Linear Stability of the High Temperature, Dense Z Pinch

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Results are presented on the linear stability of the collisionless $m = 1$ mode in a dense Z pinch. It is shown that a reduction in growth rate by a factor of about 10 (when compared to the zero Larmor radius result) is possible by initializing the Z pinch with a sufficiently low line density. With the completion of this work we conclude that linear, large Larmor radius effects cannot stabilize the high temperature, dense Z pinch. Such pinches will always exhibit linear $m = 0$ or $m = 1$ instabilities with growth times comparable to the radial Alfvén transit time.

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Recent interest in Z pinches has largely been the result of developments in pulsed power technology combined with the use of cryogenic fibers [1–3]. This has resulted in Z pinch experiments which should operate in regimes of parameter space which are far removed from the region in which ideal magnetohydrodynamics (MHD) is applicable. More specifically it can be shown that simply by varying the initial radius of the cryogenic fiber, and consequently the line density, the Z pinch can be made to operate in a variety of regimes [4]. It is well known that Z pinches for which ideal MHD is an appropriate model are exceptionally susceptible to instabilities with growth times comparable to the radial Alfvén transit time (see Ref. [5] for a detailed discussion of ideal MHD and its application to Z pinch equilibria). The particular effect investigated here is whether large ion Larmor radius (LLR) effects can stabilize the dense Z pinch. Improvements in Z pinch stability are needed if it is to be considered as a fusion device [1,2] or if 1D radiative collapse [6,7] is to be achieved.

Defining ϵ as the ratio of the average ion Larmor radius to the pinch radius, it can be shown [8] that for hydrogen $\epsilon \simeq 2.0/r_f$, where r_f is the initial fiber radius in microns. By initializing the Z pinch with a sufficiently small r_f it is therefore possible to produce a pinch in which neither the collisional localization required for ideal MHD [5] nor the small Larmor radius localization required for alternative fluid treatments (for example, [9,10]) is valid. The lack of collisional localization follows from the fact that for fixed total current the ion-ion collision time divided by the characteristic MHD time varies inversely with the line density [4]. It had been hoped that in the collisionless, LLR regime the Z pinch would show the same improvement in stability as had been found in other systems (see references in Ref. [8] for details). The stability of the Z pinch to $m = 0$ modes in the collisionless, LLR Z pinch has already been investigated [8]. In this Letter results are presented for the $m = 1$ mode.

The physics of the $m = 1$ mode in the Z pinch is different from that of the $m = 0$ mode. For $m = 0$ modes it is essential that plasma compressibility is properly included, and for internal modes it is possible to have an equilibrium which is free of $m = 0$ instabilities even in the ideal MHD limit [5]. The model which will be used here is the Vlasov fluid model [11]. This model has been used to study the $m = 0$ mode where it was shown that the class of σ stable [5] profiles is larger than in ideal MHD. However, all of these stable profiles are only stable to $m = 0$ internal modes, i.e., ones in which the outer pinch boundary is fixed, and they all have a finite pressure at this boundary. Such profiles can be stable to free boundary modes, i.e., allowing a deformable boundary, if the plasma is isolated from the vacuum by a neutral gas in such a way that the plasma pressure at the boundary is balanced by the gas pressure, i.e., there are no skin currents. Since Z pinch plasmas are expected to have an anomalous resistivity [7] in the outer region (due to lower-hybrid-drift turbulence), it is possible that this could lead to profiles which are stable to free boundary $m = 0$ modes. In this case the outer turbulent region may act as a substitute for the neutral gas described above. While this is, of course, speculative, since neither ideal MHD nor the Vlasov fluid model could consistently model both regions, it is at least *conceivable* that even when surrounded by a vacuum the Z pinch could possess a set of $m = 0$ stable equilibria.

For $m = 1$ the ideal MHD and Vlasov fluid stability thresholds are identical [11] so that all profiles are unstable to $m = 1$ perturbations, both internal and free boundary. Thus unless LLR effects are capable of stabilizing these modes, at least in the sense of σ stability, we can make general statements about the stability of the dense Z pinch without having to address the possible role

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of the turbulent region in $m = 0$ stability. This point will be clarified later in this Letter.

