Interaction of Vortex Lattice with Ultrasound and the Acoustic Faraday Effect

D. Domínguez,[1] L. Bulaevskii,[1] B. Ivlev,[1,2] M. Maley,[1] and A. R. Bishop[1]
[1]*Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[2]*Universidad Autónoma de San Luis Potosí, Instituto de Física, Álvaro Obregón 64, 78000 San Luis Potosí, San Luis Potosí, Mexico*
(Received 3 November 1994)

The interaction of sound with the vortex lattice is considered for high-T_c superconductors, taking into account pinning and electrodynamic forces between vortices and crystal displacements. At low temperatures the Magnus force results in the acoustic Faraday effect; the velocity of sound propagating along the magnetic field depends on the polarization. This effect is linear in the Magnus force and magnetic field in crystals with equivalent *a* and *b* axes for a field parallel to the *c* axis. In the thermally activated flux flow regime, the Faraday effect is caused by electric and magnetic fields induced by vortices and acting on ions.

PACS numbers: 74.60.Ge

The dynamics of vortices in the mixed state of high-T_c superconductors at low temperatures remains an intriguing question because of the absence of a microscopic description and incomplete experimental data. In the phenomenological approach, for fields $H_{c1} \ll B \ll H_{c2}$, the forces per unit volume acting on vortices are given by three terms: (a) an inertial term, $M_v\ddot{\mathbf{v}}$, where M_v is the vortex mass and $\mathbf{v}$ is the vortex displacement, (b) the Magnus force $\alpha_M[\mathbf{n} \times \dot{\mathbf{v}}]$, where $\mathbf{n}$ is the unit vector of vortex orientation, and (c) a viscous term $\eta_v \dot{\mathbf{v}}$ with viscosity coefficient η_v.

The vortex inertial mass estimated by Suhl [1] from the core kinetic energy is very small (in cuprates $M_v \approx 10^6 m_e$/vortex unit length) and is usually neglected. The theoretical prediction for the Magnus force coefficient made by Nozières and Vinen [2] by extension of ideal fluid results to superconductors is $\alpha_M = \pi\hbar n_s(B/\Phi_0)$, where n_s is the density of superconducting electrons. Kopnin and Kravtsov [3] have shown that in *s*-wave BCS superconductors the contribution of quasiparticles inside the vortex core changes this result at nonzero temperatures, but they concluded that the Nozières and Vinen result remains valid in the limit $T \rightarrow 0$. The estimate for η_v obtained by Bardeen and Stephen [4] and associated with dissipation caused by quasiparticles inside the vortex core is $\eta_v = BH_{c2}\sigma_n/c^2$, where σ_n is the conductivity in the normal state. For high-T_c superconductors these predictions may be invalid because of the very small size of the vortex core (about the superconducting correlation length ξ, and $\xi_{ab} \approx 20$ Å in YBCO and BISCO systems) and the possibly gapless spectrum of quasiparticles.

Experimental information on η_v and α_M was obtained by measurements of dc or ac resistivity ρ_{xx} and Hall angle θ_H ($\tan\theta_H = \rho_{xy}/\rho_{xx}$) in the flux flow regime attainable at high fields and currents; see [5]. The measurements of both resistivity and Hall angle are necessary to extract α_M and η_v because ρ_{xx} and ρ_{xy} depend on both parameters. Harris *et al.* [5] performed such measurements in 60 K $YBa_2Cu_3O_{6+y}$ at temperatures $T \gtrsim 13$ K; according to these data $\eta_v\Phi_0/B$ decreases from 10^{-6} g/cm s at 40 K to 60% of this value at 13 K, while $\alpha_M\Phi_0/B$ increases five times (from 10^{-7} g/cm s) in the same temperature interval. Information on the behavior of α_M and η_v below 13 K in high-T_c superconductors was absent until now because the flux flow regime can be obtained at those temperatures only at very high fields and currents.

