How Disoriented Chiral Condensates Form: Quenching versus Annealing

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We demonstrate that fluctuations, their relaxation, and the chiral phase transition are automatically incorporated in the numerical simulations of the classical equations of motion in the linear σ model when longitudinal and transverse expansions are included. We find that domains of disoriented chiral condensate with 4–5 fm in size can form through a quench while an annealing leads to domains of smaller sizes. We also demonstrate that quenching cannot be achieved by relaxing a chirally symmetric system through expansion.

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One of the proposed explanations for the Centauro events [1] in high energy cosmic ray experiments is the coherent emission of pions from a large domain of disoriented chiral condensate (DCC) [2]. However, if the system has to go through an equilibrium phase transition, quark masses, though much smaller than the intrinsic QCD scale, prevent DCC domains from reaching a size larger than $1/T_c$ [3]. As an alternative, Rajagopal and Wilczek [4] proposed that a nonequilibrium phase transition through quenching can generate large DCC domains. Their numerical simulation indeed observed the amplification of the long wavelength modes of the pion fields, but did not shed light on the size of DCC domains. Similar simulations by Gavin, Gocksch, and Pisarski [5] are, however, not conclusive about the exact size of DCC domains, since they only looked at the pion fields averaged over the transverse dimensions which cannot reveal domains smaller than the lattice size.

To model a quench, Rajagopal and Wilczek argued that one can evolve the classical fields according to the zero temperature equations of motion from a chirally symmetric initial condition with short correlation lengths. In this Letter, we shall argue that fluctuations introduced in the initial configuration actually render the effective potential to a non-zero-temperature one. The interaction between the mean fields and the fluctuations as well as their evolution can be automatically included in the numerical simulations of the equations of motion. Using an ensemble averaging technique, we demonstrate the relaxation of the fluctuations and the occurrence of the chiral phase transition due to the longitudinal [6] and transverse expansions which are consistently included in our study, whereas in earlier studies [4,5] the system only evolves in an approximately constant [7] but nonzero temperature effective potential. By choosing different initial configurations, we study the evolution of the system in both quenching [4] and annealing [8,9] scenarios.

In the standard linear σ model, the equations of motion are given by

$$\Box\phi = \lambda(v^2 - \phi^2)\phi + Hn_\sigma\,, \tag{1}$$

where $\phi \equiv (\sigma, \boldsymbol{\pi})$ is a vector in internal space, $n_\sigma = (1, \mathbf{0})$, and Hn_σ is an explicit chiral symmetry breaking term due to finite quark masses. In the following, we shall use $\lambda = 19.97$, $v = 87.4$ MeV, and $H = (119\text{ MeV})^3$, with which $m_\pi = 135$ MeV and $m_\sigma = 600$ MeV at $T = 0$.

We carry out numerical simulations of Eq. (1) including both the longitudinal and transverse expansions. We assume boost invariance in the longitudinal direction so that the longitudinal expansion is automatically included. To consider the transverse expansion, we use a cylindrical boundary condition. The initial ϕ fields are randomly distributed according to a Gaussian form with the following parameters:

$$\begin{aligned}
\langle\sigma\rangle &= [1 - f(r)](f_\pi - \sigma_0) + \sigma_0\,,\\
\langle\pi_i\rangle &= 0\,,\\
\langle\sigma^2\rangle - \langle\sigma\rangle^2 &= \langle\pi_i^2\rangle = \delta_0^2 f(r)\,,\\
\langle\dot\sigma\rangle &= \langle\dot\pi_i\rangle = 0\,,\\
\langle\dot\sigma^2\rangle &= \langle\dot{\pi_i}^2\rangle = 4\delta_0^2 f(r)\,,
\end{aligned} \tag{2}$$

where $r = (x^2 + y^2)^{1/2}$ is the radial coordinate, the dot stands for the derivative with respect to the proper time $\tau = (t^2 - z^2)^{1/2}$, and σ_0 and δ_0^2 are constants, which we can vary for different scenarios. We have introduced an interpolation function,

$$f(r) = \left[\exp\left(\frac{r - R_0}{\Gamma}\right) + 1\right]^{-1}, \tag{3}$$

to describe the boundary condition. R_0 is the radius of the initially excited region where fluctuations exist and the mean fields are different from their vacuum expectation values. Outside this region, the vacuum configuration, $\phi = (f_\pi, \mathbf{0})$, is imposed. Γ is the thickness of the transient region. The results presented in this Letter are obtained with $R_0 = 5$ fm and $\Gamma = 0.5$ fm.

