Stability Analysis of Resistive Wall Kink Modes in Rotating Plasmas

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The stability analysis of external magnetohydrodynamic modes is carried out for a cylindrical plasma in the presence of a resistive wall, plasma flow, and coupling to the sound wave continuous spectrum. It is confirmed that the resonance of the mode with the sound continuum produces an effective dissipation. The combined effects of dissipation and plasma flow open up a window of stability to the external kinks. This theory can explain the numerical results of A. Bondeson and D. J. Ward [Phys. Rev. Lett. **72**, 2709 (1994)].

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The stability of magnetohydrodynamic (MHD) external modes is an important requirement for the high-β operation of future tokamak experiments such as the tokamak physics experiment (TPX) and the international thermonuclear experimental reactor (ITER). Theoretical calculations [1] have shown that values of $\beta_N \sim 4-5$ are achievable only if ideal external kinks are stabilized by a perfectly conducting wall. However, because of the finite resistivity of the wall, it has long been thought that an external mode (the resistive wall mode) would persist, growing on the resistive wall diffusion time [2–9] (as long as the ideal MHD marginal condition with the wall at infinity is violated).

Recently, however, Bondeson and Ward [10] presented numerical calculations indicating that a combination of toroidicity and plasma rotation opens up a window of stability against both the resistive wall and ideal kinks. Here, a window refers to a range of b/a (wall radius/plasma radius) at fixed plasma parameters. In this Letter, we present an analytic theory describing the "stability window," essentially confirming the conjectures of Bondeson and Ward [10], and we show that the existence of the stability window requires both plasma rotation and a dissipation mechanism [11,12] such as resonant interaction with a continuous spectrum (sound or Alfvén).

Toroidicity is important to increase the magnitude of the dissipation, but is not necessary for the existence of the physical mechanism [11]. Thus, for simplicity, the stability analysis is carried out for a straight cylindrical pinch described by the ideal MHD model, where the main dissipative mechanism is the mode resonance with the sound continuum. We show that, for sufficiently large dissipation, the size and characteristics of the stability window are independent of the source and magnitude of the dissipation. This result can be easily extended to toroidal geometry where the dissipation has been shown to be much larger than in a cylinder [11]. Interestingly, the stability window in b/a at fixed plasma parameters transforms directly to a corresponding window in β when b/a is fixed. However, to gain access to the high-β stable window, one must first pass through a window of unstable β's as the plasma is heated. In principle, sufficiently rapid heating is required to quickly pass through the region of unstable β.

We begin our analysis with the external kink dispersion relation in the presence of a resistive wall [2–9],

$$-\frac{\delta L}{\delta W_{v\infty}} = \frac{(\delta W_{vb}/\delta W_{v\infty}) + (i/\omega\tau_w\zeta)}{1 + (i/\omega\tau_w\zeta)}. \tag{1}$$

Here δL is the MHD Lagrangian including potential plus kinetic energy

$$\delta L = \frac{2\pi^2}{\mu_0} R_0 \frac{F_a}{[k_0^2]_a}\left[\hat{F}_a + F_a\left(\frac{r\xi'}{\xi}\right)_a\right]|\xi_a|^2, \tag{2}$$

$\tau_w \approx \tau_d[1-(a/b)^{2m}]/2m$, $\tau_d = \mu_0 db/\eta$ is the resistive wall diffusion time, $\delta W_{v\infty} \approx 2\pi^2 R_0 a^2 F_a^2 |\xi_a|^2/\mu_0|m|$ is the vacuum energy with the wall at infinity, $\delta W_{vb} \approx \delta W_{v\infty}[1+(a/b)^{2m}]/[1-(a/b)^{2m}]$ is the vacuum energy with the wall at b and $\zeta = \tanh\lambda d/\lambda d$. The approximate values correspond to the limit $kb \ll 1$. Here, k is the wave number along the cylinder axis ($k = -n/R_0$), m is the poloidal wave number, a is the minor radius, R_0 is the major radius, b is the wall radius, d is the wall thickness, $F = kB_z + mB_\theta/r = k_{\|m}B$, $\hat{F} = kB_z - mB_\theta/r$, $k_0^2 = k^2 + m^2/r^2$, ξ is the mode radial displacement, and $\lambda = \sqrt{-i\omega\mu_0/\eta}$. Furthermore, Eq. (1) has been derived assuming $\lambda b \gg 1$. Equation (2) shows that the evaluation of δL requires the value of the plasma displacement and its derivative at the plasma-vacuum interface. The evaluation of $(r\xi'/\xi)_a$ is in fact the main goal of the analysis.