In this Letter we propose an alternative method to obtain information on η_v and α_M by extending a model due to Pankert [6] for interaction of ultrasound with vortices. He showed that the propagation of sound in the mixed state is affected by the vortex lattices because of their coupling via pinning centers. As a result, the elasticity and dissipation of sound are enhanced by those of the vortex lattice. Pankert calculated the change of sound velocity and dissipation in the thermally activated flux flow (TAFF) regime, valid above the irreversibility line. Experimental data [6] confirmed the main theoretical predictions of Pankert in that temperature range. Here we add in this model the effect of electrodynamic forces acting between vortices and ions, the low-temperature pinning forces, and the Magnus force (α_M) and viscous force (η_v) for vortices. By accounting for these forces we show the following below the irreversibility line.

(a) Because of the Magnus force the eigenmodes of sound waves are polarized for sound propagation parallel to the magnetic field. The difference in velocities of waves with different polarizations depends on α_M (acoustic Faraday effect). Thus the measurement of the rotation in the polarization angle provides results for α_M.

(b) Dissipation of sound is enhanced by a vortex lattice contribution proportional to η_v, while electrodynamic and Magnus forces do not affect dissipation significantly. Thus the viscosity coefficient may be extracted from measurements of sound dissipation with and without a magnetic field.

0031-9007/95/74(13)/2579(4)$06.00

(c) Elasticity of the vortex lattice contributes to crystal elasticity, and thus information on vortex lattice elastic moduli may be obtained from changes in the sound velocity. Particularly, information on the tilt modulus C_{44} is interesting because of its strong dependence on the anisotropy parameter γ in the case of large γ.

Above the irreversibility line, in addition to the effect of vortices on sound dissipation discussed by Pankert, we find also an acoustic Faraday effect, but now due to electrodynamic forces acting between vortices and ions.

We use the London model to describe vortices, and our approach is valid only well below T_c. The equation of motion for vortex displacements $\mathbf{v}(\mathbf{k})$ in $\mathbf{k}$ space is

$$i\omega\left[-\eta_v v_i + \epsilon_{ijz}(\alpha_M v_j) + \alpha_I \frac{\lambda_{ab}^2 k^2}{1+\lambda_{ab}^2 k^2} u_j\right] = [(C_{11} - C_{66})k_i k_j + C_{66} k_m k_m \delta_{ij}]v_j + C_{44} k_z^2 v_i + \alpha_p (v_i - u_i), \quad (1)$$

where $i = x, y$ are coordinates in the ab plane and we take $\mathbf{B}$ along the c (z) axis, ϵ_{ijk} is the unit antisymmetric tensor, $\mathbf{u}(\mathbf{k})$ is the Fourier component of crystal displacements in the sound wave, ω is the frequency of sound, and C_{11} and C_{66} are the compression and shear moduli of the vortex lattice. The term with coefficient $\alpha_I = \pi\hbar n_I B/\Phi_0$ takes into account the Lorentz force acting on vortices due to the current induced by moving ions and screened by superconducting electrons, n_I is the ion concentration, and λ_{ab} is the penetration length for currents along layers [7]. This term is important only for frequencies $\omega/2\pi \geq 1$ GHz, and it will be omitted in the following.

The term $\alpha_p(\mathbf{v} - \mathbf{u})$ was introduced by Pankert [6] to describe the interaction of sound waves with vortices because of pinning in the TAFF regime. In the absence of sound waves ($\mathbf{u} = \mathbf{0}$) Eq. (1) with

$$\alpha_p = \alpha_L (1 - i/\omega\tau_T)^{-1} \quad (2)$$

was used by many authors; see [8–10]. Here α_L is the Labusch constant and τ_T is the relaxation rate which takes into account thermally activated hopping of vortices between different pinning centers. Using heuristic arguments, Brandt [8] obtained