To understand the underlying physics in different scenarios, let us first examine the Hartree approximation [8] of Eq. (1). Separating ϕ into the mean fields $\langle\phi\rangle$ and

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the fluctuations $\delta\phi$ around $\langle\phi\rangle$, i.e., $\phi = \langle\phi\rangle + \delta\phi$, and taking the average of Eq. (1), we have

$$\Box\langle\phi\rangle = \lambda(v^2 - \langle\phi\rangle^2 - 3\langle\delta\phi_\parallel^2\rangle - \langle\delta\phi_\perp^2\rangle)\langle\phi\rangle + Hn_\sigma, \\ \equiv -\chi\langle\phi\rangle + Hn_\sigma, \quad (4)$$

where $\langle\phi\rangle^2 = \langle\phi_i\rangle\langle\phi_i\rangle$, $\delta\phi_\parallel$ is the component of the fluctuation parallel to $\langle\phi\rangle$, and $\delta\phi_\perp$ is the orthogonal component. As we show below, χ is related to the instability of low momentum modes. Equation (4) implies that the motion of the mean fields is determined by an effective potential,

$$V(\langle\phi\rangle) = \frac{\lambda}{4}(\langle\phi\rangle^2 + 3\langle\delta\phi_\parallel^2\rangle + \langle\delta\phi_\perp^2\rangle - v^2)^2 - H\langle\sigma\rangle, \quad (5)$$

which in the presence of the fluctuations clearly differs from the zero temperature one. By varying the level of fluctuations, chiral symmetry can be restored or spontaneously broken. The above effective potential is very generic since no assumption has been made for the fluctuations except that they are only of a classical nature. If the fluctuation terms in Eq. (5) are replaced by their counterparts in a finite temperature field theory, the well-known one loop effective potential at finite temperature [10] is recovered. Therefore, for the mean fields, and as we shall also show for the fluctuation fields, the classical equations of motion have already included the effect of fluctuations present in the effective potential. This might look surprising, but can be easily understood in a thermal equilibrium case. In a finite temperature field theory, the temperature dependence of the effective potential arises from the on-shell part of the propagator. Since no contribution from virtual particles is involved, all thermal corrections at the one loop level are purely classical.

Neglecting the corrections due to quantum fluctuations [9,11], the time evolution of $\langle\phi\rangle$ and $\delta\phi$ can be consistently solved through numerical simulations [4,5] of the classical equations of motion, Eq. (1). Since the fluctuations can evolve with time according to a given relaxation mechanism [6,8,9], the time evolution of the field configuration obtained from Eq. (1), already includes the effect of the time dependence of the effective potential. The use of equations of motion does not ensure that the effective potential takes its zero temperature form or that chiral symmetry is spontaneously broken. What matters most is the initial fluctuation of the system.

When $\delta^2 \equiv (3\langle\delta\phi_\parallel^2\rangle + \langle\delta\phi_\perp^2\rangle)/6$ is large enough, chiral symmetry is restored (approximately, due to $H \neq 0$). If the explicit chiral symmetry breaking term is neglected, the phase transition takes place at the critical fluctuation, $\delta_c^2 \equiv v^2/6$. For $\delta^2 < \delta_c^2$, the effective potential takes its minimum value at $\langle\phi\rangle = (\sigma_e, \mathbf{0})$, where σ_e depends on δ^2. When the mean fields are displaced from this equilibrium point to the central lump of the "sombrero" ($\langle\phi_i\rangle \sim 0$) and χ becomes negative, modes below the critical momentum,

$$k_c = \sqrt{-\chi}, \quad (6)$$

become unstable, and thus DCC domains can form. Since the domain size is directly related to the time scale during which these modes are unstable, it strongly depends on the initial condition, δ^2 and $\langle\phi\rangle$ of the system.

