Soft X-Ray Amplification via Resonant Backward Scattering from Relativistic Particle Beams

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A theory of a relativistic beam amplifier is presented. Owing to backscattering of radiation from a beam of relativistic atoms or ions, frequency up-conversion of the radiation from the infrared to the soft x-ray region is possible, a process that is enhanced if the incident radiation is resonant with an atomic transition. Methods for achieving amplification of the scattered radiation based on Rayleigh and Raman scattering are discussed. Requirements for beam density and field power are estimated. A comparison with requirements for the free electron laser is made.

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In this Letter we consider resonance scattering of an electromagnetic field from heavy particles (atoms or ions) as a technique for frequency up-conversion of coherent radiation from the infrared to soft x-ray range. Scattering from relativistic electrons lies at the heart of the free electron laser (FEL) and allows one to create both coherent optical [1] and x-ray spontaneous [2] sources of radiation. Recent progress in the design of ion accelerators [3,4] makes it now feasible to consider relativistic beams as a new medium for short-wavelength generation. Frequency up-conversion using resonance scattering was proposed earlier [5] as a means for obtaining spontaneous short-wavelength radiation. In this paper the possibility of creating coherent radiation is explored.

Consider an atomic beam propagating along the x axis, having average velocity $\bar{\mathbf{v}}$ and Lorentz factor $\gamma = (1 - \beta^2)^{-1/2}$, where $\boldsymbol{\beta} = \bar{\mathbf{v}}/c$. A profound difference in the scattering of radiation by atoms and electrons arises as a result of the internal atomic degrees of freedom which allow one to tune an incident pump field, $E_p(x,t) = \frac{1}{2}E_p e^{-i\Omega_p t - ik_p x} + \text{c.c.}$, having amplitude, frequency, and wave vector E_p, Ω_p, and k_p, respectively, into resonance with an atomic transition between ground and excited levels having frequency ω_a [see Fig. 1(a)]. If the incident pump field propagates in a direction opposite to that of a highly relativistic beam having $\gamma \gg 1$, the resonance condition in the atomic rest frame is $\Omega_p' = \gamma(1+\beta)\Omega_p \approx 2\gamma\Omega_p \approx \omega_a$, where primes indicate quantities evaluated in the atomic rest frame [atomic frequencies, wavelengths, scattering cross sections, and the electric susceptibility χ (see below) are evaluated in the rest frame but written without primes]. For an optical atomic transition ($\lambda_a \sim 0.5$ μm), and $\gamma \sim 10$, a pump field having wavelength $\lambda_p \approx 2\gamma\lambda_a = 10$ μm is resonant with the atom in its rest frame. The backscattered radiation has the same frequency as the incident radiation in the atom's rest frame but, in the laboratory frame, the frequency of radiation scattered in the forward direction (along the direction of the relativistic beam) is $\Omega = \gamma(1+\beta)\omega_a = [(1+\beta)/(1-\beta)]\Omega_p \approx 4\gamma^2\Omega_p$, which is in the soft x-ray region ($\lambda = 2\pi c/\Omega \sim 25$ nm). The probe beam can be separated from the atomic beam by allowing a small angle between the probe and atomic beams.

Efficiency of the frequency up-conversion depends on the scattering cross section, the value of the required pump field power, the beam density, and other parameters. Let us compare these quantities for scattering from atoms and electrons. For a resonant pump field, the efficiency of scattering is determined by the resonance fluorescence cross section which is $\sigma_a \sim \lambda_a^2 \sim c^2\hbar^6/m^2e^8$. The cross section for scattering by electrons (Thomson cross section) is $\sigma_e \sim r_e^2$, where $r_e = e^2/mc^2$ is the classical electron radius. It follows that $\sigma_a/\sigma_e \sim \alpha^{-6} \approx 7 \times 10^{12}$, where α is the fine-structure constant.