$$\tau_T = (\eta_v/\alpha_L)\exp[U(B)/T], \quad (3)$$

where $U(B)$ is the characteristic pinning potential barrier. A similar expression was obtained by Coffey and Clem for a periodic pinning potential [9]. The generalization made by Pankert is transparent; replacement of $\mathbf{v}$ by ($\mathbf{v} - \mathbf{u}$) accounts for the absence of pinning when vortices and ions move with the same velocity. We will use the same term, $\alpha_p(\mathbf{v} - \mathbf{u})$, with $\alpha_p = \alpha_L$ below the irreversibility line as well, thus neglecting jumps of vortices between pinning centers in the vortex glass phase. The Labusch parameter may be expressed in terms of the critical current J_c as $\alpha_L = J_c B/c r_p$, where r_p is the pinning interaction range ($\approx \xi_{ab}$ in high-T_c superconductors) [10]. For Bi-2:2:1:2 the critical current is in the range $5 \times 10^5 - 5 \times 10^6$ A/cm^2 at helium temperature in fields several T [11], and we estimate $\alpha_L \Phi_0/B$ in the interval $5 \times 10^4 - 5 \times 10^5$ g/cm s^2. From the resistivity data [6] in this system in the TAFF regime $U \approx 500$ K.

For small k, which are characteristic for sound waves, the elastic moduli of the vortex lattice are given as $C_{66} = B\Phi_0/(8\pi\lambda_{ab})^2$ and $C_{11} = C_{44} = B^2/4\pi$ in superconductors with small and moderate anisotropy at high magnetic fields [12]. However, in Bi-2:2:1:2 the anisotropy ratio $\gamma = \lambda_c/\lambda_{ab}$ was estimated as large as 300–1000 [13]. Here λ_c is the penetration length for currents along the c axis. For sound frequency 10 MHz and velocity $c_t = 2 \times 10^5$ cm/s we obtain $k\lambda_c \approx 1.7$ at $\gamma = 300$ and $\lambda_{ab} = 1700$ Å. In this case the dispersion of the tilt modulus (at high fields)

$$C_{44}(k) = \frac{B^2}{4\pi[1 + \lambda_c^2(k_x^2 + k_y^2) + \lambda_{ab}^2 k_z^2]} \quad (4)$$

becomes important. We will show then that measurements of sound velocity in the mixed state of highly anisotropic systems for different frequencies can provide information on γ.

The equation for crystal displacements is

$$(\rho\omega^2 + i\omega D)u_i + i\omega\alpha_I\epsilon_{ijz}(u_j - v_j) = \lambda_{im,lj} k_m k_l u_j + \alpha_p(u_i - v_i), \quad (5)$$

where $\lambda_{im,lj}$ are the elastic moduli of the crystal [$\lambda_{im,lj} = (\lambda + \mu)\delta_{lj}\delta_{mi} + \mu\delta_{lm}\delta_{ij}$ for an isotropic solid] and ρ is the crystal mass density. (The u_z component is decoupled.) The term with the coefficient $D = \eta_0 + \eta_q$ accounts for sound dissipation in the absence of vortices (η_0) and sound dissipation caused by quasiparticles inside the vortex core (η_q). The term with the coefficient α_I accounts for forces acting on ions and induced by vortices: the force determined by the electric field $\mathbf{E} = \mathbf{B} \times \dot{\mathbf{v}}/c$ and the Lorentz force $\dot{\mathbf{u}} \times \mathbf{B}/c$. We ignore here the effect of the dragging of electrons by ions, which results in the renormalization of α_I [7,14].

The Magnus force term proportional to α_M in Eq. (1) for vortices and the term proportional to α_I in Eq. (5) for sound are responsible for the mixing of the x and y components of the vortex and ion displacements, $\mathbf{v}$ and $\mathbf{u}$. Because of this mixing, the eigenwaves of the system become circularly or elliptically polarized, with different velocities for different polarizations. As a result there is an acoustic Faraday effect, a rotation of the polarization plane of sound waves caused by vortices.