Previous work on linear Vlasov fluid theory has usually been restricted to internal modes (see references in [8]). It is worth describing in detail how the free boundary modes were included in this work as this is important for a correct interpretation of the results to be presented later. As in previous work [8] two independent techniques have been used. One is an initial value code (FIGARO) which has been used for internal modes and has been found to be in agreement with the second method to within a few percent for all profiles and ϵ tried. This second method, a variational dispersion functional approach, has been extended to include free boundary modes (as well as internal modes) by using the appropriate plasma-vacuum boundary condition. To ensure that the full Vlasov fluid model satisfies the boundary condition it is necessary to guarantee that the expansion functions, used in forming the dispersion functional matrix, all satisfy this condition separately. In this way any linear combination of expansion functions must also satisfy the correct boundary condition. Finally the surface and vacuum contributions are added to the dispersion matrix. This is straightforward as both the surface and vacuum contributions depended only on the radial displacement of the boundary.

The linear, kinetic boundary condition can be derived in the usual way by integrating the exact equations across the plasma-vacuum interface. This differs from the usual fluid approach in that the total perturbed pressure P^* inside the boundary contains a kinetic contribution from $\int m_i v_r^2 f_1 d\mathbf{v}$, where m_i is the ion mass, v_r is the radial component of the velocity, and f_1 is the perturbed ion distribution function. Taking f_1 from Ref. [11] after some simple transformations it can be shown that

$$P^* = \left\{\frac{2B_0^2}{\mu_0 r}\right\}\xi_r - \frac{B_0^2}{\mu_0}\nabla \cdot \boldsymbol{\xi} - \frac{i\omega m_i^2}{k_B T_i}\int v_r^2 \left\{\int_{-\infty}^{t} (i\omega\boldsymbol{\xi} \cdot \mathbf{v} - \mathbf{v}\mathbf{v} : \nabla\boldsymbol{\xi})d\tau\right\} f_0 d\mathbf{v}\,.$$

In this equation B_0 is the equilibrium magnetic field, $\boldsymbol{\xi}$ is the linear fluid displacement from equilibrium, ω is the complex eigenvalue, and the $\int d\tau$ integral is along the unperturbed ion orbits. The dependence of $\boldsymbol{\xi}$ on $(\mathbf{r}, t)$ through $\exp\{i(m\theta + kz - \omega t)\}$ has been omitted for brevity. This expression for P^* has the same role as in MHD theory and is only needed at the plasma-vacuum boundary. In the limit of the pressure at the boundary tending to zero, i.e., no skin current, the term involving f_0 vanishes and the expression for P^* is identical to the ideal MHD expression. For this case the ideal MHD free boundary eigenfunctions can be used as expansion functions thereby guaranteeing that the Vlasov fluid boundary condition is also satisfied.

For comparison with the $m = 0$ case [8] results are presented for both the parabolic and Bennett profiles. As before the Bennett profile results are calculated for internal modes with $\delta = 3$, where δ is the ratio of the pinch radius to the radius at which the magnetic field has its maximum value. Free boundary modes are simulated by taking $\delta = 10$ so that the profile extends into a sufficiently low density region that the vacuum can be ignored. This is less of an approximation than for $m = 0$ as the $m = 1$ free boundary modes of the Bennett profile are actually localized near the axis, with only a relatively small radial displacement of the boundary. Indeed for $\delta \geq 3$ the free boundary and internal modes have the same growth rates and eigenfunctions, to within about 1%, for $m = 1$ in the Bennett profile. For the parabolic pressure profile a small finite pressure is still required at the boundary to avoid a singular radial electric field in the equilibrium. The small skin current associated with this pressure should strictly affect the boundary condition as described above. However, varying this residual boundary pressure (P_a) between $10^{-6}P_0$ and $10^{-3}P_0$, where P_0 is the equilibrium pressure on the axis, did not affect the results, and we conclude that providing the pressure at the boundary is sufficiently small it can be assumed to be zero. A third important equilibrium is also included here for the first time. This is the Kadomtsev $m = 0$ marginally stable profile [5]. The results presented here are for such a profile with $P_a \simeq 0.01 P_0$. Like the Bennett profile this equilibrium cannot be surrounded by a vacuum unless P_a is balanced by a skin current or neutral gas. This would violate the exact Vlasov fluid boundary condition when using MHD expansion functions as outlined above. Unstable $m = 1$ modes for the Kadomtsev profile are, however, localized near the axis. This is true in ideal MHD [5] and we have found that it is also true in the Vlasov fluid theory. Thus, like the Bennett profile, treatment of internal modes also implicitly covers the free boundary analysis.

