Solution of the Two-Impurity, Two-Channel Kondo Model

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We solve the two-impurity, two-channel Kondo model using a combination of conformal invariance and bosonization techniques. In the odd-even symmetric case, the RKKY interaction is exactly marginal, resulting in a line of non-Fermi-liquid fixed points. Explicit formulas are given for the critical exponents and for the finite-size spectrum, which depend continuously on a single parameter. The marginal line spans a range of values of the RKKY coupling I which goes from the infinitely strong ferromagnetic point $I = +\infty$ to a finite antiferromagnetic critical value $I_{\min} < 0$ beyond which a Fermi liquid is recovered. When the odd-even symmetry is broken, the marginal line is unstable for ferromagnetic I, while for antiferromagnetic I it extends into a manifold of fixed points.

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The effect of interimpurity interactions on quantum impurity models possessing a non-Fermi-liquid ground state is of crucial importance for the possible experimental realizations of such systems [1], and for understanding non-Fermi-liquid behavior in lattice models of correlated fermions starting from a local point of view.

The two-impurity Kondo model with two channels of conduction electrons is one of the simplest models where this problem can be addressed. For a single impurity, this model is controlled by a nontrivial fixed point [2], resulting in a specific heat coefficient C/T and susceptibility $\chi_{\rm imp}$ diverging logarithmically as $T \to 0$, and a universal finite-size spectrum of excitation energies differing from the free-fermion form [3]. The corresponding two-impurity model is the simplest situation which brings in the competition between the formation of this nontrivial Kondo state and the ordering of the impurities via the RKKY interaction. It has been recently studied by numerical renormalization-group (NRG) methods [4–6].

In this Letter, we present an analytic solution of the low-energy universal properties of this model using a combination of conformal field theory [3,7] and bosonization methods [8,9]. We find that the RKKY interaction (as well as other interimpurity couplings) is a marginal perturbation, giving rise to a continuous family of non-Fermi-liquid fixed points. The finite-size spectrum and the critical properties vary continuously with the strength of the interaction. We obtain analytic formulas for this dependence. These results are in excellent agreement with recent NRG results [4–6]. They should be contrasted with the single-channel case in which, in the presence of particle-hole symmetry, Kondo screening always dominates over RKKY ordering or vice versa, resulting in two stable Fermi liquid fixed points separated by an unstable nontrivial critical point [10,7].

We formulate the model in terms of left-moving (chiral) fermions $\psi_{li\alpha}(x)$ on the full axis $-\infty < x < +\infty$. $l = 1,2$ is an index labeling the two impurity sites, $i = 1,2$ is a channel index, and α is a spin index. The Hamiltonian is written as

$$\begin{aligned} H = {} & i v_F \sum_{li\alpha} \int_{-\infty}^{+\infty} dx\, \psi_{li\alpha}^\dagger(x) \frac{\partial}{\partial x} \psi_{li\alpha}(x) \\ & + J_+(\vec{S}_1 + \vec{S}_2) \cdot [\vec{J}_1(0) + \vec{J}_2(0)] \\ & + J_m(\vec{S}_1 - \vec{S}_2) \cdot [\vec{J}_1(0) - \vec{J}_2(0)] \\ & + J_-(\vec{S}_1 + \vec{S}_2) \cdot \sum_{i,\alpha\beta} [\psi_{1i\alpha}^\dagger(0) \tfrac{\vec{\sigma}_{\alpha\beta}}{2} \psi_{2i\beta}(0) \\ & + \psi_{2i\alpha}^\dagger(0) \tfrac{\vec{\sigma}_{\alpha\beta}}{2} \psi_{1i\beta}(0)] \\ & - I \vec{S}_1 \cdot \vec{S}_2 \,. \end{aligned} \tag{1}$$

In this formula, $\vec{J}_l(x) = \sum_{i,\alpha\beta} \psi_{li\alpha}^\dagger(x) \frac{\vec{\sigma}_{\alpha\beta}}{2} \psi_{li\beta}(x)$ denotes the spin current at position x for species l. Our notations follow closely those of Ref. [7]. The combinations $\psi_{1,2}$ correspond to $(\psi_e \pm \psi_o)/\sqrt{2}$, where $\psi_{e,o}$ stand for the fermion fields which are even or odd with respect to the midpoint between impurities. A parity transformation exchanges the indices $l = 1,2$ for both impurity spins and conduction electrons. In order to make contact with the couplings $\Gamma_{e,o,m}$ used in Ref. [4], let us note the identifications $J_m \propto \Gamma_m$, $J_+ \propto \Gamma_e + \Gamma_o$, and $J_- \propto \Gamma_e - \Gamma_o$.