One can also compare the requirements on pump field power for the FEL and the relativistic beam amplifier (RBA). For the FEL, saturation is achieved for an undulator parameter $K \sim e[E_p^{(e)}]'\lambda_p'/mc^2 \sim 1$. If the pump field frequency in the electronic rest frame is of order of ω_a, then $[E_p^{(e)}]' \sim (mc/e)\omega_a$. On the other

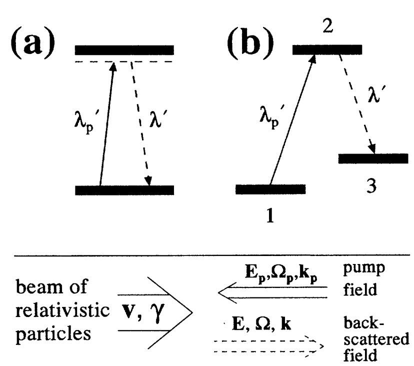

FIG. 1. Relativistic atomic beam scatters a counterpropagating pump field E_p into a copropagating probe field mode E. (a) *Rayleigh scattering,* in the atomic rest frame, the fields' wavelengths are nearly equal to the wavelength of the transition between ground and excited states, but, in the laboratory frame, $\lambda \approx \lambda_p/4\gamma^2$, if $\gamma \gg 1$. (b) *Raman scattering,* in the atomic rest frame, the pump and probe field are resonant with the $1 \to 2$ and $2 \to 3$ transitions, respectively, but, in the laboratory frame, $\lambda \approx \lambda_p/4\eta\gamma^2$, $\eta = \lambda_p'/\lambda'$.

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hand, to saturate an atomic transition, one needs a Rabi frequency $d[E_p^{(a)}]'/\hbar$ of order of the excited state decay rate $\Gamma \sim \alpha^3\omega_a$, or $[E_p^{(a)}]' \sim \alpha^3\hbar\omega_a/ea_B$ ($d \sim ea_B$ is the dipole moment matrix element between the ground and excited states, a_B is the Bohr radius). The ratio of the fields' Poynting vectors $S_p' = c|E_p'|^2/8\pi$ in the two cases is $[S_p^{(a)}]'/[S_p^{(e)}]' \sim ([E_p^{(a)}]'/[E_p^{(e)}]')^2 \sim \alpha^8$. Thus, the saturation pump field power for a RBA is 17 orders weaker than that for the FEL. The Poynting vector in the laboratory frame $S_p \approx [S_p^{(a)}]'/4\gamma^2$ is of order $S_p \sim \alpha^9(mc^2)^2/32\pi\hbar\gamma^2a_B^2 \sim 10\ \mathrm{W/cm^2}$, which is easily achievable even for cw pump infrared radiation.

Finally, the ion density cannot exceed its space charge limited value. Moreover, the density must be sufficiently small to guarantee that the pump field absorption does not exceed several percent.

Using Maxwell's equations and either Minkowski equations [6], or constitutive equations in three-dimensional form [7], or constitutive equations in four-dimensional form, one finds that the amplitude $\mathbf{E}(\mathbf{r})$ of the field $\mathbf{E}(\mathbf{r},t) = \frac{1}{2}\mathbf{E}(\mathbf{r})e^{-i\Omega t + i\mathbf{k}\cdot\mathbf{r}}$ + c.c., considered as a slowly varying function of $\mathbf{r}$, satisfies

$$(\mathbf{n}\cdot\nabla)\mathbf{E} = 2\pi ik\chi\gamma^2\{\mathbf{E}(\mathbf{n}-\boldsymbol{\beta})^2 - [\boldsymbol{\beta} - \mathbf{n}(\mathbf{n}\cdot\boldsymbol{\beta})] \times (\boldsymbol{\beta}\cdot\mathbf{E}) - (\boldsymbol{\beta}\times\mathbf{n})[\mathbf{E}\cdot(\boldsymbol{\beta}\times\mathbf{n})]\}, \quad (1)$$

