic resonancelike behavior in a weakly ionized magnetoplasma system is experimentally observed by applying external white noise without periodic driving. The study covers a fixed-point region and its neighboring limit-cycle region connected through an inversed supercritical bifurcation. The noise deteriorates the coherent motion of the limit cycle. However, it also stimulates the coherent motion which is damped by the dissipation process on the fixed-point side, and in turn induces stochastic resonance.

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The dynamical behavior of nonlinear systems subjected to noise has been an interesting subject of recent studies [1−10]. A great deal of work has been devoted to stochastic resonance (SR), i.e., the phenomenon of generating coherent motion by adding noise to a dynamical system [1−9]. Most of the analytical and experimental studies concentrate on systems with bistable states and external periodic forcing, e.g., periodically modulated double-well systems, two-state electronic Schmitt triggers, bistable ring laser systems, etc. [1−8]. The addition of random noise can improve the periodic transition probability between the two stable states, namely, the signal-to-noise ratio (SNR) of the periodic component is greatly amplified by noise. SR occurs when the SNR passes through a maximum with increasing driving noise level. However, in many nonlinear autonomous systems, coherent motion can be self-organized without external periodic forcing. The effect of external noise on the coherent motion of this kind of system is much less well understood. To our knowledge, only the recent computer simulation by Gang *et al.* has demonstrated that SR occurred for the fixed points near a limit cycle [9]. Therefore it is interesting to see whether general SR behavior exists in other different real physical autonomous systems. In this Letter, we report the first experimental evidence of SR-like behavior around the fixed-point regime adjacent to the limit-cycle−fixed-point bifurcation point in a weakly ionized magnetoplasma system by stochastically modulating the system control parameter without periodic forcing.

The limit cycle is a fundamental object of various autonomous nonlinear systems. It exhibits self-organized periodic motion. A limit cycle can be converted to a fixed point through a supercritical bifurcation process by adjusting the system control parameter [11]. That is, due to the strong dissipative process on the fixed-point side, the circulation motion is damped and occurs only in the transient process. The addition of noise turns the transient circulation into asymptotic coherent oscillation which, on the other hand, can be disrupted at large noise level. SR occurs under the competition between the above two processes.

Our weakly ionized magnetoplasma system is a nonlinear continuous system. It supports spontaneously generated nonlinear ionization drift waves. The fundamental principle of drift waves and nonlinear phenomena can be found in our previous reports [12−14]. The state of the system is controlled by the complicated nonlinear balance equations of particle number, momentum, and energy, and the Poisson equation. The system exhibits different regular and chaotic motions through a cascade of bifurcations. In this study, we choose a parameter regime in which the system exhibits a simple inversed supercritical fixed-point−limit-cycle bifurcation as shown in Fig. 1. The specific form of the equation of motion for this complicated plasma system is not important. The key point is that self-organized limit-cycle motion is eliminated in the fixed-point region by the dissipation process through adjusting the system control parameter $V_0$. The general bifurcation behavior is similar to that of a Van der Pol oscillator except the bifurcation is inversed [11]. In this experiment, wideband Gaussian noises with different intensities are added onto $V_0$ at three different positions $A$, $B$, and $C$ around the bifurcation point. Coherent oscillation induced by noise is observed with a low-frequency

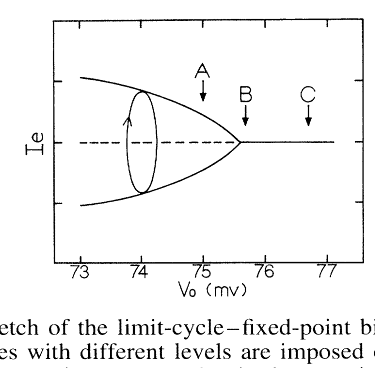


FIG. 1. Sketch of the limit-cycle−fixed-point bifurcation diagram. Noises with different levels are imposed on the control parameter $V_0$ at points $A$, $B$, and $C$ in the experiment.

  

noise background on the fixed-point side. The SNR shows a maximum as the noise level increases. It is more difficult to induce SR for points farther away from the bifurcation point on the fixed-point side. On the limit-cycle side, the coherent oscillation is gradually deteriorated by noise with increasing intensity but without any frequency shift.