We shall start by identifying the global symmetries of the Hamiltonian. For most of this Letter we shall concentrate on the case $J_- = 0$, corresponding to a Hamiltonian invariant under odd-even exchange ($\Gamma_e = \Gamma_o$). H has a higher symmetry in that case, with independent charge and channel (or "flavor") transformations allowed for $l = 1,2$,

$$\psi_{li\alpha} \to e^{i\theta_l} \psi_{li\alpha} \,, \tag{2}$$

$$\psi_{li\alpha} \to \sum_j U_{ij}^{(l)} \psi_{lj\alpha}, \qquad U^{(l)} \in \mathrm{SU}(2) \,. \tag{3}$$

When $J_- \neq 0$, only $\theta_1 = \theta_2$ and $U^{(1)} = U^{(2)}$ are allowed. Let us look first at the case of two *decoupled* impurities

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(each one interacting with two conduction channels) obtained by setting $I = 0$ and $J_+ = J_m$ in addition to $J_- = 0$. In that case, H has independent spin-rotation symmetry,

$$\psi_{li\alpha} \to \sum_\beta V^{(l)}_{\alpha\beta}\psi_{li\beta}, \qquad S^a_l \to \sum_b R^{ab}(V^{(l)})S^b_l\,, \tag{4}$$

where $R^{ab}(V) = 1/2\mathrm{tr}(\sigma^a V \sigma^b V^\dagger)$ $(a,b = x,y,z)$ is the adjoint representation of $V \in \mathrm{SU}(2)$. Hence, two decoupled two-channel Kondo models have global symmetry $[\mathrm{SU}(2)_{\mathrm{spin}} \otimes \mathrm{SU}(2)_{\mathrm{flavor}} \otimes \mathrm{U}(1)_{\mathrm{charge}}]^2$. Coupling the two impurities ($I \neq 0, J_+ \neq J_m$) while keeping $J_- = 0$ leaves unchanged the independent charge and flavor symmetries, but reduces the spin symmetry to the diagonal SU(2) corresponding to $V^{(1)} = V^{(2)}$ in Eq. (4).

At a fixed point, these global symmetries are promoted to local conformal symmetries [11]. For decoupled impurities, the symmetry algebra consists in two copies of a product of Kac-Moody algebra for spin, channel, and charge: $[\widehat{\mathrm{SU}}_2(2)_s \otimes \widehat{\mathrm{SU}}_2(2)_f \otimes \widehat{\mathrm{U}}(1)_c]^2$, where $\widehat{\mathrm{SU}}_k(2)$ stands for the level-k SU(2) Kac-Moody algebra. When coupling the impurities with $J_- = 0$, the diagonal SU(2) symmetry of the spin sector gives rise to a $\widehat{\mathrm{SU}}_4(2)$ algebra. The generators of this algebra are the sum of the generators of the two $\widehat{\mathrm{SU}}_2(2)_s$ for each impurity, that is, the sum of the spin currents $\vec{J}_1(x) + \vec{J}_2(x)$. Hence we must understand how the product $\widehat{\mathrm{SU}}_2(2)_s \otimes \widehat{\mathrm{SU}}_2(2)_s$ can be decomposed into $\widehat{\mathrm{SU}}_4(2)_s$ plus some residual degrees of freedom. The answer is given by the so-called coset construction [12],

$$\widehat{\mathrm{SU}}_2(2)_s \otimes \widehat{\mathrm{SU}}_2(2)_s = \widehat{\mathrm{SU}}_4(2)_s \otimes \mathrm{A}(2,2)\,. \tag{5}$$

The algebra A(2,2) turns out to be an $N = 1$ superconformal unitary model [11,13] corresponding to the $m = 4$ member of the discrete series with central charge $c = \frac{3}{2}[1 - 8/m(m+2)]$, and thus has $c = 1$. This construction generalizes to the two-channel case, the one made by Affleck and Ludwig in their solution of the one-channel two-impurity problem [7]. There, the coset construction is $\widehat{\mathrm{SU}}_1(2)_s \otimes \widehat{\mathrm{SU}}_1(2)_s = \widehat{\mathrm{SU}}_2(2)_s \otimes \mathrm{A}(1,1)$, where the algebra A(1,1) is actually an Ising model with $c = 1/2$.

