Microwave Billiards with Broken Time Reversal Symmetry

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First results on microwave billiards with broken time reversal symmetry are presented. The billiards are quasi two dimensional with an attached microwave isolator acting as a unidirectional transmission line. Spectral level dynamics was studied by changing the billiard length. For all spectral properties observed, i.e., nearest-neighbor distance, asymptotic curvature, and closest-approach distance at avoided crossings, we have found behavior characteristic of the Gaussian unitary ensemble.

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Classically chaotic systems with time reversal invariance display linear level repulsion in their quantum spectra, just as if the Hamiltonian were a matrix drawn from the so-called Gaussian orthogonal ensemble (GOE) of random matrices [1–7]. If time reversal invariance could be broken one would expect, again under conditions of classical chaos, quadratic level repulsion, as is typical for matrices from the Gaussian unitary ensemble (GUE). However, no spectra with GUE statistics have been seen in any experiment so far.

To enforce quadratic level repulsion for, e.g., a molecular spectrum one would have to apply a magnetic field which is (i) substantially inhomogeneous across the molecule and (ii) sufficiently strong to shift a typical level by at least a typical level spacing [1]. Such requirements are still a bit outrageous even in the Rydberg regime of excitation.

We report here experiments on microwave resonators of billiard shape with broken time reversal invariance. We achieve the desired destruction of the anticanonical symmetry for our "photon billiard" by extending the billiard by a "handle" with one-way propagation characteristics. Unidirectional propagation through the handle is due to the insertion of ferrites which have been used for many years in standard microwave components such as bridges and isolators.

A microwave isolator is a device which in the ideal case has perfect microwave transmission in one direction and none in the reverse one. It works as follows [see Fig. 1(a)]. A stripe of ferrimagnetic material is placed off-centered in a microwave guide. The electron spins precess about a magnetic field perpendicular to the direction of microwave propagation. Because of the off-center position of the ferrite the propagating microwaves produce a rotating $\vec{B}$ field at the site of every spin, clockwise or counterclockwise, depending on the propagation direction. In the resonance case microwaves whose $\vec{B}$ rotates in the same sense as the spins are absorbed, whereas waves with their $\vec{B}$ turning in the opposite sense pass the isolator unattenuated [8].

The experiments were performed in a microwave cavity with height $h = 0.78$ cm and a fixed width of $b = 23.71$ cm [see Fig. 1(b)]. The length a could be varied between 35 and 42 cm. On one short side a waveguide was attached with a built-in microwave isolator (model 62 IGR, Isoguide Co.). Its attenuation was >30 dB in one direction and <1 dB in the reverse one in the frequency range 12 to 19 GHz. The movable part of the wall could be exchanged to form either an asymmetric Sinai billiard (formed by a rectangle and a quarter circle), a Sinai billiard with mirror symmetry (formed by a rectangle and two quarter circles), or a rectangular billiard.

The measuring technique is the same as described earlier [4,9]. Microwaves are fed into the resonator via an antenna, and the microwave reflection spectrum is taken. In the applied frequency range 12 to 19 GHz the resonator is quasi two dimensional, and there is a complete equivalence between the Helmholtz equation and the time-independent

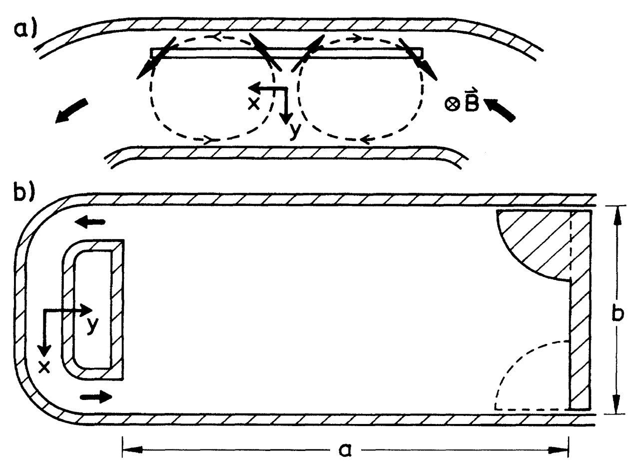

FIG. 1. (a) Scheme of a microwave isolator: a stripe of ferrimagnetic material is placed off-centered within a waveguide. A transverse magnetic field is applied. (b) Sketch of the apparatus: on one short side of a microwave resonator a waveguide with a built-in isolator is attached. The other short end can be moved to study level dynamics.