The experiment is conducted in a cylindrical rf magnetoplasma system, as described elsewhere [12,13]. Briefly, the system consists of two 30-cm-long concentric cylindrical stainless-steel electrodes. The center one is grounded and the outer one is capacitively coupled to a rf power system with 14-MHz rf frequency. With low-pressure (13 mTorr) argon gas, a steady-state weakly ionized rf discharge confined by a moderate uniform axial magnetic field (47 G) is sustained between the two electrodes. With a positively biased Langmuir probe located 7 mm away from the wall of the outer cylinder, the state of the system can be controlled by varying the rf power and monitored by measuring the probe current $Ie$ ($-Ie$ is proportional to the electron density). The signal is digitally processed. The rf power system consists of a rf power amplifier with 60-dB gain and two programmable function generators, PFG-I and PFG-II. The amplitude of the rf voltage across the two electrodes varies linearly with the amplitude $V_{\mathrm{rf}}$ of the 14-MHz oscillation of PFG-I. $V_{\mathrm{rf}}$ is modulated by a signal with a dc offset and a flat wide band noise with 5 MHz bandwidth generated by PFG-II. That is, $V_{\mathrm{rf}} = V_0 + V_n(t)$, where $V_0$ is the averaged value, and $V_n(t)$ is the noise with zero mean and root mean square intensity $V_{\mathrm{rms}}$. The correlation time of the noise is less than 0.5 ms. $V_0$ and $V_{\mathrm{rms}}$ can be computer controlled during the experiment. Points $A$, $B$, and $C$ in Fig. 1 correspond to the points with different $V_0$.

Figure 2 shows the effect of noise on the power spectra (the power spectra are averaged over 14 time series) and the typical time evolution of the limit cycle corresponding to point $A$ in Fig. 1. At $V_{\mathrm{rms}} = 0$ [Fig. 2(a)], the system shows a stable oscillation with a sharp peak located at 15.6 kHz in the power spectrum. The noise floor in the power spectrum is low and flat. Figure 2(b) shows the modulation of the coherent oscillation by noise at $V_{\mathrm{rms}} = 3.7$ mV. The power spectrum picks up a low frequency background with a hump around the peak position. Further increasing $V_{\mathrm{rms}}$ causes lowering and broadening of the sharp peak and increase of the low frequency noise background [Fig. 2(c)]. The SNR (SNR is defined as the height of the peak above the noise floor in the power spectrum) monotonically decreases but is associated with no frequency shift of the peak position.

Figure 3 shows the SR-like behavior at point $B$ which is on the fixed-point side adjacent to the bifurcation point. At $V_{\mathrm{rms}} = 0$, there is no coherent oscillation [Fig. 3(a)]. The small noise of the real time signal is mainly due to the small intrinsic noise of the system and the quantization noise of the digital scope. SR is manifested by the appearance of a peak centered at 16.3 kHz with increasing intensity as $V_{\mathrm{rms}}$ [Figs. 3(b) and 3(c)]. Similarly to that at point $A$, further increasing $V_{\mathrm{rms}}$ also induces noisy disruption of the coherent motion and increase of the low-frequency noise floor in the power spectra. It subsequently destroys the coherent motion at large $V_{\mathrm{rms}}$ [Fig. 3(d)].

The SR phenomenon becomes less pronounced as we operate on the fixed-point side away from the bifurcation point. Figure 4 shows the behavior at point $C$. Comparing to point $B$, a larger $V_{\mathrm{rms}}$ is required to make the peak (centered at 19.7 kHz) visible in the power spectrum [Fig. 4(b)]. The peak has a smaller SNR and is easily buried by the noise background as $V_{\mathrm{rms}}$ increases. It is hard to induce any SR for $V_0 > 78$ mV. The inclusion of the stochastic driving induces only a noise background

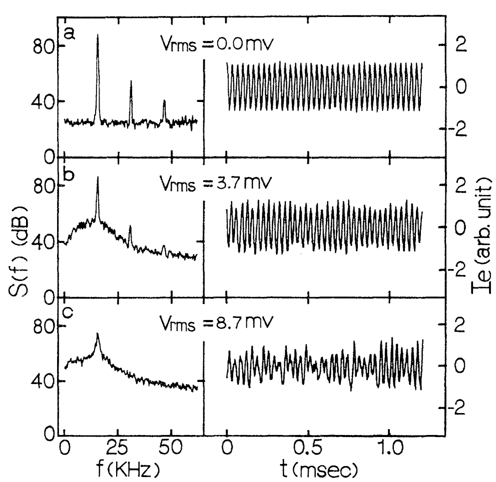


FIG. 2. The power spectra and time evolutions of probe current at point $A$ ($V_0 = 75$ mV) with different $V_{\mathrm{rms}}$.

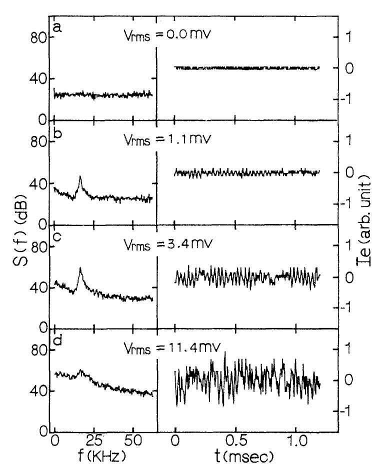


FIG. 3. The power spectra and time evolutions of probe current at point $B$ ($V_0 = 75.7$ mV) with different $V_{\mathrm{rms}}$.