Let us discuss first the case when the acoustic Faraday effect is maximum: sound propagation along the direction of the magnetic field, $\mathbf{k} \parallel \mathbf{B} \parallel \mathbf{c}$ in crystals where a and b axes are equivalent (Bi- and Tl-based superconductors, $YBa_2Cu_4O_8$, and $La_{2-x}Sr_xCuO_4$). We take $k_x = k_y = 0$, $k_z = k$. We solve for $\mathbf{v}$ in (1) and obtain the effective equations for transverse sound waves (in first order in α_M, α_I),

$$[U - \alpha_p(1-g)]u_x + i\omega[\alpha_l(1-g) + \alpha_M g^2]u_y = 0,$$
$$-i\omega[\alpha_l(1-g) + \alpha_M g^2]u_x + [U - \alpha_p(1-g)]u_y = 0, \tag{6}$$

where $U = \rho\omega^2 - \lambda_{44}k^2 - i\omega D$, $V = C_{44}k^2 + \alpha_p + i\omega\eta_v$, with $g = \alpha_p/V$ and $\lambda_{44} = \lambda_{xz,xz}$. The conditions $U = 0$ and $V = 0$ give the unperturbed dispersion relations for sound and vortices, respectively. In the absence of a vortex lattice the transverse sound velocity is $c_t = (\lambda_{44}/\rho)^{1/2}$. From Eqs. (6), we can see that $u_x/u_y = \pm i$; i.e., the sound is circularly polarized. The modified dispersion relation can be obtained from solving $U - \alpha_p(1-g) = \pm\omega[\alpha_l(1-g) + \alpha_M g^2]$. The solution can be written in the general form

$$\rho\omega^2 - \rho\tilde{c}_t^2k^2 - i\omega\tilde{D} = \pm\omega(F + i\omega G), \tag{7}$$

where $\tilde{c}_t$ is the modified transverse sound velocity, $\tilde{D}$ is the modified dissipation coefficient, and F and G account for the circular polarization effect in sound velocity and dissipation (the latter effect is negligible, $\omega G \ll \tilde{D}$). The split in sound velocity for different circular polarizations is $\tilde{c}_{t,\pm} = \tilde{c}_t(1 \pm F/2\rho\omega)$. If ultrasound with a given polarization is introduced at one boundary of the sample, after traveling a length ℓ, the polarization plane will rotate. This is the acoustic Faraday effect. The rotation angle per unit length will be given by $\theta/\ell = \omega/2(1/\tilde{c}_{t,-} - 1/\tilde{c}_{t,+}) \approx F/2\rho\tilde{c}_t$.

With the definitions $Q = (C_{44}k^2 + \alpha_T)^2 + \omega^2(\eta_v + \eta_T)^2$, $\alpha_p = \alpha_T + i\omega\eta_T$, $\alpha_T = \alpha_L\omega^2\tau_T^2/(1+\omega^2\tau_T^2)$, and $\eta_T = \alpha_L\tau_T/(1+\omega^2\tau_T^2)$, the general solution for the modified transverse sound velocity is

$$\tilde{c}_t^2 = c_t^2 + \frac{C_{44}}{\rho}\frac{\omega^2\eta_T^2 + \alpha_T(C_{44}k^2 + \alpha_T)}{Q} + \frac{\alpha_T\omega^2\eta_v^2}{\rho k^2 Q}, \tag{8}$$

for the dissipation,

$$\tilde{D} = D + \frac{\eta_T C_{44}^2k^4 + \eta_v[\alpha_T^2 + \omega^2\eta_T(\eta_v + \eta_T)]}{Q}, \tag{9}$$

and for the circular polarization coefficient,

$$F = \alpha_M\Big\{[\alpha_T(C_{44}k^2 + \alpha_T) + \omega^2\eta_T(\eta_v + \eta_T)]^2 - \omega^2[\eta_T C_{44}k^2 - \alpha_T\eta_v]^2\Big\}Q^{-2} + \alpha_l[C_{44}k^2(C_{44}k^2 + \alpha_T) + \omega^2\eta_v(\eta_v + \eta_T)]Q^{-1}. \tag{10}$$