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Schrödinger equation. Experimentally a density of states was obtained in agreement with Weyl's law up to about 5%. Thus the isolator seems not to reduce the density of states. However, the area of the attached waveguide amounts to only 10% of the total area. A reduction in the density of states of this order of magnitude might just escape detection. We tried to repeat the measurements with a larger ratio of areas of waveguide and billiard. As the area of the waveguide was more or less fixed by the dimensions of the isolator, we reduced the length of the billiard to 20 cm. Unfortunately, the number of eigenvalues in the accessible frequency region was then so much reduced that the results obtained were not usable.

For the experimental determination of the density of states we had to study the change of the spectra with the cavity length. It was demonstrated earlier [9] that it is thus possible to get all eigenvalues, even those missed in one or the other of the individual spectra. The fractional loss of levels in the individual spectra amounted to $\alpha = 0.39$ for the asymmetric Sinai and even $\alpha = 0.51$ and $\alpha = 0.49$ for the symmetric Sinai and the rectangular billiard, respectively. Such deplorably large loss is the price to be paid for the insertion of the microwave isolator.

Because of the loss of levels the measured histogram $P(s)$ of nearest-neighbor spacings must be interpreted with care (to avoid prejudices we refrained from replacing lost levels by invoking level dynamics here). If an eigenvalue is missed with probability α then only the fraction $(1 - \alpha)$ of all experimentally found nearest neighbors really corresponds to nearest neighbors, whereas the fraction $\alpha(1 - \alpha)$ corresponds to next nearest neighbors, etc. Let $p(n, s)$ denote the probability to find an eigenvalue at a distance s of a given other eigenvalue, with exactly n additional eigenvalues in between. Then the experimentally found $P(s)$ can be expressed as

$$P(s) = (1 - \alpha)[p(0,s) + \alpha p(1,s) + \alpha^2 p(2,s) + \cdots]. \tag{1}$$

As there is no simple analytical expression available for the $p(n, s)$, we made an ansatz in the form

$$p(n,s) = \gamma s^{\mu} e^{\beta s^2}. \tag{2}$$

The repulsion exponent μ equals 1 for nearest, 4 for next-nearest, and 8 for third-nearest neighbors in the GOE case. For GUE systems the corresponding numbers are 2, 7, and 14, respectively [10]. The coefficients γ and β are obtained by normalizing as

$$\int p(n,s)\, ds = 1, \qquad \int s p(n,s)\, ds = n + 1. \tag{3}$$

Distributions of the form (2) are known as Wigner's surmises for the densities of nearest-neighbor spacings of which they actually are excellent approximations [1]. Since the next-nearest-neighbor distribution for the GOE is identical to the nearest-neighbor distribution for the Gaussian symplectic ensemble (up to $s \rightarrow 2s$, see Chap. 10 of Ref. [10]), the approximation (1) must be fine for $p_{\mathrm{GOE}}(1,s)$ as well. To check the faithfulness of (2) to the remaining cases, $p_{\mathrm{GOE}}(2,s)$, $p_{\mathrm{GUE}}(1,s)$, and $p_{\mathrm{GUE}}(2,s)$, we evaluated these functions both from (2) and (3) and the corresponding power series given in Ref. [10]. The agreement is slightly worse than for the nearest-neighbor distributions of the GOE, GUE, and GSE but acceptable for our purpose.

Figure 2 shows the nearest-neighbor distribution for the asymmetric Sinai billiard with broken time reversal symmetry. The solid line was calculated from (1) assuming GUE statistics and $\alpha = 0.39$. The inset shows the histogram for small s and clearly exhibits quadratic repulsion. It should be noted that because of the high repulsion exponent for next-nearest neighbors the experimental loss does not influence the histogram at small distances. The loss becomes effective only for $s > 1$ and is responsible for the bumps observed in the histogram at larger values of s.