Let us now consider three different scenarios. (i) In a quenching scenario, the initial fluctuation is below the critical value, $\delta^2 \leq \delta_c^2$, and $\langle\phi_i\rangle \sim 0$. As the mean fields roll down from the central lump of the potential, pion modes below k_c will be amplified. In the meantime, as the system cools down and the fluctuation decreases via, e.g., an expansion, the effective potential will also change and the equilibrium point of the potential $(\sigma_e, \mathbf{0})$ moves toward the zero temperature value $(f_\pi, \mathbf{0})$. This will increase the roll-down time and lead to a larger domain size. (ii) On the other hand, if we initially choose $\langle\phi\rangle$ to be very close to the equilibrium point of the effective potential, $(\sigma_e, \mathbf{0})$, the mean fields and the effective potential may both evolve so that the system always oscillates around the equilibrium position. We refer to this scenario as a cold annealing since the system starts with an effective potential in which chiral symmetry is spontaneously broken. (iii) What we shall call a hot annealing scenario is similar to the cold annealing except that the initial fluctuation is much larger than the critical value, $\delta^2 \gg \delta_c^2$, so that chiral symmetry is almost restored. In both annealing cases, the mean fields can evolve almost synchronously with the effective potential so that the system only oscillates around the equilibrium point, i.e., $\langle\phi\rangle^2 \approx v^2 - 3\langle\delta\phi_\parallel^2\rangle - \langle\delta\phi_\perp^2\rangle$. One can then expect that the low momentum modes are less amplified and that the domain size is smaller than in the quenching case.

For the three scenarios we consider here, we take (i) $\delta_0^2 = \delta_c^2 = v^2/6$, with which the system is about to go through a phase transition. The initial field configuration is set to $\sigma_0 = 0$ for a quenching case. (ii) For a cold annealing case, we take $\sigma_0 = \sigma_e = 44$ MeV, in order to have an equilibrium initial configuration with the given fluctuation. (iii) For a hot annealing case, we take $\delta_0^2 = v^2/4$ and $\sigma_0 = \sigma_e = 20$ MeV.

We define a correlation function $C(r,\tau)$ as

$$C(r,\tau) = \frac{\sum_{i,j}\boldsymbol{\pi}(i)\cdot\boldsymbol{\pi}(j)}{\sum_{i,j}|\boldsymbol{\pi}(i)||\boldsymbol{\pi}(j)|}, \quad (7)$$

where the sum is taken over those grid points i and j such that the distance between i and j is r. In Fig. 1(a), we show the time evolution of the correlation function for the quenching case. Throughout our calculations, we take the initial time $\tau_0 = 1$ fm and a lattice spacing $a = 0.25$ fm. In our numerical simulations, we have used the two-step Lax Wendroff method, a version of the leap frog method [12]. We have used an initial correlation length $\ell_{\text{corr}} = 0.5$ fm. Usually the lattice spacing a has been identified to ℓ_{corr} [4,5]. To include the initial correlation

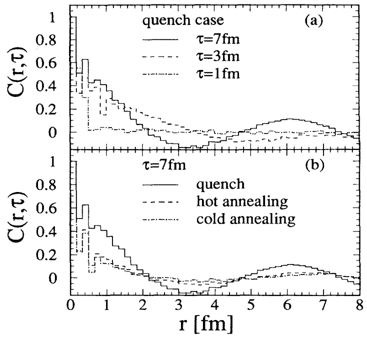

FIG. 1. Correlation functions (a) at $\tau = 1$ fm (initial state), 3 fm, and 7 fm for the quenching initial condition; (b) for the quenching, hot and cold annealing scenario at $\tau = 7$ fm.

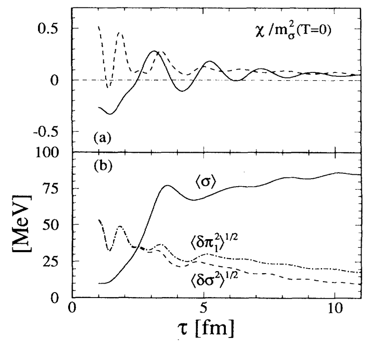

FIG. 2. Evolution of (a) $\chi/m_\sigma^2(T = 0)$, where the solid and dashed lines are quenching and hot annealing cases, respectively, (b) $\langle\sigma\rangle$, $\langle\delta\sigma^2\rangle^{1/2}$, and $\langle\delta\pi_1^2\rangle^{1/2}$ for a hot annealing case. The averages are made over 100 events and within $r \leq 3$ fm in both.