Figure 1 presents the results for each of these equilibria with the growth rate of the instability plotted against ϵ for $ka = 10$, where k is the wave number of the instability. All growth rates are normalized to the ideal MHD value (with $\Gamma = 5/3$, where Γ is the ratio of specific heats) for that equilibrium. The parabolic profile results are for free boundary modes with $P_a = 10^{-6}P_0$ so that the exact boundary condition above is satisfied. The Bennett profile results are for internal modes with $\delta = 3$ so that comparison with earlier $m = 0$ results is possible. When δ is varied, $\delta = 20$ is the largest which has been tested, there is no change in the nature of this curve, and we concluded that the free boundary and internal mode Bennett profile stability results are the same. The Kadomtsev results are for internal modes, but as discussed above the eigenfunction is localized near the axis and the free boundary growth rates and eigenfunctions are the same as presented here. Thus a wide range of internal

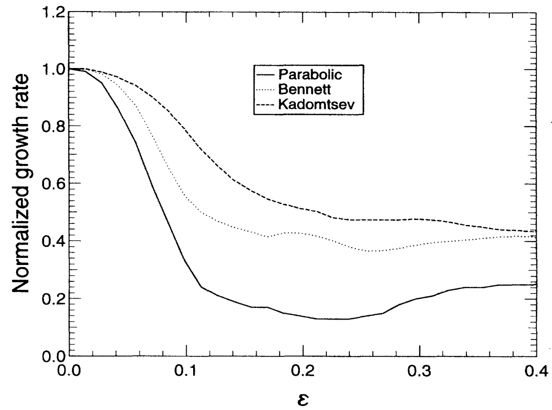

FIG. 1. Growth rate (normalized to the MHD value) against ϵ with $ka = 10$ for the parabolic profile free boundary modes, Bennett equilibrium ($\delta = 3$), and Kadomtsev $m = 0$ marginally stable profile.

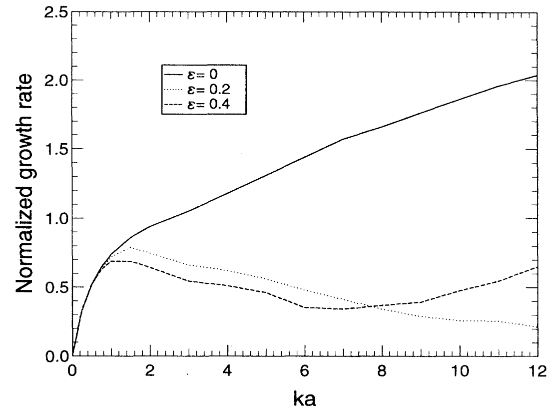

FIG. 2. Growth rate (normalized to the ion thermal transit time) against ka for the free boundary modes of the parabolic profile with $\epsilon = 0.01$, $\epsilon = 0.2$, and $\epsilon = 0.4$.

and free boundary modes have been studied for different equilibria. In all cases results similar to those presented in Fig. 1 were found. Results for $\epsilon > 0.4$ have not been produced since this corresponds to $r_f < 5$ μm and cryogenic hydrogen fibers this thin are unobtainable.