Additional tests of random-matrix theory are possible by employing level dynamics. It was shown by Gaspard *et al.* [5] that the curvature distribution $P(K)$ is given asymptotically by $K^{-(\nu+2)}$ where ν is the universality index. For ordinary billiards one therefore expects an asymptotic K^{-3} behavior which was found both in calculations for a stadium billiard [11] and experimentally in a microwave billiard [7]. The curvature of an eigenvalue is given by $K_n = \ddot{x}_n / \langle (\dot{x}_n)^2 \rangle$, where $\langle (\dot{x}_n)^2 \rangle$ is a local average of the squared "velocity" and the dot means derivative with respect to the billiard length. Figure 3 shows curvature distributions for the asymmetric Sinai billiard on a double logarithmic scale. The asymptotic slope obtained from a fit to the experimental data is somewhat arbitrary, since the measurement gives no clear indication for the beginning of the asymptotic region. If the first two histogram values in Fig. 3 are omitted from the fit, one gets a slope of −4.22, again in good agreement with the GUE expectation.

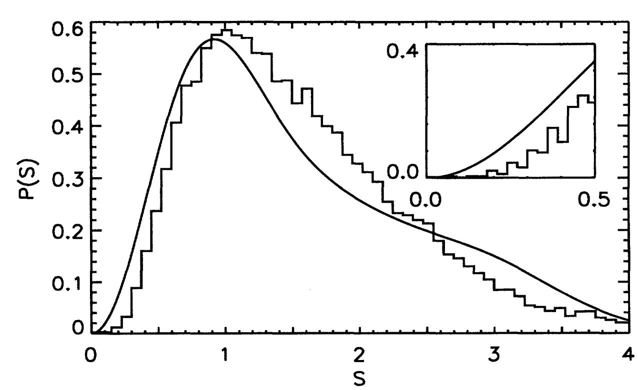

FIG. 2. Distribution of nearest-neighbor distances for the asymmetric Sinai billiard with attached isolator. The solid line displays GUE behavior. The inset shows quadratic level repulsion at small distances.

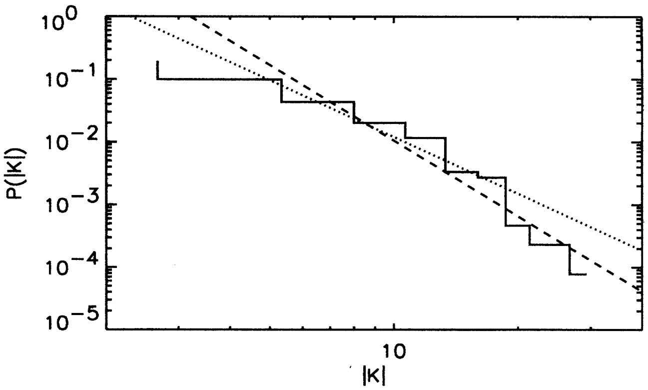

FIG. 3. Curvature distributions for the asymmetric Sinai billiard. The slopes of the straight lines are -3 and -4, respectively.

The last point to be discussed is the distribution $P(c)$ of closest-approach distances c at avoided crossings. Zakrzewski and Kuś [6] derived the distribution $P(c) = (\pi/2)c\exp(-\pi c^2/4)$ for the GUE, where the c are normalized to a mean distance of one. Figure 4 shows our experimental results for the asymmetric Sinai billiard. The full line corresponds to the GUE behavior just mentioned. Again, good agreement between experiment and random-matrix behavior is found, except for the smallest values of c. If the distance between neighboring eigenvalues becomes smaller than the mean linewidth, the resonances cannot be separated, whereupon avoided crossings with very close encounters are underestimated in their number.

We now present our results for the symmetric Sinai and the rectangular billiard. Intuitively, one might expect that the break of time reversal symmetry by the isolator is compensated by the apparent mirror symmetry, so as to give rise to some generalized time reversal invariance. Such expectations are unambiguously refuted by the quadradic level repulsion we have measured for the symmetric Sinai, shown in Fig. 5. Curiously enough the rectangular billiard, which is no more symmetric than the symmetric Sinai, yields data suggesting linear repulsion, as also shown in Fig. 5. It is actually not only for $P(s)$ but also for the curvatures and the closest approaches that the rectangular billiard seems to present itself as a GOE system.