This coset construction can be understood more explicitly when dealing with spin currents (i.e., for the adjoint representation of the algebra). Let us first recall [11] that the $\widehat{\mathrm{SU}}_2(2)$ spin current $J^a_l(x)$ $(a = x,y,z)$ for a given $l = 1,2$ can be represented in terms of three Majorana (i.e., real) fermions $\chi_l^{x,y,z}$ as follows:

$$J^a_l(x) = i\epsilon_{abc}\chi^b_l\chi^c_l\,. \tag{6}$$

This is particularly transparent when using the Emery-Kivelson bosonization approach to the two-channel Kondo model [8,9], in which case $\chi^x = \sin\Phi_s$, $\chi^y = \cos\Phi_s$, $\chi^z = \cos\Phi_{sf}$ where Φ_s, Φ_{sf} are the boson fields introduced in Ref. [8] corresponding to spin and spin-flavor degrees of freedom. Here, we are dealing with two copies of $\widehat{\mathrm{SU}}_2(2)$ and hence with six Majorana fermions. We combine them into three complex fermions and bosonize these new degrees of freedom as

$$\chi^a_1(x) + i\chi^a_2(x) \propto e^{i\Phi_a(x)}\,, \qquad a = x,y,z\,. \tag{7}$$

Our conventions are such that $e^{ik\Phi}$ has dimension $k^2/2$. In terms of these fields, the total spin current corresponding to the diagonal $\widehat{\mathrm{SU}}_4(2)$ algebra reads: $J^x = J^x_1 + J^x_2 = \cos(\Phi_y - \Phi_z)$ (and cyclic permutations). It is convenient to introduce three linear combinations of boson fields, $\Phi = (\Phi_x + \Phi_y + \Phi_z)/\sqrt{3}$, $\mu = (\Phi_x - \Phi_y)/\sqrt{2}$, and $\nu = (\Phi_x + \Phi_y - 2\Phi_z)/\sqrt{6}$. In terms of these combinations, the components of the total spin current read:

$$J^x = \cos\left(\frac{\mu - \sqrt{3}\nu}{\sqrt{2}}\right), \qquad J^y = \cos\left(\frac{\mu + \sqrt{3}\nu}{\sqrt{2}}\right),$$
$$J^z = \cos(\sqrt{2}\mu)\,. \tag{8}$$

Note that Φ does not enter these expressions. Hence the two bosons μ, ν are sufficient to describe the $\widehat{\mathrm{SU}}_4(2)$ algebra (as expected from its central charge $c = 2$) and Φ corresponds to the residual A(2,2) degree of freedom ($c = 1$). The central charge $c = 3/2 + 3/2$ has thus been distributed as $c = 2 + 1$ in the coset construction Eq. (5). Thus, a very useful explicit realization of the algebra A(2,2) as a free field theory of a single compact chiral boson Φ of radius $R = \sqrt{3}/2$ has been found (Φ has periodicity $2\pi R$). This algebra is actually an "orbifold" theory, meaning that Φ and $-\Phi$ must be identified [11]. This implies [11] that, in addition to the operators $e^{\pm i(n\sqrt{3}+m/2\sqrt{3})\Phi}$ and $\partial^n\Phi$ (with n, m integers), A(2,2) contains two operators of dimension 1/16 which do not have a simple boson representation. The full set of primary operators of the A(2,2) algebra thus reads: (0) [1], $(\frac{1}{24})$ $[e^{\pm i\Phi/2\sqrt{3}}]$, $(\frac{1}{16})$, $(\frac{1}{16})$, $(\frac{1}{6})$ $[e^{\pm i\Phi/\sqrt{3}}]$, $(\frac{3}{8})$ $[e^{\pm i\sqrt{3}\Phi/2}]$, $(\frac{1}{16} + \frac{1}{2})$, $(\frac{9}{16})$, $(\frac{1}{6} + \frac{1}{2})$ $[e^{\pm i2\Phi/\sqrt{3}}]$, (1) $[\partial\Phi]$. The number in parentheses labels the operator by its dimension, and the boson representation is given in brackets, when it exists.