where $\mathbf{n} = \mathbf{k}/k$, and χ is the beam susceptibility in the rest frame. The assumption $|\chi| \ll 1$, corresponding to a weakly amplifying medium, is necessary for the validity of Eq. (1). The field Poynting vector $\mathbf{S}(\mathbf{r}) = (c/8\pi)|\mathbf{E}(\mathbf{r})|^2$ obeys the equation $(\mathbf{n}\cdot\nabla)\mathbf{S}(\mathbf{r}) = -4\pi k\,\mathrm{Im}(\chi)\gamma^2(1-\mathbf{n}\cdot\boldsymbol{\beta})^2\mathbf{S}(\mathbf{r})$. For a wave propagating along the x axis, the connection between the field intensities $S_{\mathrm{in}}(x=0)$ and $S_{\mathrm{out}}(x=L)$ can be written $S_{\mathrm{out}} = \exp[A]S_{\mathrm{in}}$, where the amplification A is given by

$$A = -4\pi k\gamma^2(1-\mathbf{n}\cdot\boldsymbol{\beta})^2\int_0^L dx\,\mathrm{Im}[\chi(x)]. \quad (2)$$

Thus, one needs to calculate the susceptibility χ when atoms interact with a sum of pump and probe fields $\mathcal{E}'(x',t') = \frac{1}{2}E_p'e^{-i\Omega_p't' - ik_p'x'} + \frac{1}{2}E'e^{-i\Omega't' + ik'x'}$ + c.c. For an interaction length $L \lesssim 10$ m and $\gamma \sim 10$, the interaction time in the rest frame $\tau = L/\gamma\bar{v} \lesssim 3$ ns is smaller than a typical excited state lifetime ($\tau_s \sim 10$ ns). As a consequence, one must consider the problem in the transient rather than steady-state domain [8].

We assume that the particles' energy distribution in the laboratory frame has a width $\delta\epsilon$ which is sufficiently small to guarantee that the width u of the velocity distribution in the rest frame satisfies

$$u = \epsilon_e c/\beta \ll \bar{v}, \quad (3)$$

where $\epsilon_e = \delta\epsilon/\epsilon$. Note that the term "rest frame" will be used for the reference system moving with velocity $\bar{v}$ along the x axis. Condition (3) permits one to use a nonrelativistic description of the atomic density matrix. Even for an energy spread as small as $\epsilon_e \sim 10^{-5}$, one is in the large Doppler broadening limit since the ratio of the Doppler width $k_p'u/2\pi \sim 5$ GHz to transit-time width $1/2\pi\tau$ is

$$k_p'u\tau \sim 100 \gg 1. \quad (4)$$

Since the probe field is tuned to the transition between ground and excited states, one needs to choose a regime where amplification owing to two-quantum interactions dominates probe absorption. Different ways have been proposed to achieve this goal [8–16] in the nonrelativistic case. For instance, a dispersionlike two-quantum line shape, centered at the pump field frequency, resonant with an open two-level system, can result [9–13] in probe amplification if $\Delta_p' \gg k_p'u$ [13]. One can use this possibility for multiply ionized atoms, having an excited state lifetime that is shorter than the time of interaction. We plan to consider this regime in the future. To achieve a similar effect in a closed system one can use the recoil induced resonance (RIR) [17,18], which has also been proposed recently as a mechanism for achieving gain in frequency up-conversion schemes [15,16]. If the RIR is used, the required time of interaction τ must be greater than ω_k^{-1}, where $\omega_k = \hbar k'^2/2M$ is the recoil frequency and M the atomic mass. For typical values $\omega_k \sim 10^5\ \mathrm{s}^{-1}$, $\bar{v} \sim c$, $\gamma \sim 1$, the length of the interaction zone L must be greater than $\bar{v}\gamma\tau \gtrsim \bar{v}\gamma\omega_k^{-1} \sim 3\times10^5$ cm, which looks rather unrealistic.