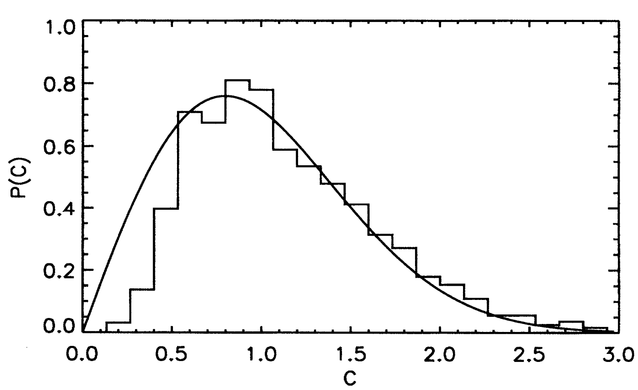

FIG. 4. Distribution of closest-approach distances at avoided crossings for the asymmetric Sinai billiard. The solid line represents the expected GUE distribution.

A few words about theory are now in order. First, we should indicate how well-defined resonances with GUE statistics can arise in spite of strong unidirectional damping. In principle, one could ascertain such resonances by solving the wave equation with ideal-conductor boundary conditions at the walls and boundary conditions at the ports of the isolator expressing unidirectional damping. Arguing more phenomenologically [12] we may think of the eigenvalues of a non-Hermitian "Hamiltonian" of the form $H(\gamma) = H_0 + i\gamma\Gamma$ with Hermitian $N \times N$ matrices H_0 and Γ of large dimension N and a positive dimensionless coupling constant γ. It is important that among the N eigenvalues of Γ some, say N', be negative and the remaining ones vanish. The negative eigenvalues of Γ can be associated with decay channels. The complex eigenvalues $E(\gamma) = \mathrm{Re}E + i\mathrm{Im}E$ of $H(\gamma)$ then have negative real parts which can be interpreted as the widths of the resonances located at the real parts $\mathrm{Re}E$.

Now in the limit of strong damping, $\gamma \gg 1$, H_0 is but a small perturbation of Γ. The N eigenvalues of $H(\gamma)$ thus fall in two clouds in the lower half of the complex E plane. One cloud consists of the perturbed versions of the nonvanishing eigenvalues of $i\gamma\Gamma$ which move downward toward $-i\infty$ with increasing γ. The more interesting

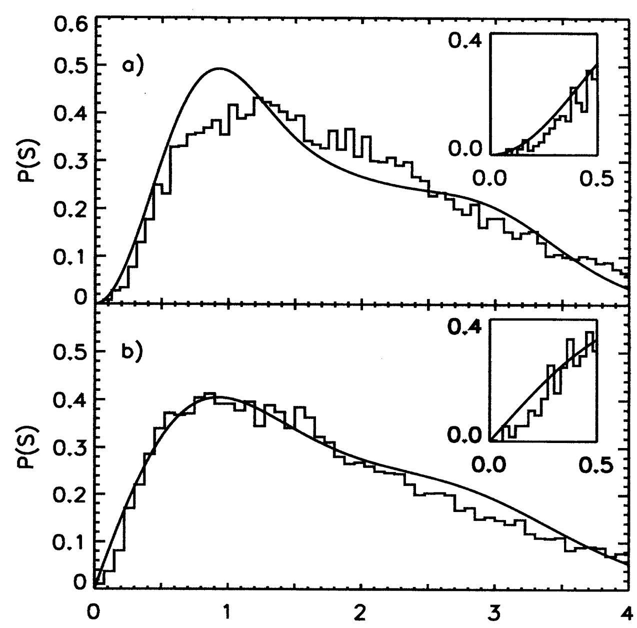

FIG. 5. Nearest-neighbor distance distributions for the symmetric Sinai (a) and the rectangular billiard (b) with attached isolator. The solid lines were calculated assuming GUE and GOE behavior, respectively.

cloud of $N - N'$ points crouches immediately below the real axis such that the imaginary parts ImE are much smaller in modulus than the spacing between the real parts ReE. In fact, these ReE can be obtained by diagonalizing H_0 in the $N - N'$ dimensional subspace spanned by the eigenvectors of Γ with vanishing eigenvalues. Clearly, the statistics of these $N - N'$ resonances ReE must reflect the symmetries of H_0: GOE statistics prevails in the case of time reversal invariance while GUE statistics arises when that invariance is broken [13].