Let us consider the low-temperature regime ($\omega\tau_T \gg 1$, meaning $T < 40$ K in $Bi_{1.6}Pb_{0.4}Sr_2Cu_3O_y$ studied in [6]). In this case $\alpha_T \approx \alpha_L$ is the largest parameter, $\alpha_L \gg \omega\alpha_M, \omega\eta_v, C_{11}k^2, C_{66}k^2, C_{44}k^2$, and η_T is negligible. In this limit vortices are completely involved in sound oscillations. The modified sound velocity is simply $\tilde{c}_t^2 = c_t^2 + C_{44}/\rho$, and the modified dissipation is $\tilde{D} = D + \eta_v$. Therefore a measurement of the change in sound velocity and dissipation will give direct information on C_{44} and η_v at low temperatures. The Faraday effect is dominated by the Magnus force, $F \approx \alpha_M$. The rotation angle, $\theta/\ell = \alpha_M/2\rho c_t$, is about 7 deg/cm for $\alpha_M\Phi_0/B = 10^{-6}$ g/cm s, $\rho \approx 5$–7 g/cm^3, and $B = 5$ T. Therefore a measurement of θ can provide direct information on the Magnus force constant α_M below 40 K. The advantage here is that for sound propagation the effects of vortex dissipation η_v and Magnus force α_M are decoupled, in contrast to their effect on resistivity and Hall angle. For standing sound waves the Magnus force lifts the degeneracy of the resonance frequency with respect to polarization, with a splitting of $\Delta\omega = \alpha_M/\rho$. This value is $\approx$0.01 MHz in a field of 5 T. It can be observed for small dissipation, $(D + \eta_v)/\rho \ll \Delta\omega$.

Note that θ can also be observed by measurements of the amplitudes B_{x0} and B_{y0} of the ac magnetic field induced by oscillating vortices dragged by sound waves. If sound polarized along the x axis and with amplitude u_0 is introduced at one side of the sample with thickness ℓ, on the other side of the sample the field amplitudes will be $B_{x0} = Bku_0\cos\theta$ and $B_{y0} = Bku_0\sin\theta$. For sound with a typical $u_0 \approx 10$ nm at $\ell = 0.1$ cm and $B = 5$ T we estimate $B_{x0} \approx 15$ G and $B_{y0} \approx 1.15$ G.

Now we consider the high-temperature limit $\omega\tau_T \ll 1$, where $\alpha_T \ll C_{44}k^2$ and $\eta_T \approx \eta_v\exp(U/T)$. In this limit $\tilde{c}_t$ and $\tilde{D}$ in Eqs. (8) and (9) reduce to the values calculated by Pankert [6]. There is a peak for sound dissipation when $C_{44}k^2 \approx \omega\eta_T$ (in Ref. [6] it occurs at about 60 K for $\omega/2\pi = 3$ MHz and $B = 5$ T). Below the peak, $C_{44}k^2 \ll \omega\eta_T$, the circular polarization coefficient is given by $F \approx \alpha_M$. Above the peak, $C_{44}k^2 \gg \omega\eta_T$, we have $F \approx \alpha_l$. Therefore at the dissipation peak there is a crossover in the Faraday effect from a regime dominated by the Magnus force to a regime dominated by the electrodynamic forces acting on the ions (α_l term) at high temperatures. This temperature behavior is schematically shown in Fig. 1.

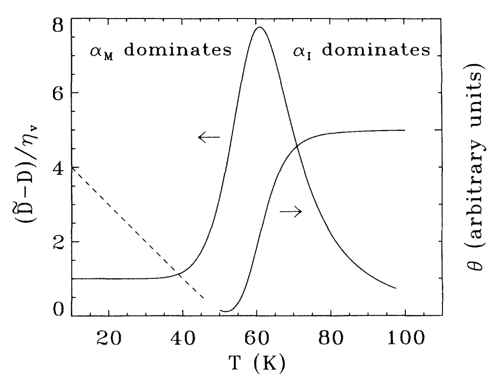

FIG. 1. The sound dissipation $\tilde{D} - D$ due to vortex dynamics and the Faraday angle θ as a function of temperature, calculated using Eqs. (9) and (10), with the experimental parameters of Ref. [6] for $B = 5$ T and $\omega/2\pi = 3$ MHz. Here we assume that α_M is linear in T (dashed line) and η_v is temperature independent. This is only meant to be a guide to the reader, in order to show that θ and $\tilde{D} - D$ at low T will give direct information on $\alpha_M(T)$ and $\eta_v(T)$.