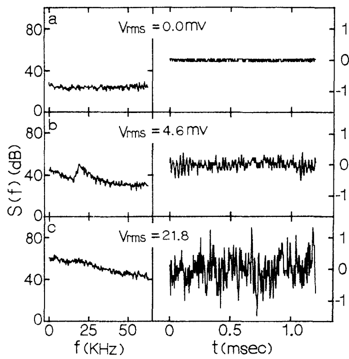


FIG. 4. The power spectra and time evolutions of probe current at point $C$ ($V_0 = 76.7$ mV) with different $V_{\rm rms}$.

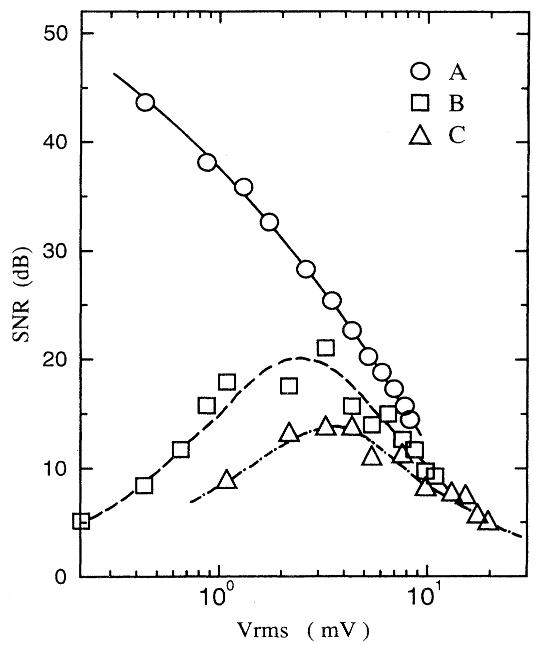


FIG. 5. The SNR vs $V_{\rm rms}$ at points $A$, $B$, and $C$.

with a stronger low-frequency part in the power spectrum (not shown).

It is interesting to point out that, in the ordinary bistable SR system, the periodic transition is strongly driven by the external periodic forcing in conjunction with noise. The former locks the response frequency and leads to a deltalike peak in the power spectrum. In our system, the coherent motion is self-organized. Our results indicate that stochastic driving has two different contributions to dynamical behavior. In the fixed-point regime where the oscillation is damped by the dissipative process, the noise drives the system away from the stable fixed point. That is, it plays the role of an effective pumping source to stimulate the damped coherent oscillation. On the other hand, it disturbs the coherent motion. The competition between the two stochastic processes drastically modulates the coherent motion. It causes broadening of the peak and induces low-frequency noise in the power spectrum when $V_{\rm rms}$ increases.

The effects of the above two processes can also be observed in Fig. 5 where the SNR vs $V_{\rm rms}$ at points $A$, $B$, and $C$ is plotted. The increasing $V_{\rm rms}$ monotonically decreases the SNR at point $A$. The SR on the fixed-point side near the bifurcation is evidenced by the maxima of curves $B$ (about 20 dB) and $C$ (about 13 dB). The lower height of the maximum of curve $C$ and the larger noise level required indicate the difficulty of driving SR due to the larger dissipation. The larger $V_{\rm rms}$ for exciting coherent motion under the stronger dissipation also deteriorates the coherent motion and makes the SR more difficult. It is thereby hard to observe SR for $V_0 > 78$ mV.

The general SR behaviors in our system are similar to the results from the computer simulation by Gang *et al.* [9], although their system has four fixed points (two saddle points and two stable nodes) before the bifurcation to the limit-cycle region and is quite different from our system. The presence of a SNR hump as in Fig. 5, the increasing coherent peak width, and the increasing low-frequency noise level as the driving noise level increases were also observed. However, our SR behavior is more pronounced. Our maximum SNR at resonance is about 25 times larger than theirs and our resonant width is also much narrower. In contrast to the many shifts of the peak frequency $f$ with increasing noise level in their work, the increasing $V_{\rm rms}$ induces almost no shift of $f$ on the limit-cycle side and a small increase of $f$ (less than 1 kHz) on the fixed-point side in our system. That is, the external noise does not strongly affect the intrinsic characteristic frequency of the self-organized motion. This is similar to the behavior of a Van der Pol oscillator with its control parameter stochastically modulated by white noise with increasing intensity around the bifurcation point according to our recent computer simulation [15].

In conclusion, we demonstrate experimentally a SR-like phenomenon for an autonomous magnetoplasma system subjected to white Gaussian noise modulation without periodic driving in the control parameter. On the limit-cycle side, the modulation deteriorates the coherent motion of the limit cycle. On the fixed-point side adjacent to the bifurcation, where the coherent motion is damped by the dissipative process, the coherent motion with low frequency noisy background can be stimulated by the modulation. The SNR varies with noise amplitude and SR is observed. It is harder to induce SR for fixed points far away from the bifurcation point due to the stronger dissipation. The noise modulation only slightly affects the frequency of the coherent motion but broadens its peak width in the spectrum.

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