Pair Susceptibilities and Gap Equations in Non-Fermi Liquids

A. Sudbø
Institutt for Fysikk, Universitetet i Trondheim, Norges Tekniske Høgskole, 7034 Trondheim-NTH, Norway
(Received 5 October 1994)

The pair susceptibility and superconducting gap equation are obtained when a branch-cut spectrum, with either spin-charge separation or an anomalous Fermi surface exponent $\alpha > 0$ is introduced for the normal state. For $\mathbf{k}$-nondiagonal pairing interactions, spin-charge separation leads to an *enhancement* of T_c and $\Delta_{\mathbf{k}}(0)$ compared to a Fermi liquid. For $\alpha > 0$, a critical coupling is required for a solution. For spin-charge separation or $0 < \alpha < \frac{1}{2}$, an arbitrarily small $\mathbf{k}$-diagonal contribution to the pairing interaction still leads to a solution, as in a Fermi liquid.

PACS numbers: 74.20.Mn

Finding a theoretical foundation for novel non-Fermi-liquid metallic ground states in higher-dimensional ($D > 1$) strongly correlated fermion systems has been a subject of intense research and much controversy over the last few years, following suggestions that the normal state of the high-T_c cuprates is an unconventional metal [1,2]. A corollary problem of interest in that context is to what extent the superconducting properties are affected by such an unconventional normal ground state, and indeed if the high T_c itself is a consequence of just this. Recently, a superconducting gap equation resulting from the Josephson pair tunneling mechanism of high-T_c cuprate superconductivity [3] was considered in some detail [4]. These investigations relied on two nontrivial assumptions. (i) In the normal state, Fermi liquid theory was postulated to have broken down as a result of strong correlations, leading to quenched, or at least incoherent, single-particle tunneling between closely coupled CuO_2 layers. (ii) In explicit calculations, the non-Fermi-liquid aspect was assumed to manifest itself only through the quenching of the coherent single-particle tunneling between layers, and the dominance of coherent pair tunneling in the interlayer coupling: The *pair susceptibility* of each individual layer was taken to be of Fermi liquid form.

Assumption (i) has been reexamined and further justified [5,6]. To provide such a justification, it is crucial to allow for the fact that, *in the normal state* at low energies, the spectral weight of the fermionic two-point correlation function satisfies a non-Fermi-liquid homogeneity relation close to the Fermi surface [4], given by

$$A(\Lambda k, \Lambda \omega) = \Lambda^{-1+\alpha} A(k, \omega), \qquad (1)$$

with an exponent $\alpha > 0$. A Fermi liquid has $\alpha = 0$. The fact that a finite deviation from Fermi liquid theory appears to be required to justify the assumption of quenched or incoherent single-particle tunneling between layers raises a question about the validity of the assumption made in point (ii) in calculating the intralayer pair susceptibility. In this Letter, we reconsider this problem using explicitly non-Fermi-liquid normal propagators. The breakdown of the Fermi liquid theory and the establishment of a novel fixed-line theory characterized by one single nonuniversal exponent is a highly nontrivial assumption. At present, providing a rigorous foundation for this for $D > 1$ remains a formidable challenge in condensed matter theory. Nonetheless, it appears to be of interest to investigate what the consequences for superconductivity at the mean-field level in higher dimensions are, given such an assumption.

Our starting point is therefore the single-particle fermion propagator for the normal state, *assumed* to be given by

$$G_{ne}(\mathbf{k}, \omega) = \frac{1}{\omega_c^{\alpha}} \frac{1}{[\omega - u_\sigma k]^{1/2} [\omega - u_\rho k]^{1/2-\alpha}}, \qquad (2)$$

which defines the anomalous Fermi surface exponent α, with a corresponding expression for the hole propagator $G_{nh}(\mathbf{k}, \omega)$. u_ρ and u_σ are the velocities for charge- and spin-density excitations, respectively, and ω_c is a frequency cutoff which is introduced to make the Green's function dimensionally correct. Physically, we may view ω_c as an energy cutoff (some fraction of the width of the band crossing the Fermi level) beyond which the above non-Fermi-liquid framework, being an assumed asymptotically correct *low-energy* description of the normal state, becomes inapplicable. The above normal-state propagator may be viewed as an *ad hoc* generalization to higher dimensions of a fermion propagator in a 1D interacting system [7]. Note that the detailed form of Eq. (2) is not identical to the forms of Ref. [7]. Equation (2) is purely phenomenological in character, and there is no reason to employ identically the structure of purely 1D objects [8]. Equation (2) has the basic property of lacking poles, i.e., there are no low-energy quasiparticle excitations in the normal state. It is the consequence of this phenomenology we wish to investigate. For a Fermi liquid we have, in addition to $\alpha = 0$, also $u_\rho = u_\sigma = v_F$, where v_F is the Fermi velocity.