To derive the mode eigenfunction in the plasma, consider an ideal plasma, flowing along the z axis with velocity $U = \Omega R_0$ (Ω is the rotation frequency). We treat the important regime where Ω is much smaller than the Alfvén frequency, and we use the standard radial equation for the perturbed magnetic flux $\Psi = r\xi$ given in Ref. [3],

$$\frac{d}{dr}A(r)\frac{d\Psi}{dr} - C(r)\Psi = 0, \tag{3}$$

where

$$A(r) = \rho \frac{V_s^2 + V_a^2}{r} \frac{\overline{\omega}^2 - \omega_a^2}{\overline{\omega}^2 - \omega_f^2} \frac{\overline{\omega}^2 - \omega_h^2}{\overline{\omega}^2 - \omega_s^2}, \tag{4}$$

$$C(r) = -\frac{\rho}{r}(\overline{\omega}^2 - \omega_a^2) - \frac{4k^2 V_a^2 B_\theta^2}{\mu_0 r^3} \times \frac{\omega_g^2 - \overline{\omega}^2}{(\omega_f^2 - \overline{\omega}^2)(\omega_s^2 - \overline{\omega}^2)} + \frac{d}{dr}\left(\frac{B_\theta^2}{\mu_0 r^2}\right) + \frac{d}{dr}\left(\frac{2kB_\theta G}{\mu_0 r^2} \frac{V_a^2 + V_s^2}{\omega_f^2 - \overline{\omega}^2} \frac{\overline{\omega}^2 - \omega_h^2}{\overline{\omega}^2 - \omega_s^2}\right), \tag{5}$$

$\omega_a^2 = F^2/\mu_0\rho$ is the Alfvén frequency, $\omega_{(f,s)}^2 = 0.5k_0^2(V_s^2 + V_a^2)[1 \pm \sqrt{1-\alpha^2}]$ are the fast and slow magnetoacoustic frequencies $[\alpha^2 = 4V_s^2\omega_a^2/k_0^2(V_s^2 + V_a^2)^2 \sim \epsilon^2\beta \ll 1,\ \epsilon = a/R_0]$, $\overline{\omega} = \omega + n\Omega$ is the Doppler-shifted mode frequency, $\omega_g^2 = \omega_a^2 V_s^2/V_a^2$, $\omega_h^2 = \omega_a^2 V_s^2/(V_a^2 + V_s^2)$, and $G = mB_z/r - kB_\theta$. We consider external modes, for which $F(r)$ never vanishes inside the plasma, having a frequency on the order of the plasma rotation frequency ($\omega \sim \Omega$). The sound frequency is assumed to vanish at the plasma edge $[\omega_g(a) = \omega_h(a) = \omega_s(a) = 0]$ and has its peak value at the center of the plasma column satisfying $\omega_s(0) > \overline{\omega}_r$. For a marginally stable mode ($\omega_i = 0$), the function $A(r)$ vanishes only at the radial position r_0, where $\omega_h^2(r_0) = \overline{\omega}_r^2$ and the eigenfunction $\xi(r)$ becomes singular. At such a location, the mode resonates with the sound continuum and thus loses energy by the excitation of sound waves.