We now consider the effect of turning on the couplings I and $\delta J \equiv J_+ - J_m$ away from the decoupled impurity fixed point (keeping $J_- = 0$, i.e., $\Gamma_e = \Gamma_o$). I is marginal to lowest order (since $\vec{S}_{1,2}$ have dimension 1/2), while δJ has dimension 3/2 and is irrelevant. Furthermore, it can be shown [14] using the approach of Ref. [8] that the operator of lowest dimension corresponding to $\vec{S}_1$ ($\vec{S}_2$) can be written at the decoupled fixed point as $a_1\vec{\chi}_1$ ($a_2\vec{\chi}_2$), where $a_{1,2}$ are *local* real fermions (i.e., of dimension 0) needed to ensure proper commutations. Hence the perturbing term of lower dimension associated with the RKKY interaction reads $\int dt\, a_1 a_2 \vec{\chi}_1 \cdot \vec{\chi}_2$. In the bosonic language above, this translates into an induced boundary term in the A(2, 2) sector of the Hamiltonian,

$$H_{A(2,2)} = \frac{v_F}{4\pi}\int dx\left(\frac{\partial\Phi}{\partial x}\right)^2 + \tilde{I}\left(d^\dagger d - \frac{1}{2}\right)\frac{\partial\Phi}{\partial x}(0), \quad (9)$$

where we have set $d^\dagger \equiv (a_1 + ia_2)/\sqrt{2}$, and $\tilde{I}$ is some (nonuniversal) function of I and $J_+ - J_m$, with $\tilde{I} = I + O\{I^2, (J_+ - J_m)^2\}$ to lowest order. Hence the RKKY coupling is associated with a dimension 1 operator and is an *exactly marginal perturbation.* As shown below, this implies the existence of a *line of fixed points* extending on both sides of the decoupled impurity point.

The Hamiltonian (9) is similar to the x-ray edge Hamiltonian in the bosonized form [15]. Here also the interacting Hamiltonian is related to the noninteracting one by a canonical transformation, $H_{A(2,2)} = U_\delta^\dagger H_{\tilde{I}=0} U_\delta$, where $U_\delta \equiv \exp[i(2\delta/\pi)(d^\dagger d - \frac{1}{2})\Phi(0)]$ corresponding to a phase shift $\delta/\pi = \tilde{I}/2v_F$. Maximal scattering is obtained for $\delta/\pi = \delta_{\max}/\pi \equiv 1/2\sqrt{3}$, as appropriate for a boson of radius $R = \sqrt{3}/2$. In the following we shall use the normalized parameter $x \equiv \delta/\delta_{\max} = 2\sqrt{3}\,\delta/\pi$. $x > 0$ ($x < 0$) corresponds to ferromagnetic (antiferromagnetic) RKKY.