We now write the finite temperature 2×2 Green's function in Nambu-Gor'kov matrix formulation for the superconducting state as

$$G^{-1}(\mathbf{k}, i\omega_n) = \begin{pmatrix} G_{ne}^{-1}(\mathbf{k}, i\omega_n) & -\Delta_{\mathbf{k}}^{\dagger} \\ -\Delta_{\mathbf{k}} & G_{nh}^{-1}(\mathbf{k}, i\omega_n) \end{pmatrix}, \qquad (3)$$

where $\omega_n = (2n + 1)\pi/\beta$; $\beta = 1/k_B T$, and $\Delta_{\mathbf{k}} \sim \langle c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow} \rangle$ is the mean-field gap, to be determined self-consistently. Given the inverse matrix propagator $G^{-1}(\mathbf{k}, i\omega_n)$, we define the anomalous propagators $F(\mathbf{k}, i\omega_n)$ and $F^\dagger(\mathbf{k}, i\omega_n)$ in the usual way as the off-diagonal elements of $[G^{-1}(\mathbf{k}, i\omega_n)]^{-1}$. The gap equation is then given by the self-consistency condition

$$\Delta_{\mathbf{k}} = \frac{1}{\beta} \sum_{\mathbf{k}', \omega_n} V_{\mathbf{k},\mathbf{k}'} F(\mathbf{k}', i\omega_n) = \sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \chi_0(\mathbf{k}') \Delta_{\mathbf{k}'},$$

$$\chi_0(\mathbf{k}) = -\frac{1}{\beta} \sum_{\omega_n} \frac{1}{\omega_c^{2\alpha} f_\sigma^{1/2} f_\rho^{1/2-\alpha} - \Delta_{\mathbf{k}}^2}, \tag{4}$$

which defines the pair susceptibility $\chi_0(\mathbf{k})$, and where $f_\nu \equiv (i\omega_n)^2 - (u_\nu k)^2$. Note that Eq. (4) follows from (i) the phenomenology of Eq. (2), and (ii) the use of the Gor'kov equations. This gives a pair susceptibility which is different from the 1D results, perhaps not unexpectedly given our use of the Gor'kov equations.

We consider the simple case $V_{\mathbf{k},\mathbf{k}'} = V, |e_{\mathbf{k}}|, |e_{\mathbf{k}'}| < \omega_D$, where ω_D is a frequency cutoff on the spectrum of the bosons responsible for the attractive interaction. In this Letter, we make the assumption that $\omega_D < \omega_c$. Now the gap equation $0 \le T \le T_c$ may be written in the form

$$\frac{1}{\lambda} = 2 \int_0^{\omega_D} de\, \chi_0(e). \tag{5}$$

Here $\lambda = VN_n(0)$, and $N_n(0)$ is the normal state Fermi level density of states. For the Fermi liquid case, the Matsubara sum in χ_0 may be evaluated explicitly at all temperatures to give the well-known result $\chi_0(\mathbf{k}) = \tanh(\beta E_{\mathbf{k}}/2)/2E_{\mathbf{k}}$, where $E_{\mathbf{k}} = \sqrt{e_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$, with the linearized spectrum $e_{\mathbf{k}} = v_F k$ close to the Fermi surface. For $T = 0$, the integrated susceptibility $\int_0^{\omega_D} de\, \chi_0(e) \sim \ln(\omega_D/\Delta\mathbf{k})$, and $\Delta(0) \sim \omega_D \exp(-1/\lambda)$. At $T = T_c$, the integrated susceptibility $\int_0^{\omega_D} de\, \chi_0(e) \sim \ln(\beta\omega_D)$, and $T_c \sim \omega_D \exp(-1/\lambda)$. The gap equation can be satisfied for an arbitrarily small coupling constant $\lambda > 0$ by exploiting the logarithmic divergence that develops in the integrated susceptibility at (a) small $\Delta(0)$, $T = 0$, or (b) small T_c, $\Delta = 0$.