In our analysis, we assume that $\alpha^2 \sim \epsilon^2\beta \ll 1$. We expand ω_s^2, A, C, and the eigenfunction Ψ in powers of $\delta = \alpha^2/4 \ll 1$ ($\Psi = \Psi_0 + \delta\Psi_1 \cdots$). To lowest order, the eigenfunction equation reduces to

$$\frac{d}{dr}\hat{A}\frac{d\Psi_0}{dr} - [\hat{C}_1 + \hat{C}_2']\Psi_0 = 0, \tag{6}$$

where $r\hat{A} = \rho(\omega_a^2 - \overline{\omega}^2)H$, $\hat{C}_2(r) = 2kB_\theta GH/\mu_0 r^2$, $H = (V_a^2 + V_s^2)/(\omega_f^2 - \overline{\omega}^2)$, and $\hat{C}_1(r)$ is the regular part of the first three terms on the right-hand side of Eq. (5). Equation (6) is regular for $0 < r < a$ and in the limit of small β is identical to the standard eigenvalue equation for Alfvén waves [3]. However, because of the singularity of Eq. (3) at $r = r_0$, we retain the first order correction to the eigenfunction (Ψ_1), which is only important for $\overline{\omega}^2 \approx \omega_h^2 \simeq \omega_s^2$. Near the point of resonance,

$$\Psi_1(r \approx r_0) = \frac{\overline{\omega}^2}{(\omega_s^2)'_{r_0}} \sigma \overset{\ln}{\log}[(1-\delta)\omega_s^2(r) - \overline{\omega}^2], \tag{7}$$

where

$$\sigma = \left(\Psi_0' - \frac{2}{r}\frac{kB_\theta G}{F^2}\frac{V_a^2 + V_s^2}{V_a^2}\Psi_0\right)_{r_0}. \tag{8}$$

Observe that for a purely real eigenfrequency, Ψ_1 has a logarithmic singularity at r_0. However, if the mode has a finite growth rate, the eigenfunction is continuous throughout the plasma. A general quadratic form can be derived by multiplying Eq. (3) by Ψ^* and integrating over the plasma volume. Assuming $\omega_i > 0$, we find that the eigenfunction and its derivative at the plasma vacuum interface satisfy the relation $[\Psi^* A(r)\Psi']_a = (1 + a\xi_a'/\xi_a)aA(a)|\xi_a|^2$, where

$$\frac{a\xi_a'}{\xi_a} = -1 + \frac{\int_0^a [A|\Psi'|^2 + C|\Psi|^2]\,dr}{aA(a)|\xi_a|^2}. \tag{9}$$

Since A and C are complex functions, $a\xi_a'/\xi_a$ has both a real and an imaginary part. Because of the singularity of Ψ_1 at the point of resonance with the sound continuum, the imaginary part is finite even in the limit of $\omega_i \to 0$. Substituting Eq. (7) into Eq. (9), the lowest imaginary part of $a\xi_a'/\xi_a$ can be written in the following form:

$$\mathrm{Im}\left(\frac{a\xi_a'}{\xi_a}\right) = -\frac{\overline{\omega}_r}{|\overline{\omega}_r|}\frac{\pi}{[\omega_f^2]_{r_0}}\frac{\overline{\omega}_r^4}{|r_0(\omega_s^2)'|_{r_0}}\left(\frac{k_0^2}{F^2}\right)_a\left(\frac{F^2}{k_0^2}\right)_{r_0} \times \frac{B^2(r_0)}{B^2(a)}\left(\frac{V_a^2}{V_a^2 + V_s^2}\right)_{r_0}\frac{|\sigma|^2}{|\xi(a)|^2}. \tag{10}$$

The real part of $a\xi_a'/\xi_a$ can be calculated perturbatively by ordering $\overline{\omega}^2/\omega_a^2 \sim \epsilon^2 \ll 1$, and expanding the eigenfunction in powers of (ϵ^2, δ); $\Psi = \Psi_0^0 + \epsilon^2\Psi_0^1 + \delta\Psi_1^0 + \cdots$. Because of the self-adjointness of the MHD operator, the calculation of $\mathrm{Re}(a\xi_a'/\xi_a)$ to first order in (ϵ^2, δ) requires only the zeroth order eigenfunction $\Psi_0^0 = r\xi_0$ which satisfies the well-known equation $[f(r)\xi_0']' - g(r)\xi_0 = 0$, where $f(r) = \rho r\omega_a^2/k_0^2$, and $g(r)$ is given in Ref. [3]. Combining these results leads to the desired expression for $a\xi_a'/\xi_a$:

$$\frac{a\xi_a'}{\xi_a} = i\,\mathrm{Im}\left(\frac{a\xi_a'}{\xi_a}\right) + \frac{\hat{\omega}_F^2 - \overline{\omega}^2}{\omega_0^2}, \tag{11}$$

where $\hat{\omega}_F^2 = \delta W_F/K_M$, $\delta W_F = 2\pi^2 R_0 \int_0^a dr\,[f|\xi_0'|^2 + g|\xi_0|^2]$, $\omega_0^2 = 2\pi^2 R_0 F_a^2|\xi_a|^2/\mu_0 k_0^2(a) K_M$, and

$$K_M = 2\pi^2 R_0 \int_0^a \left\{\frac{\rho r}{k_0^2}|\xi_0'|^2 + \left[\rho r\frac{k_0^2 r^2 - 1}{k_0^2 r^2} - r^2\left(\frac{\rho}{k_0^2 r^2}\right)'\right]|\xi_0|^2\right\}dr. \tag{12}$$

The imaginary term in Eq. (11) is new and, as will be shown later, is responsible for the existence of the stability window.

Using Eqs. (1), (2), and (11), we can write the dispersion relation as

$$\frac{\omega_b^2}{\omega_\infty^2} + \frac{i}{\omega\tau_w\zeta} + \left(\frac{\omega_F^2 - \overline{\omega}^2}{\omega_\infty^2} - i\frac{\omega_D^2}{\omega_\infty^2}\right)\left(1 + \frac{i}{\omega\tau_w\zeta}\right) = 0, \tag{13}$$

where $\omega_b^2 = \delta W_{vb}/K_M$ and $\omega_\infty^2 = \delta W_{v\infty}/K_M$ are the vacuum frequencies with and without the wall, $\omega_F^2 = \hat{\omega}_F^2 + (2\pi^2 R_0 F\hat{F}/\mu_0 k_0^2 K_M)_a|\xi_a|^2$ is the frequency corresponding to the complete fluid potential energy, and $\omega_D^2 = -\omega_0^2\,\mathrm{Im}(a\xi_a'/\xi_a)$ is the new dissipative term. The interesting regime is $\omega_F^2 < 0$, which is required for an instability to occur. Since the imaginary part of δL has been derived in the limit $\omega_i \to 0$, Eq. (13) is only valid near marginal stability. The dispersion relation in toroidal geometry has a form and properties similar to the ones of Eq. (13). In particular, ω_F^2 is negative for unstable modes, K_M is positive, and ω_D^2 vanishes for $\overline{\omega}_r = -n\Omega$. Furthermore, as shown in Ref. [11], the dissipative term ω_D^2 is larger in toroidal geometry by a factor $1/\epsilon^2$, although it is still small compared to the real terms. Thus, we can solve the dispersion relation for arbitrary ω_D^2 and ω_F^2, without making specific reference to their cylindrical representation.

Depending upon the magnitudes of the dissipation, plasma rotation, and wall diffusion time, we identified two regimes where the dispersion relation can be solved analytically: (A) the small dissipation and large rotation regime $\Omega^2/\omega_D^2 \gg 1$, $1 \ll b/d \ll 1/\hat{\omega}_{D0}^2 \ll (\Omega\tau_w b/d)^{1/2}$ and (B) the thin wall and large dissipation regime $1 \ll 1/\hat{\omega}_{D0}^2 \ll \Omega\tau_w \ll b/d$. Here $\hat{\omega}_{D0}^2 = \omega_D^2(\omega_r = 0)/(\omega_b^2 - \omega_\infty^2)$ is a dimensionless form of the dissipated energy. Regime (A) is appropriate for a cylindrical high-β plasma. The solution of the dispersion relation at marginal stability yields three values of the eigenfrequency. The first is a very low frequency mode with $\omega_{r1}\tau_w = -\hat{\omega}_{D0}^2 \ll 1$. The second and third have oscillation periods larger than the wall diffusion time $\omega_{r2}\tau_w \simeq -\text{sgn}(\Omega)(\tau_d/2\tau_w)(d/b)(1/\hat{\omega}_{D0}^4)$, $\omega_{r3} \simeq -n\Omega$. In regime (A), the value of the low frequency root (ω_{r1}) is so low that a dispersion relation valid for $\lambda b < 1$ is required for that root. However, the stability window (as shown later) occurs between the marginally stable points corresponding to ω_{r2} and ω_{r3}. Thus we omit a lengthy calculation for the less important root ω_{r1}.