The gain mechanism considered below is based on the dynamic Stark splitting in a strong pump field which separates the spectral regions for probe absorption and amplification [8,14]. In terms of the x coordinate in the laboratory frame, one finds a susceptibility [8],

$$\chi(x) = N'(|d|^2/\hbar)\langle r(t_x)\rangle, \quad (5a)$$

where

$$r(t) = \sum_{j=1}^{3}\left[\sum_{k=1}^{3}\xi_{jk}f_{jk}(t) + \eta_j h_j(t)\right], \quad (5b)$$

$$f_{jk}(t) = \{\{1-\exp[i(\tilde{\delta}-\lambda_j)t]\}/(\tilde{\delta}-\lambda_j) + \{\exp[i(\tilde{\delta}-\lambda_j)t] - \exp[-i\lambda_k t]\}/(\tilde{\delta}-\lambda_j+\lambda_k)\}/\lambda_k, \quad (5c)$$

$$\xi = |\chi_p'|\sin(\theta)\begin{bmatrix}\sin^2(\theta/2) & 0 & 0\\ 0 & -\cos^2(\theta/2) & 0\\ \cos^2(\theta/2) & -\sin^2(\theta/2) & 0\end{bmatrix}, \quad (5d)$$

$(\lambda_1,\lambda_2,\lambda_3) = (\omega_{AB}, -\omega_{AB}, 0)$, $h_j = \{\exp[i(\tilde{\delta}-\lambda_j)t]-1\}/(\tilde{\delta}-\lambda_j)$, $(\eta_1,\eta_2,\eta_3) = [\sin^4(\theta/2), \cos^4(\theta/2), \sin^2(\theta)/2]$, $N' = N/\gamma$, N is the beam density in the laboratory frame, $t_x = x/\gamma\bar{v}$, $\tilde{\delta} = \delta - 2kv$, v is the atomic velocity in the rest frame, $\delta = \Omega' - \Omega_p'$ is the detuning of the probe field from the pump in the rest frame, $\hbar\omega_{AB} = \hbar(\Delta_p'^2 + 4|\chi_p'|^2)^{1/2}$ is the energy interval between A- and B-type dressed states [8], $\Delta_p' = \Omega_p' - \omega_a$ and $\chi_p' = -dE_p'/2\hbar$

are the pump field detuning and Rabi frequency, respectively, and $\cos(\theta) = 2\Delta'_p/\omega_{AB}$.

Equation (5b) contains components that oscillate on a time scale of order $1/\omega_{AB}$, as well as components that are (nearly) constant in time. If $\omega_{AB}\tau \gg 1$, the oscillating components are negligible. Analysis shows that the smooth part contains two components, $r_\pm(t_x)$, centered at $\Omega' = \Omega'_p \pm \omega_{AB}$. For $\Delta'_p > 0$, the r_+ component gives amplification. Assuming a Maxwellian distribution of velocities, one gets for amplification in the Doppler limit (4)

$$A = [\pi^{3/2}/18]N\lambda_p'^2 L(\Gamma/\Omega'_p)\beta[\epsilon_e(1+\beta)\gamma^2]^{-1} f(|\chi'_p|/\Delta'_p) \times \exp\{-[(\delta - \omega_{AB})/2k'_p u]^2\}, \quad (6a)$$

$$f(|\chi'_p|/\Delta'_p) = 27\cos(\theta)\sin^4(\theta/2), \quad (6b)$$

where $\Gamma = \frac{4}{3}|d|^2/\hbar\lambda_p'^3$ is the excited state radiative decay rate. The corresponding formula for absorption, arising from the r_- component, can be obtained from (6b) with the replacement $\theta \to \theta + \pi$. The dependence of the amplification on the field intensity is given by the function f, having its maximum value, $f = 1$, at

$$\theta = \arccos\left(\tfrac{1}{3}\right), \qquad |\chi_p| = 2^{1/2}\Delta_p. \quad (7)$$

For this field strength, maximum probe absorption is 4 times larger than maximum probe amplification, but the distance between the absorption and amplification lines is $2\omega_{AB} = 6\Delta'_p$. If the parameter $\epsilon_\Delta = k'_p u/|\Delta_p|$ is sufficiently small, then the wing of the absorption line is negligible in the range $|\Omega - [(1+\beta)/(1-\beta)]\Omega_p - \omega_{AB}/\gamma(1-\beta)| \lesssim \beta^{-1}\epsilon_e[(1+\beta)/(1-\beta)]\Omega_p$ where one expects amplification.