and to reduce the finite size effect, we have adopted the lattice spacing smaller than $\ell_{\rm corr}$ [13]. The initial fields are therefore uniform within $\ell_{\rm corr} \times \ell_{\rm corr}$ squares. Since domain formation is caused by the amplification of low momentum modes, the domain size should not be very sensitive to the value of $\ell_{\rm corr}$. We have actually confirmed this [12]. At $\tau = \tau_0$, there is no correlation beyond $\ell_{\rm corr}$. We can clearly see that a long range correlation emerges at later times. Note that the apparent shrinking of correlation length at $\tau = 7$ fm is a little misleading. Since the resultant pion distributions only depend on π_i^2, $C(r,\tau) < 0$ should be regarded as the manifestation of the correlation as long as $C(r,\tau) \neq 0$. At $\tau = 7$ fm, the typical correlation length is as long as $r \sim 2.5$ fm. In Fig. 1(b), we compare the results of the quenching, hot, and cold annealing scenarios at $\tau = 7$ fm. We observe that quenching gives the largest correlation among the three cases. We have also checked that changing σ_0 to 0 in the hot annealing case does not help much to create larger domains, since the system has moved to the equilibrium position before the chiral phase transition takes place. In other words, a quenching condition can never be realized through a hot annealing. The situation does not improve much even if a second order phase transition is assumed ($H = 0$), because the expansion time scale is too short for any long range correlation to develop. In the quenching case, the expansion of the system reduces the fluctuation, and as a result, the evolving effective potential provides a longer roll-down time for the system to form larger domains. We have, in fact, checked that for larger τ_0 cases, where relative longitudinal expansion is slower, a smaller correlation is generated [12].

In Figs. 2(a) and 2(b), we show the time evolution of $\chi/m_\sigma^2(T = 0)$ and that of the average σ field, $\langle\sigma\rangle$, and the average fluctuations, $\langle\delta\sigma^2\rangle^{1/2}$ and $\langle\delta\pi_1^2\rangle^{1/2}$, respectively. $\langle\delta\pi_2^2\rangle^{1/2}$ and $\langle\delta\pi_3^2\rangle^{1/2}$ are similar to $\langle\delta\pi_1^2\rangle^{1/2}$. We have generated 100 events and averaged over the central region $r \leq 3$ fm. In Fig. 2(a), we have taken a very small initial fluctuation, $\delta_0^2 = v^2/16$, and $\sigma_0 = 0$ to simulate a very strong quenching case for the solid line. For the dashed line, $\delta_0^2 = 3v^2/8$, which is much larger than the critical value, and $\sigma_0 = \sigma_e = 9$ MeV have been taken to simulate a very hot annealing case. We clearly see that in the quenching case χ stays negative longer than the annealing case and that χ becomes negative again even after it once became positive. This also explains the large correlation length in the quenching case as observed in Fig. 1. We note that in a smaller region in a single event the duration when χ is negative can be even longer due to fluctuation. In Fig. 2(b), we have used the latter set of δ_0^2 and σ_0. We see that $\langle\delta\sigma^2\rangle$ and $\langle\delta\pi_1^2\rangle$ decrease on the average with time due to the longitudinal and transverse expansion. On the other hand, $\langle\sigma\rangle$ increases, following the equilibrium position of the evolving effective potential. In principle, $\langle\sigma\rangle$ approaches its vacuum value, f_π, as δ^2 goes to zero. A very interesting and important point is that $\langle\delta\sigma^2\rangle$ and $\langle\delta\pi_1^2\rangle$ decouple from each other at about $\delta = 35$ MeV, which is about the value of the critical fluctuation δ_c. This decoupling is nothing but the manifestation of the mass splitting during the chiral phase transition: the pion mass becomes smaller and the sigma mass becomes larger. Obviously, the fluctuations also experience an evolving effective potential as well as the mean fields.

Finally, to demonstrate domain formation, we show the contour plot of π_2 in Fig. 3 for one event in our quenching case as a function of the transverse coordinates at the initial time $\tau = \tau_0 = 1$ fm and $\tau = 5$ fm. We can clearly see two large domains with opposite signs in π_2 at $\tau = 5$ fm in this event, whereas initially there exists no structure inside the chirally restored region. We also note an apparent transverse expansion and decreasing fluctuations in the inner region. The domain formation is more dramatic for a smaller system [12].

In summary, we have shown that the usual prescription for a quench actually already includes the effect of fluctuations. The relaxation of the fluctuations and their effect on the effective potential are automatically included