Consider the next regime (B) which is typical of a toroidal high-β plasma surrounded by a very thin wall. All three marginally stable eigenfrequencies satisfy the assumptions leading to Eq. (1). After some straightforward manipulations, the imaginary part of Eq. (1) yields the following values for the eigenfrequencies: $\omega_{r1}\tau_w \simeq -\hat{\omega}_{D0}^2$, $\omega_{r2}\tau_w \simeq -1/\hat{\omega}_{D0}^2$, and $\omega_{r3} \simeq -n\Omega$. Since identical considerations apply to regimes (A) and (B), no distinctions between the two need be made in the marginal stability analysis that follows. It is easy to show that $|\omega_{r1}| \ll |\omega_{r2}| \ll |\omega_{r3}|$, $|\omega_{r1}| \ll 1/\tau_w$, $1/\tau_w \ll |\omega_{r2}| \ll n\Omega$, and $\omega_{r3} \simeq -n\Omega$. The first root ω_{r1} represents a mode essentially locked to the wall, the third root ω_{r3} represents a mode locked to the plasma and the second root represents a mode that is tied neither to the wall nor the plasma. The marginal stability conditions corresponding to the three frequencies are easily derived from the real part of Eq. (13). A short calculation yields

$$\omega_r = \omega_{r1} \rightarrow \omega_\infty^2 + \omega_F^2 - n^2\Omega^2 = 0, \qquad (14)$$

$$\omega_r = \omega_{r2} \rightarrow \omega_b^2 + \omega_F^2 - n^2\Omega^2 = 0, \qquad (15)$$

$$\omega_r = \omega_{r3} \rightarrow \omega_b^2 + \omega_F^2 = 0. \qquad (16)$$

Observe that Eq. (16) represents the ideal marginal stability condition with the wall at b and can be explained by noting that a mode rotating at a frequency $\omega_r = -n\Omega$ would not dissipate energy ($\omega_D^2 = 0$). Therefore it is identical to an ideal mode in the absence of rotation and dissipation. Equation (14) shows that a mode locked to the wall ($\omega_r = \omega_{r1} \ll 1/\tau_w$) is stable if its total fluid energy (potential + kinetic) is less than the vacuum energy with the wall at infinity. The physical interpretation of the second marginally stable point [Eq. (15)] is less intuitive than the others. A perturbative analysis of the dispersion relation about the three marginally stable points shows that the external kink is unstable for $\omega_\infty^2 - n^2\Omega^2 < -\omega_F^2 < \omega_b^2 - n^2\Omega^2$ and $-\omega_F^2 > \omega_b^2$. Consequently, a stability window in ω_F^2 (which is proportional to β) exists for

$$\omega_b^2 - n^2\Omega^2 < -\omega_F^2 < \omega_b^2. \qquad (17)$$

For the interesting case of an equilibrium unstable without the wall, this is both a necessary and sufficient condition for stability.