To determine the pump field power associated with the Rabi frequency (7) we assume that the field is a Gaussian beam having aperture a_p (field amplitude dependence on the transverse coordinates $\mathbf{r}_\perp$ is $\exp[-(\mathbf{r}_\perp/a_p)^2)$ larger than the atomic beam size, and a sufficiently large confocal parameter $a^2/2\lambda_p \gg L$ to guarantee that the center of the amplification line is the same for the different points in the interaction zone. In terms of the field amplitude, the power is calculated as $W_p = c|E_p|^2 L\lambda_p/8\epsilon_c$, where $\epsilon_c = 2L\lambda_p/a_p^2 \ll 1$. Using Eq. (7) and taking into account the transformation law for the field, $E_p = \gamma(1-\beta)E'_p$, one arrives at the expression for optimum pump field power

$$(W_p)_{\text{opt}} = \tfrac{4}{3}[\hbar\Omega_p'^4 L/\bar{v}\Gamma\gamma\beta(1+\beta)](\epsilon_e^2/\epsilon_\Delta^2\epsilon_c). \quad (8)$$

The probe field now drives a transition $2 \to 3$ coupled to the pump field transition $(1 \to 2)$ [see Fig. 1(b)]. Level 3 is initially unpopulated. If $\tau/\tau_s \ll 1$, level 3 is negligibly populated via spontaneous decay from level 2 and probe field amplification occurs for arbitrary pump detunings and field strength. As before, the probe field propagates in a direction opposite to the pump.

In the transient regime one can use equations for state amplitudes a_i [19]. Solving them to first order in the probe field Rabi frequency $\chi' = -d_{23}E'/2\hbar$ (d_{23} is the $2 \to 3$ matrix element of the dipole moment operator) and calculating the beam susceptibility $\chi(x) = 2N'd_{32}\langle a_2(t_x)a_3(t_x)^* \exp[i\Delta' t_x]\rangle/E'$, $t_x = x/\gamma\bar{v}$, one finds for amplification in the Doppler limit (4)

$$A = \tfrac{3}{2}\pi^{1/2}N\lambda'^2(\Gamma_{23}/\Omega')\beta[\epsilon_e(1+\beta)\gamma^2]^{-1}L\eta q, \quad (9a)$$

$$q = k'_p\tau|\chi'_p|^2\int_{-\infty}^{\infty}dv \times \left|\int_0^\tau dt\,\sin(\tilde{\omega}_{AB}t/2)\exp[i(\tilde{\Delta}' - \tilde{\Delta}'_p/2)t]\right|^2, \quad (9b)$$

where Γ_{2i} is the decay rate $2 \to i$ ($i = 1$ or 3), $\tilde{\Delta}' = \Delta' - k'v$, $\tilde{\Delta}'_p = \Delta'_p + k'_p v$, Δ' and Δ'_p are the probe and pump field detunings from transition frequencies ω_{21} and ω_{23}, respectively, $\tilde{\omega}_{AB} = [\tilde{\Delta}'_p + 4|\chi'_p|^2]^{1/2}$, $\eta = k'/k'_p$, and $|\Delta'_p| \ll k'_p u$, $|\chi'_p| \ll k'_p u$. In the weak pump field limit one arrives at the expression

$$q = \tfrac{2}{3}\pi(1+\eta)^{-1}|\chi'_p\tau|^2\varphi[(\Delta' + \eta\Delta'_p)\tau/(1+\eta)], \quad (10a)$$

$$\varphi(x) = 6x^{-2}[1 - \sin(x)/x], \quad (10b)$$

which describes a transit-time broadened, Doppler free (owing to velocity selection), Raman resonance. The line center and linewidth $\Gamma_{1/2} \approx 3.7(1+\eta)/\tau$ are in qualitative agreement with former results [20], obtained in a steady-state limit, if one assumes that the level 2 decay rate is of order of τ^{-1}; however, the line shape (10b) differs from the previous results.