Unlike the $m = 0$ results [8] there is little difference between the equilibria and the destabilizing effect of increasing ϵ for the Bennett profile is absent. The parabolic profile shows the greatest reduction in growth rate, by a factor of about 10 for $ka = 10$, with the minimum growth rate occurring at $\epsilon \simeq 0.2$. This is similar to the $m = 0$ results and we concluded that as far as linear stability theory is concerned there is an optimal line density N for dense Z pinches of $N \simeq 8 \times 10^{18} A$, where the relation between N and ϵ is taken from Ref. [8] with $\epsilon = 0.2$ and A is the atomic mass number. For $m = 0$ it is known that the zero Larmor radius limit must reproduce the Chew, Goldberger, and Low growth rate. For $m = 1$ the stability threshold is the same as ideal MHD but the growth rate need not be the ideal MHD value. Since, in principle, the zero Larmor radius Vlasov fluid model can be written as a closed set of fluid equations, the only difference between the resulting equations and ideal MHD being the nonscalar pressure and consequently more complex equation of state, one would expect the zero Larmor radius growth rate to be close to the ideal MHD value. This is because the $m = 1$ ideal MHD growth rates are insensitive to the choice of Γ.

Figure 2 shows the effect of increasing ϵ for the parabolic profile over a range of values of ka. Growth rates in this figure are normalized to the ion thermal transit time. The best overall reduction in growth rate occurs for $\epsilon \simeq 0.2$. Figure 2 also shows that the fastest growing mode will have a wave number such that $ka \simeq 1$ for the optimal line density. This should be contrasted with the $m = 0$ case where the fastest growing mode of the parabolic profile was at shorter wavelengths, i.e. $ka \simeq 6$. For long wavelength modes, i.e., $ka < 1$, LLR effects have no effect on the $m = 1$ growth rates for this equilibrium. The Bennett and Kadomtsev profile results are similar to those shown here in Fig. 2.

For the Z pinch to achieve fusion, or 1D radiative collapse, it needs to persist for approximately 10^3 radial thermal transit times. The reductions in growth rate found here for $m = 1$, and previously for $m = 0$, are therefore of little practical consequence for fusion. The fact that the Kadomtsev profile is not stabilized by LLR effects does, however, allow us to reach a general conclusion about Z pinches. While the LLR calculations were restricted to $m = 0$ there was always the possibility that the pinch may evolve into a Kadomtsev stable profile with an outer resistive region acting as a neutral gas in the sense outlined above. We can now conclude that even if this were true the pinch would still be unstable to the $m = 1$ mode. The viability of any magnetic confinement configuration relies on the absence of large amplitude instabilities. The present work has shown that in the case of the high temperature Z pinch linear instabilities are never absent. Irrespective of the equilibrium profile or the size of the average Larmor radius the Z pinch will certainly be linearly unstable to the $m = 1$ mode and probably to the $m = 0$ mode also. We can now conclude that the only way in which a high temperature, i.e., collisionless, Z pinch could persist for longer than a few radial Alfvén transit times is if physical processes omitted from the present model dominate the pinch evolution. The possible nonlinear saturation of unstable modes is the most significant of these omissions, nonlinear effects obviously being beyond the scope of the present analysis. The nonlinear saturation of ideal MHD, $m = 1$ modes has been demonstrated analytically for modes close to marginal stability [12]. However, this result cannot be extended to the general stability (i.e., far from the marginal points) of the collisionless, LLR Z pinch and is only indicative of the kind of nonlinear effect which *may* pertain to this regime. In the fully collisionless regime, i.e., with kinetic LLR ions *and* electrons, a full 3D electromagnetic simulation of the Z pinch has been implemented in the SPLASH code [13]. The assumption of an ion to electron mass ratio of 16 prevented quantitative measurements from this code although the onset of the $m = 1$ mode was clearly observed. The nonlinear evolution and possible mode saturation of a LLR Z pinch are thus still unknown.

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