The second marginally stable point can be interpreted as follows. As the potential energy increases ($-\omega_F^2 > \omega_\infty^2 - n^2\Omega^2$), there is enough energy to induce the rotation of the mode ($|\omega_r| > |\omega_{r1}|$). Increasing mode rotation reduces the dissipated energy (in fact $\omega_D^2 = 0$ when the mode rotates with the plasma). However, as the rotation frequency becomes larger than the inverse wall time ($|\omega_{r2}\tau_w| \gg 1$), the resistive wall behaves like a superconducting wall and the mode is stabilized (ac wall stabilization). By keeping the plasma parameters fixed and the total fluid energy above the minimum value required for instability ($-\omega_F^2 + n^2\Omega^2 > \omega_\infty^2$), Eq. (17) also describes a stability window in b/a. By denoting as b_i the wall position corresponding to ideal marginal stability [$\omega_b^2(b = b_i) = -\omega_F^2$], and as b_r the one corresponding to resistive wall marginal stability [$\omega_b^2(b = b_r) = -\omega_F^2 + n^2\Omega^2$], Eq. (17) shows that any wall position satisfying $b_r < b < b_i$ is stable to both the ideal and resistive modes. By assuming that $b_i - b_r \ll b_i$, the following approximate formula relating the plasma rotation frequency and the amplitude of the stability window can be derived for the cylinder, or large-aspect-ratio circular torus:

$$\frac{b_i - b_r}{b_i} \approx \left[\frac{(m-1)^{1/2}}{2m}\,\frac{1-(a/b_i)^{2m}}{(a/b_i)^m}\,\frac{n\Omega}{|k_{\|m}(a)\,V_a|}\right]^2. \qquad (18)$$

In deriving Eq. (18), the simple trial function $\xi = \xi_a(r/a)^{m-1}$ has been used to estimate K_M. Equation (18) is only an estimate of the size of the stability window. Observe that the amplitude of the stability window is sensitive to the magnitude of the parallel wave number at the plasma edge [$k_{\|m}(a)$]. A more accurate evaluation of $b_i - b_r$ requires the numerical solution for $\xi_0(r)$ and subsequent calculation of the kinetic energy K_M. Furthermore, there is a critical minimum rotation frequency (Ω_c) below which the window does not exist. The magnitude of Ω_c depends on the dissipation and satisfies the following equation: $n\Omega_c\tau_w \simeq (\nu + 1)^{\nu+1}/(\nu^\nu \hat{\omega}_{D0}^2)$. The latter has been derived for $\lambda d \ll 1$ and ω_{D0}^2 scaling as $\overline{\omega}_r^\nu$, where $\nu \simeq 3$ for a dissipation induced by sound wave resonance in cylindrical plasmas. Observe that $\Omega_c \rightarrow \infty$ for $\hat{\omega}_{D0} \rightarrow 0$. A simple check of the analytic theory with the numerical simulation can be performed by comparing the size of the stability window illustrated in Fig. 1 of Ref. [10] and the one obtained using Eq. (18). For the parameters $\Omega R_0/V_a = 0.06$, $q_a = 2.55$, $n = 1$, and $b_i \simeq 1.7a$, Ref. [10] shows a stability window of approximately $(b_i - b_r) \simeq 0.17 b_i$. For the same parameters and for $m = 3$ ($m = 3$, $n = 1$ is the obvious cylindrical mode to be compared with the toroidal $n = 1$ external kink when $q_a = 2.55$), Eq. (18) yields a stability window $b_i - b_r \simeq 0.15 b_i$, in reasonable agreement with the numerical results.

Figure 1 shows the effect of the wall time and plasma dissipation on the size of the stability window for an $n = 1$, $m = 3$ mode and a cylindrical plasma in regime (A). The rotation frequency is very large, the wall time is very long and the dissipation is artificially increased. Observe how the window widens as the dissipation and the wall time increases. Because of the small dissipation in a cylinder ($\omega^2_{D(\mathrm{cyl})} = 0.0009\omega_F^2$), a very large rotation velocity and long wall time are needed to open up the stability window.