We can investigate the operator content and low-temperature critical behavior of physical quantities for nonzero $I, \delta J$ (corresponding to a specific value of x), using the fact that a given operator $\mathcal{O}$ is changed to $U_\delta \mathcal{O} U_\delta^\dagger$ under the action of U_δ. Spin correlations can be obtained from the identification $S_1^a \pm S_2^a \propto de^{\pm i\Phi^a} + \text{H.c.}$ After the canonical transformation, these operators acquire a dimension $1/3 + (1 \mp x)^2/6$, leading to a singular behavior of the uniform susceptibility $\chi_{\text{imp}} \propto T^{-\theta(x)}$ on the ferromagnetic side $x > 0$ with a continuously varying exponent $\theta(x) = x(2 - x)/3$. Similarly, the staggered susceptibility behaves as $\chi_{\text{st}} \simeq T^{-\theta(-x)}$ on the antiferromagnetic side $x < 0$. χ_{st} (χ_{imp}) is finite for $x > 0$ ($x < 0$). Hence, we find that critical exponents depend continuously on x, establishing the existence of a line of fixed points. In order to find the precise extension of this line, and the low-temperature behavior of the specific heat along it, we look for the leading irrelevant perturbations compatible with all symmetries of the model. At the decoupled impurities point, one has two such operators of dimension 3/2 which read $\mathcal{O}_{+,m} \equiv (\vec{S}_1 \pm \vec{S}_2)\cdot(\vec{J}_\infty \pm \vec{J}_\epsilon)$. Their bosonized form involves $de^{-i\sqrt{3}\Phi} + \text{H.c.}$ for $\mathcal{O}_m$ and $d^\dagger e^{-i\Phi/\sqrt{3}}\sum_a \xi_a J_a + \text{H.c.}$ for $\mathcal{O}_+$, where ξ_a is the adjoint operator of the $\widehat{\text{SU}_4(2)}$ algebra. For a nonzero x, they are changed into $de^{-i(\sqrt{3}+x/\sqrt{3})\Phi} + \text{H.c.}$ and $d^\dagger e^{-i\Phi(1-x)/\sqrt{3}}\sum_a \xi_a J_a + \text{H.c.}$, respectively. Hence the leading irrelevant operator corresponds to $\mathcal{O}_+$ of dimension $\Delta_+ = 4/3 + (1 - x)^2/6$ for $x > 0$, and to $\mathcal{O}_m$ of dimension $\Delta_m = 3(1 + x/3)^2/2$ for $x < 0$. This leads to $C/T \simeq T^{-\theta(x)}$ and to a universal (x-dependent) Wilson ratio for $x > 0$, while a different behavior $C/T \simeq T^{-\alpha(x)}$ with $\alpha(x) = -x(6 + x)/3$ is found on the antiferromagnetic side $x < 0$. Also, the marginal line extends all the way from $x = 0$ up to infinitely strong ferromagnetic coupling $I = +\infty$ corresponding to maximum scattering $x = +1$ (since $\mathcal{O}_+$ never becomes relevant). On the antiferromagnetic side $x < 0$, however, $\mathcal{O}_m$ becomes relevant for $x < x_{\min} \equiv \sqrt{6} - 3 \simeq -0.55$. The marginal line ends at $x = x_{\min}$, and the system flows to the "strong-antiferromagnetic" (Fermi liquid) fixed point where the two impurities bind into a singlet state [4]. Note that, at the infinitely strong ferromagnetic fixed point $x = +1$, the critical behavior derived above coincides with that [3] of the spin-1, four-channel Kondo problem (with spin dimension 1/3 and leading irrelevant operator of dimension 4/3), in agreement with the conjecture made in Ref. [4] and with the physical picture that the two impurities bind into an $S = 1$ triplet state at this point.

We have also investigated the finite-size spectrum of the model at a given fixed point along the marginal line, as a function of x. The method consists in first classifying the states of the decoupled impurity fixed point according to the $\widehat{\text{SU}_4(2)}_s \otimes \text{A}(2,2) \otimes (\widehat{\text{SU}_2(2)}_f \otimes \text{U}(1)_c)^2$ decomposition, and then acting on each state with the transformation U_δ. This modifies the contribution of the A(2, 2) sector to the total energy of the state. The dimension 1/16 of the twist operators can be shown [14] to be unchanged by the action of U_δ. Under multiplication by U_δ, the dimension of an operator $e^{ik\Phi}$ is changed to $\frac{1}{2}(k \pm \delta/\pi)^2$, for $d^\dagger d - 1/2 = \pm 1/2$, respectively. Hence, we also need to associate with each state an eigenvalue of $d^\dagger d - 1/2 = \pm 1/2$ to decide which of the two possible new dimensions is produced. This can be done, when constructing the spectrum at the decoupled point, by keeping track of the relative sign between the *impurity spin* and the *total spin* of the state. More precisely, the impurity spin is proportional to the adjoint operator of $\widehat{\text{SU}_2(2)}$ up to a sign which depends on the state. The product of these two signs for $l = 1, 2$ yields the eigenvalue of $2d^\dagger d - 1$. In particular, this "selection rule" is essential to insure that the spectrum of the $S = 1$ four-channel model is obtained at $x = +1$.