The data presented above suggest that the effective number N' of channels is small, $N' \ll N$; otherwise, the resonances found could not obey Weyl's law as well as is actually the case. Possibly, the small area of the isolator is responsible for $N' \ll N$. On the other hand, the experiments certainly pertain to the limit of large γ since the one-way character of the isolator is well pronounced.

We finally turn to the effect of the isolator on the symmetries of the billiard. We mimic the isolator by a boundary condition in the plane $x = 0$ and look at a single plane wave whose amplitude $E(x, y, t)$ is given by

$$E = \begin{cases} (Ae^{ikx} + Be^{-ikx})e^{-i\omega t} + \text{c.c.}, & x < 0 \\ (Ce^{ikx} + De^{-ikx})e^{-i\omega t} + \text{c.c.}, & x > 0 \end{cases} \tag{4}$$

(the x axis is oriented parallel to the short side of the billiard with the origin in the center of the isolator). Since only the lowest transverse magnetic excitations are of interest the only nonvanishing component of the electric-field vector is the one normal to the plane of the billiard; the amplitudes A, B, C, D are thus complex numbers (which depend on k and y) rather than vectors. The isolator can then be described by a 2×2 transfer matrix M or, equivalently, by a 2×2 scattering matrix S as

$$\begin{pmatrix} C \\ D \end{pmatrix} = M\begin{pmatrix} A \\ B \end{pmatrix}, \qquad \begin{pmatrix} C \\ B \end{pmatrix} = S\begin{pmatrix} A \\ D \end{pmatrix}. \tag{5}$$

Obviously, M and S are uniquely related to one another. For stable systems no eigenvalue of S can exceed unity in modulus. Dissipation leads to eigenvalues of S with moduli smaller than unity. Unidirectionality requires unequal diagonal elements of S. Any symmetry of the billiard would yield further restrictions on M and S.

The operation of conventional time reversal T, $T(E(x,y,t)) = E(x,y,-t)$, acts on the two-component vectors $\binom{A}{B}$ and $\binom{C}{D}$ as $T = \sigma_1 C$ where C is complex conjugation and $\sigma_1 = \binom{01}{10}$ a Pauli matrix. Therefore T invariance is equivalent to the relation $M = \sigma_1 M^* \sigma_1$ where M^* is the complex conjugate of M. However, in a system with broken T invariance, such as the isolator under consideration, this equation will not hold.

It is also interesting to discuss reflections in the plane $x = 0$, $P(E(x,y,t)) = E(-x,y,t)$. P invariance, which actually does not hold in the presence of the isolator even if the shape of the billiard is otherwise symmetric under $x \rightarrow -x$, would yield the requirement $M = \sigma_1 M^{-1} \sigma_1$. Were invariance under the generalized time reversal TP present we would thus have $MM^* = 1$.

In fact, unidirectional dissipative isolators break all three symmetries mentioned, T, P, as well as TP. In particular, it is an easy game with 2×2 matrices to show that TP invariance, $MM^* = 1$, implies unitarity of the S matrix which precludes any dissipation.

In view of these arguments one would expect GUE behavior for both the symmetric Sinai and the rectangular billiard. For the symmetric Sinai billiard this is indeed found in the experiment. For the rectangular billiard, however, our data seem to suggest GOE statistics. But the above theoretical reasoning irrefutably shows that TP invariance and unidirectionality are incompatible. Since unidirectional damping is manifest we must rule out TP invariance. The way out of the conflict might be this: If the perturbation by the isolator can be considered as small, pseudointegrable behavior is expected [14]. Nearest-neighbor distributions in the transition regime from Poisson to GUE behavior [15] may then be hardly discernible from the corresponding GOE quantities. In any case, more theoretical efforts seem indicated toward a quantitative treatment of billiards with one-way components.

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