Figure 2 shows the stability window for an $n = 1$, $m = 3$ mode and a set of realistic equilibrium parameters (similar to the ones of Ref. [10]) and values of the dissipated energy typical of a toroidal plasma. The dissipation in toroidal geometry [11] is larger because the singular part of the eigenfunction scales as β (instead of $\epsilon^2\beta$). This can be shown by deriving the Alfvén wave equation in toroidal geometry and retaining only the singular terms. In toroidal geometry, $f(r) = \rho r^3[\omega^2_{am} - \Omega^2_{s(m-1)} - \Omega^2_{s(m+1)}]$, where $\Omega^2_{s(m)} = (\Omega_E \omega_{s(m)} - \Omega\overline{\omega})^2/(\omega^2_{s(m)} - \overline{\omega}^2)$ and $\Omega_E = (v_s/R) + (R\Omega^2/2v_s)$. Observe that the eigenfunction is singular where f vanishes. Since $\omega_s^2/\omega_a^2 \sim \beta \ll 1$, there are at least two singular points due to the sound wave resonance at $\omega^2_{s(m-1)} \simeq \overline{\omega}^2$ and $\omega^2_{s(m+1)} \simeq \overline{\omega}^2$, and it can be easily deduced that the singular part of the toroidal eigenfunction scales as β. Including the multiple resonances of toroidal sidebands with the sound as well as the Alfvén wave continua, the dissipated energy for a toroidal high-β plasma scales as $\omega^2_{D(\mathrm{torus})} \sim (N/\epsilon^2)\,\omega^2_{D(\mathrm{cyl})}$, where N is the number of resonances. In Fig. 2, the dissipation is varied from $\omega_D^2(\omega_r = 0) \simeq 0.01\omega_F^2$ (dashed curve) to $\omega_D^2(\omega_r = 0) \simeq 0.05\omega_F^2$ (solid curve). Observe how the window size is weakly dependent on the magnitude of the dissipation when the latter is sufficiently large. This result is in agreement with Eq. (18) which is independent of the dissipation.

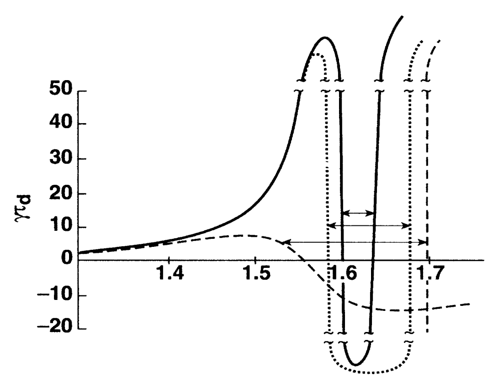

FIG. 1. Plot of the normalized growth rate $\gamma\tau_d$ versus the wall position b/a for an $n = 1$, $m = 3$ mode in cylindrical plasmas with $q_a = 2.55$, $\Omega = 0.08V_a/R_0$, $\Omega = 4k_{\parallel m}(a)\,V_s(0)$, $\epsilon = \frac{1}{3}$, $\omega_F^2 = -1.1\omega_\alpha^2$, $b_i = 1.7a$, and $\tau_d = 10^5/\Omega$ (solid curve). The wall time is increased up to $10^6/\Omega$ (dotted curve) and the dissipation is increased 50 times (dashed curve).

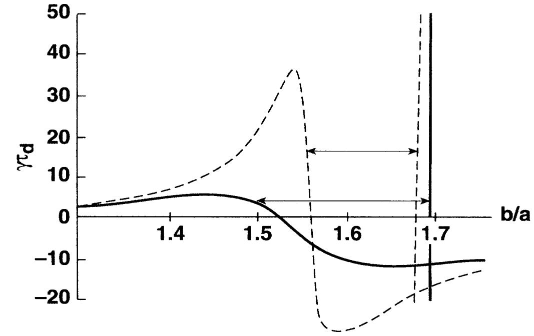

FIG. 2. Plot of the normalized growth rate $\gamma\tau_d$ versus the wall position b/a for an $n = 1$, $m = 3$ mode, $\Omega = 0.06V_a/R_0 = k_{\parallel m}(a)\,V_s(0)$, and $\tau_d = 10^4/\Omega$. The dissipated energy is typical of toroidal plasmas: $\omega_D^2(\omega_r = 0) \simeq 0.01\omega_F^2$ (dashed curve) and $\omega_D^2(\omega_r = 0) \simeq 0.05\omega_F^2$ (solid curve).

We have presented an analytic theory demonstrating the existence of a stability window (in b/a and β) against external kinks surrounded by a resistive wall.

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