One can expect that optimum conditions for amplification (9a) are achieved at the center of the $2 \to 3$ transition ($\Delta' = 0$) for resonant pumping $\Delta'_p = 0$ and a Rabi frequency $|\chi'_p| \sim \tau^{-1}$. The field dependence of the amplification factor q has been calculated numerically, and it appears that q achieves its maximum value q_m only at infinite field strength. For this kind of dependence we determine $|\chi'_p|_{\text{opt}}$ from $q = (1-\alpha)q_m$, with $\alpha \ll 1$. Values of the maximum amplification and optimum Rabi frequencies for different wave vector ratios are shown in Fig. 2 for $\alpha = 0.2$. From this figure one can find an optimum pump field power

$$(W_p)_{\text{opt}} = \tfrac{1}{6}(|\chi'_p|_{\text{opt}}\tau)^2\hbar\Omega_p'^2 c/L\Gamma_{21}\epsilon_c \quad (11)$$

in the same manner as was described above for Rayleigh scattering.

Let us estimate the amplification and pump field power requirements. Consistent with state of the art relativistic beams [3], we take $\gamma = 10$, $\beta \approx 1$, relative width of the energy distribution in the beam $\epsilon_e = 10^{-5}$, and interaction length $L = 10$ m. Other parameters chosen are $\lambda' = 0.1$ μm, $\Gamma = 10^8$ s^{-1}, and $\epsilon_\Delta = \epsilon_c = 0.25$. Substituting these values into Eqs. (6) and (8) one finds that Rayleigh scattering can result in amplification $A = 5 \times 10^{-13}N$, achievable for pump field power $W_p = 1.2$ kW. For Raman scattering we choose $\eta = 0.5$ and $q_m \approx 3.2$,

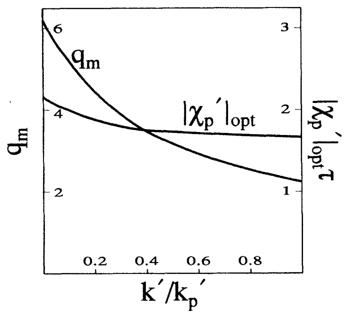

FIG. 2. Maximum amplification q_m and dimensionless optimum pump field Rabi frequency $|\chi_p'|_{opt}\tau$ for different values of the wave vector ratio.

$|\chi_p|_{opt} \approx 1.7/\tau$. From Eqs. (9) and (11) one finds $A = 7.1 \times 10^{-12}N$, $W_p \approx 2.2$ mW. Consequently, beam densities of order of 10^9 to 10^{10} cm^{-3} are required for a gain of order of 1%.

Raman scattering also can be observed using a relativistic H^0 beam [4,21] having density $N = 8 \times 10^{10}$ cm^{-3}, kinetic energy 800 MeV ($\beta = 0.84$, $\gamma = 1.86$), and relative momentum width $\delta p/p = 5 \times 10^{-4}$, corresponding to $\epsilon_e = \beta^2 \delta p/p = 3.5 \times 10^{-4}$. Choosing, for instance, the three-level scheme $1S \rightarrow 3P \rightarrow 2S$ [$\lambda_p' = 102.5$ nm, $\lambda' = 656.2$ nm, $k'/k_p' = 0.16$, $q_m = 4.7$, $|\chi_p'|_{opt}\tau = 1.9$ (see Fig. 2)], where $\Gamma_{21} = 1.7 \times 10^8$ s^{-1}, $\Gamma_{23} = 2.2 \times 10^7$ s^{-1}, length of the interaction zone $L = 6.3$ cm (coinciding with the length of the H^0 beam micropulse [21]) one can expect amplification at a wavelength $\lambda = \lambda'/\gamma(1 + \beta) = 191.6$ nm under action of a pump field having wavelength $\lambda_p = \lambda_p'\gamma(1 + \beta) = 351.3$ nm. Amplification $A = 1.2\%$ can be achieved for an optimum pump field power $W_p \approx 2.4$ W, focused to a spot with aperture $a_p = \sqrt{2L\lambda_p/\epsilon_c} = 170$ μm.

These estimates show that RBA can provide observable amplification in the soft x-ray region using readily available cw and pulse laser sources.

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