We now discuss briefly the case of sound propagation perpendicular to the magnetic field $\mathbf{k}\perp\mathbf{B}$, where we can take $k_y = k_z = 0$, $k_x = k$. The longitudinal and transverse components u_x and u_y are mixed because of the Magnus and electrodynamics forces, and the sound becomes "elliptically" polarized. The effect is quadratic in α_M and α_l because longitudinal and transverse waves have different velocities even without Magnus and electrodynamic forces. Thus, the effect of Magnus force at low temperatures is negligibly small in this case, of order $(\alpha_M/\rho\omega)^2$. In the low-temperature limit ($\omega\tau_T \gg 1$), the modified sound longitudinal velocity is $\tilde{c}_l^2 = c_l^2 + C_{11}/\rho$, the modified transverse velocity along the y direction is $\tilde{c}_t^2 = c_t^2 + C_{66}/\rho$, and modified dissipation $\tilde{D} = D + \eta_v$ [where, for a tetragonal solid, $c_t = (\lambda_{66}/\rho)^{1/2}$ along the y direction and $c_l = (\lambda_{11}/\rho)^{1/2}$ along the x direction, with $\lambda_{66} = \lambda_{xy,xy}$, $\lambda_{11} = \lambda_{xx,xx}$]. At high temperatures, $\omega\tau_T \ll 1$, $\tilde{c}_t$, $\tilde{c}_l$, and $\tilde{D}$ reduce to the values calculated in Ref. [6].

In principle, experimental data on sound attenuation at low temperatures $T \ll T_c$, measured as the difference $\Delta D(B) = \tilde{D}(B) - \tilde{D}(0) = \eta_q + \eta_v$, can provide information on η_v, if $\eta_q \ll \eta_v$. In superconductors with a gap, $\eta_q \approx \eta_n B/H_{c2}$ (η_n is the normal-state attenuation coefficient), and η_q can be neglected since $\eta_n c^2/\sigma_n \ll H_{c2}^2$. In a gapless superconductor a stronger field dependence of η_q is possible. In a clean gapless superconductor the quasiparticle density of states (DOS) $N_s(\epsilon)$ is suppressed as compared with the DOS in the normal state N_F for energies ϵ below T_c, and $N_s(\epsilon) \propto \epsilon$ in the case of zero gap along the line on the Fermi surface. As shown by Volovik [15], in the presence of vortices the DOS becomes finite at zero energy, $N_s(\epsilon) \approx N_F(B/H_{c2})^{1/2}$. Then sound dissipation caused by quasiparticles in the presence of vortices η_q is given as $\eta_n(B/H_{c2})^{1/2}$. Now η_q is larger than η_v, approximately by 1 order of magnitude in magnetic fields of several tesla, and therefore the quasiparticle contribution dominates in the dissipation $\Delta D(B)$ induced by a magnetic field (via vortices). In dirty gapless superconductors $N_s(\epsilon)$ is finite at $\epsilon = 0$ because of impurities; see [16] and references therein. The effect of the magnetic field on the DOS and η_q is now weaker, and both contributions, η_v and η_q, may be of the same order of magnitude. Here more study is necessary to obtain a theoretical description for $\eta_q(B)$.

The sound dissipation in $Bi_{1.6}Pb_{0.4}Sr_2Ca_2Cu_3O_y$ at low temperatures was measured in experiments by Pankert *et al.* [6] in fields of 3 and 5 T. Almost the same $\Delta D(B)$ was found in those fields. Assuming that $\Delta D(B) = \eta_v$, we estimate from these data $\eta_v \Phi_0/B \approx 10^{-7}$ g/cm s. This is comparable with η_v obtained by Bi-2:2:1:2 from flux flow resistivity at 58 and 70 K [17], but in contradiction with an $\eta_v \propto B$ dependence. The experimental values of $\Delta D(B)$ in Ref. [6] are, however, much smaller than η_q as estimated above for clean gapless superconductors with nodes on a line. This result may be attributed either to a superconductor with a gap or to a weaker field dependence of η_q in a dirty gapless superconductor.