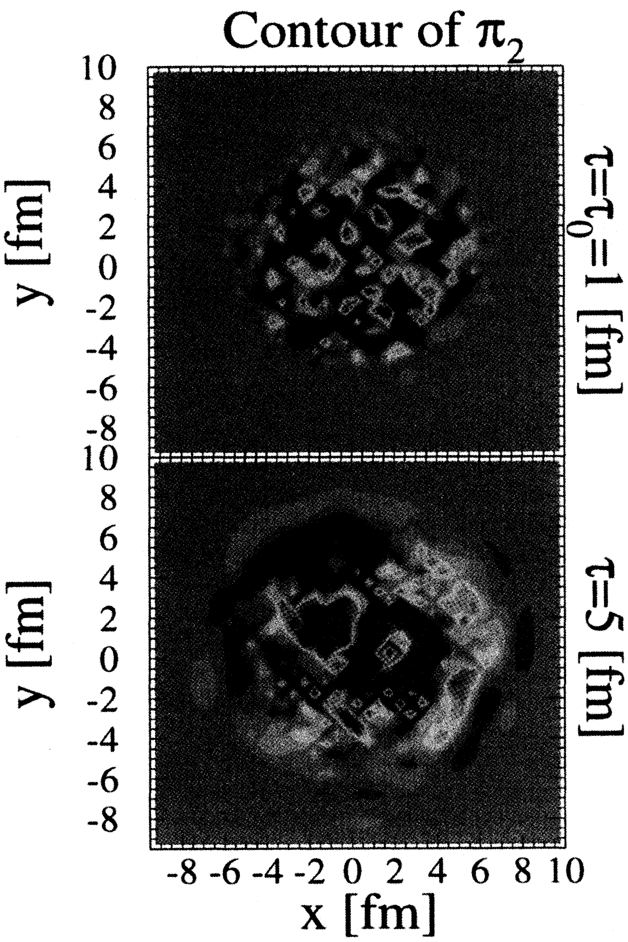

FIG. 3. Contour plot of π_2 field in an event at $\tau = \tau_0 = 1$ fm and $\tau = 5$ fm. A quenching initial condition is used as in Fig. 1. The π_2 field has opposite signs in the two large domains.

in the evolution of the system which undergoes both longitudinal and transverse expansions. Mass splitting of the pion and sigma fields at the classical level during the chiral phase transition has been clearly demonstrated. We have also shown in our numerical calculations that DCC domains of a typical size up to 4–5 fm can form for realistic parameters in the linear σ model if the quenching initial condition is realized. Furthermore, we have demonstrated that such a quenching condition cannot be achieved by relaxing a system from a chirally symmetric phase through expansions. We will discuss elsewhere [12] whether and how a quenching initial state can be realized in hadronic or nuclear collisions.

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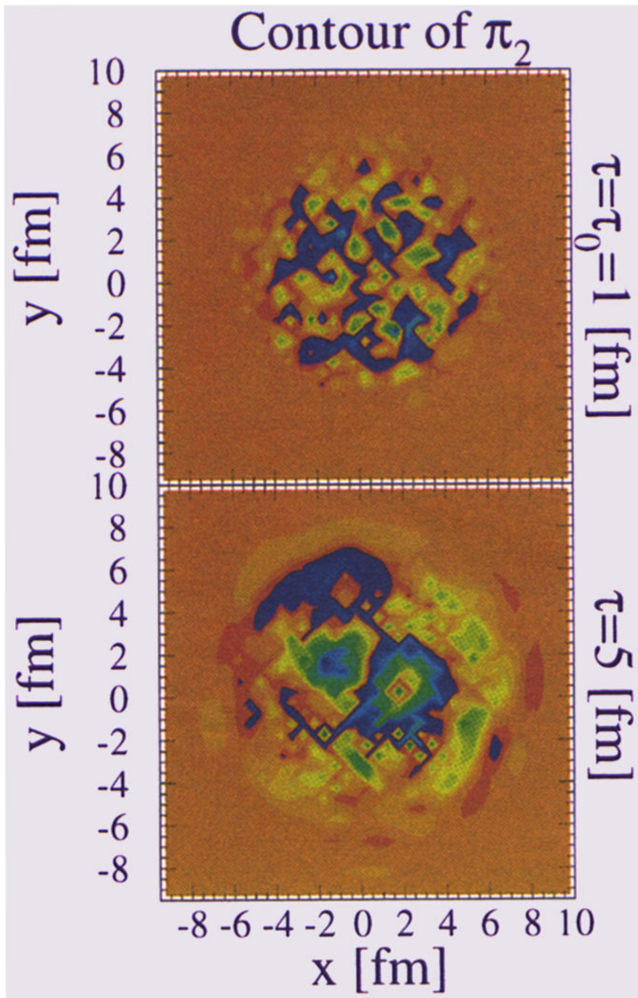
[13] This will lead to an artificial cooling of the fluctuations. However, we have checked, by using a lattice spacing equal to ℓ_{corr}, that our conclusions, especially on the domain formation, remain the same. We thank B. Müller for bringing this to our attention.



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