The resulting spectrum for the first few low-lying states is displayed in Table I. Note that the ground state is the triplet of lowest energy for $x > 0$ (ferromagnetic coupling), and the singlet of lowest energy for $x < 0$ (antiferromagnetic coupling). Accordingly, the normalized excitation energy of a given state, $L\Delta E/\pi v_F$ (with L the radial length of the bulk system and v_F the Fermi velocity), is obtained from the total dimension Δ_{tot} given in Table I, by substracting from Δ_{tot} the dimension Δ_{tot}^{gs} associated with the appropriate ground state. The resulting formulas are in excellent agreement with recent numerical renormalization group results of Ingersent and Jones obtained by the numerical renormalization group method [6]. A detailed comparison with these results will be made in a future publication. We believe that the procedure above gives a way of continuously deforming the

TABLE I. Finite-size spectrum of low-lying states. j is the total spin quantum number, j_1 (j_2) is the $SU(2)_{\text{flavor}}$ quantum number for $l = 1$ ($l = 2$), and Q_1 (Q_2) is the charge. σ is the sign of $d^\dagger d - 1/2 = \pm 1/2$. The third column displays the A(2, 2) operator associated with each eigenstate at the decoupled impurities fixed point ($I = 0$, i.e., $x = 0$), whereas the sixth column displays the corresponding operator at the strong ferromagnetic fixed point ($I = +\infty$, i.e., $x = +1$). The degeneracy of each state is displayed in the last column, while Δ_{tot} is the total conformal dimension at arbitrary x. The normalized excitation energies are given by $L\Delta E/\pi v_F = \Delta_{\text{tot}} - \Delta_{\text{tot}}^{gs}$.

j	(j_1, Q_1, j_2, Q_2)	A(2, 2) decoupled	σ	Δ_{tot}	A(2, 2) strong ferro.	Deg.
0	$(0,0,0,0)$	$(\frac{3}{8})$	+	$\frac{3}{8}(1 + \frac{x}{3})^2$	$(\frac{1}{6} + \frac{1}{2})$	1
1	$(0,0,0,0)$	$(\frac{1}{24})$	−	$\frac{1}{3} + \frac{(1-x)^2}{24}$	(0)	3
$\frac{1}{2}$	$(\frac{1}{2}, \pm 1, 0, 0)$ and $1 \leftrightarrow 2$	$(\frac{1}{16})$		$\frac{1}{2}$	$(\frac{1}{16})$	16
0	$(\frac{1}{2}, \pm 1, \frac{1}{2}, \pm 1)$, $(\frac{1}{2}, \pm 1, \frac{1}{2}, \mp 1)$	(0)	+	$\frac{5}{8} + \frac{x^2}{24}$	$(\frac{1}{24})$	16
0	$(0, \pm 2, 0, 0)$, $(1,0,0,0)$, and $1 \leftrightarrow 2$	$(\frac{3}{8})$	−	$\frac{1}{2} + \frac{3}{8}(1 - \frac{x}{3})^2$	$(\frac{1}{6})$	10
1	$(0, \pm 2, 0, 0)$, $(1,0,0,0)$, and $1 \leftrightarrow 2$	$(\frac{1}{24})$	+	$\frac{5}{6} + \frac{(1+x)^2}{24}$	$(\frac{1}{6})$	30

boundary condition of the full conformal field theory involved. These deformations do not correspond, however, to a fusion with an operator inside the spectrum. Even for going directly from $x = 0$ to $x = +1$, we have to follow the orbifold fusion rule with (1/24) *supplemented by a projection* which corresponds to the above "selection rule."

Finally, we mention that the effect of a nonzero coupling J_- (even-odd asymmetry) can be analyzed along the same lines [14]. It gives rise to a *relevant* perturbation of dimension $5/8 + 3(1 - 2x/3)^2/8$ on the ferromagnetic side ($x > 0$), thus destabilizing the marginal line in favor of a spin-1, two-channel Fermi liquid fixed point, as found in [4]. For antiferromagnetic RKKY interactions ($x < 0$), the leading operators generated are marginal, and the line extends into a surface of non-Fermi-liquid fixed points with continuously varying properties depending on *two* phase shifts, in agreement with recent NRG findings [5].

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