In conclusion, ultrasound propagation in the mixed state at low temperatures can provide information on (i) the Magnus force, (ii) elastic moduli, and (iii) viscosity for the vortex lattice. Measurements of sound dissipation in a magnetic field probe also the density of states for quasiparticles in the vortex state, and they may be an effective tool used to obtain information on the gap in clean superconductors.

We acknowledge useful discussions with A. V. Balatsky and V. G. Kogan.

[1] H. Suhl, Phys. Rev. Lett. **14**, 226 (1965); E. Šimánek, Phys. Lett. A **154**, 309 (1991).
[2] P. Nozières and W. F. Vinen, Philos. Mag. **14**, 667 (1966); P. Ao and D. J. Thouless, Phys. Rev. Lett. **70**, 2158 (1993).
[3] N. B. Kopnin and V. E. Kravtsov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 631 (1976) [JETP Lett. **23**, 578 (1976)]; Zh. Eksp. Teor. Fiz. **71**, 1644 (1976) [Sov. Phys. JETP **44**, 861 (1976).
[4] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965); M. Tinkham, Phys. Rev. Lett. **13**, 804 (1964).
[5] A. V. Samoilov, Z. G. Ivanov, and L.-G. Johansson, Phys. Rev. B **49**, 3667 (1994); J. M. Harris *et al.*, Phys. Rev. Lett. **73**, 1711 (1994).
[6] J. Pankert, Physica (Amsterdam) **165–166B**, 1273 (1990); Physica (Amsterdam) **168C**, 335 (1990); J. Pankert *et al.*, Phys. Rev. Lett. **65**, 3052 (1990); M. Saint-Paul *et al.*, Physica (Amsterdam) **180C**, 394 (1991); M. Yoshzawa *et al.*, Solid State Commun. **89**, 701 (1994).
[7] D. P. Belozorov and E. Kaner, Zh. Eksp. Teor. Fiz. **55**, 642 (1968) [Sov. Phys. JETP **28**, 334 (1969)]; I. E. Bulyzhenkov and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **71**, 1172 (1968) [Sov. Phys. JETP **44**, 613 (1969)]; G. Blatter and B. Ivlev, Report No. ETH-TH/94-34, 1994.
[8] E. H. Brandt, Phys. Rev. Lett. **67**, 2219 (1991).
[9] M. W. Coffey and J. R. Clem, Phys. Rev. Lett. **67**, 386 (1991); Phys. Rev. B **45**, 9872 (1992).
[10] C. J. van der Beek, V. B. Geshkenbein, and V. M. Vinokur, Phys. Rev. B **48**, 3393 (1993).
[11] C. J. van der Beck *et al.*, Physica (Amsterdam) **195C**, 307 (1992); C. J. van der Beck and P. Kes, Phys. Rev. B **43**, 13 032 (1991).
[12] E. H. Brandt, J. Low Temp. Phys. **26**, 709 (1977); **28**, 291 (1977); A. Houghton, R. A. Pelkovits, and A. Sudbo, Phys. Rev. B **40**, 6763 (1989); L. I. Glazman and A. E. Koshelev, Phys. Rev. B **43**, 2835 (1991); Physica (Amsterdam) **173C**, 180 (1991).
[13] R. Kleiner and P. Müller, Phys. Rev. B **49**, 1327 (1994); J. H. Cho *et al.*, Phys. Rev. B **50**, 6493 (1994); A. Schilling *et al.*, Phys. Rev. Lett. **71**, 1899 (1993); Y. Iye *et al.*, Physica (Amsterdam) **199C**, 154 (1992); J. C. Martinez *et al.*, Phys. Rev. Lett. **69**, 2276 (1992).
[14] A. B. Pippard, Philos Mag. **46**, 1104 (1955).
[15] G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **58**, 457 (1993) [JETP Lett. **58**, 468 (1993)].
[16] A. V. Balatsky, A. Rosengren, and B. L. Altshuler, Phys. Rev. Lett. **73**, 720 (1994).
[17] L. N. Bulaevskii *et al.*, Phys. Rev. B **50**, 3507 (1994).