We will in the following consider two further special cases of Eq. (4). The first case is a quantum liquid with assumed spin-charge separation and Fermi liquid scaling [7]. The second case is a quantum liquid with assumed spin-charge confinement and non-Fermi-liquid scaling [5]. As far as pair susceptibilities and gap equations are concerned, the second case will be shown to exhibit more drastic deviations from the Fermi liquid results than the first. The details are as follows.

Case (i): $\alpha = 0$, $u_\rho > u_\sigma$.—In this case, the pole singularities in the Green's function are determined from the condition $[(i\omega_n)^2 - (u_\rho k)^2]^{1/2}[(i\omega_n)^2 - (u_\sigma k)^2]^{1/2} - \Delta_{\mathbf{k}}^2 = 0$, with the solution

$$(i\omega_n)^2 = E_{\mathbf{k}}^2 \equiv \frac{1}{2}\left[\nu_+ + \sqrt{\nu_-^2 + 4\Delta_{\mathbf{k}}^4}\right], \tag{6}$$

where $\nu_\pm = (u_\rho k)^2 \pm (u_\sigma k)^2$. In addition, we have the branch points at $i\omega_n = \pm u_\rho k$ and $i\omega_n = \pm u_\sigma k$. The poles give a Fermi-liquid-like contribution to $\chi_0(\mathbf{k})$, but with modified residues, whereas the branch cuts give rise to a novel non-Fermi-liquid contribution. We get for the pair susceptibility

$$\chi_0(\mathbf{k}) = \frac{\Delta_{\mathbf{k}}^2}{\sqrt{\nu_-^2 + 4\Delta_{\mathbf{k}}^4}} \frac{\tanh(\beta E_{\mathbf{k}}/2)}{E_{\mathbf{k}}} + \chi_0^{\mathrm{cut}}(\mathbf{k}),$$

$$\chi_0^{\mathrm{cut}}(\mathbf{k}) = \frac{1}{\pi} \int_{u_\sigma k}^{u_\rho k} dx\, \tanh\left(\frac{\beta x}{2}\right) \frac{h_\sigma h_\rho}{h_\sigma^2 h_\rho^2 + \Delta_{\mathbf{k}}^4}, \tag{7}$$

where $h_\nu \equiv \sqrt{|x^2 - (u_\nu k)^2|}$, and $u_\rho + u_\sigma = 2v_F$. The $T = 0$ integrated pair susceptibility $2\int_0^{\omega_D} de\, \chi_0(e)$ as a function of $\Delta_{\mathbf{k}}$ for various values of $[u_\rho - u_\sigma]/v_F$ is shown in Fig. 1. It increases monotonically as $\Delta_{\mathbf{k}}$ is reduced, and is enhanced over the Fermi liquid case given by the solid line. Note that the enhancement is finite for $\Delta_{\mathbf{k}} \neq 0$, even when $u_\sigma/u_\rho = 0$, as is also seen from Eq. (7) by inspection.

The cut contribution vanishes as $u_\sigma \to u_\rho$, whereas the pole contribution reduces to the Fermi liquid result. The increase of the cut contribution as $u_\rho - u_\sigma$ increases is seen to overcompensate the reduction of the pole contribution. *No critical value of the coupling constant λ is required for a solution for all $0 < T < T_c$*, and $\Delta_{\mathbf{k}}(0)$ is enhanced over the Fermi liquid case. Spin-charge separation essentially amounts to a renormalization of the coupling constant (see below). Notice also that the pair susceptibility on the Fermi surface at $T = 0$ diverges as $\Delta_{\mathbf{k}}^{-1}$, since spin-charge separation is not felt at $k = 0$ (here measured with respect to k_F). The divergence of χ_0 on the Fermi surface is thus precisely as in a Fermi liquid. At $T = 0$, the Fermi liquid limit is approached smoothly as $u_\sigma \to u_\rho$. At $T = T_c$, $u_\rho > u_\sigma$, only the cut contribution survives. However, at $T = T_c$, $\lim_{u_\sigma \to u_\rho} \chi_0^{\mathrm{cut}}(e)$ reduces to the Fermi liquid result

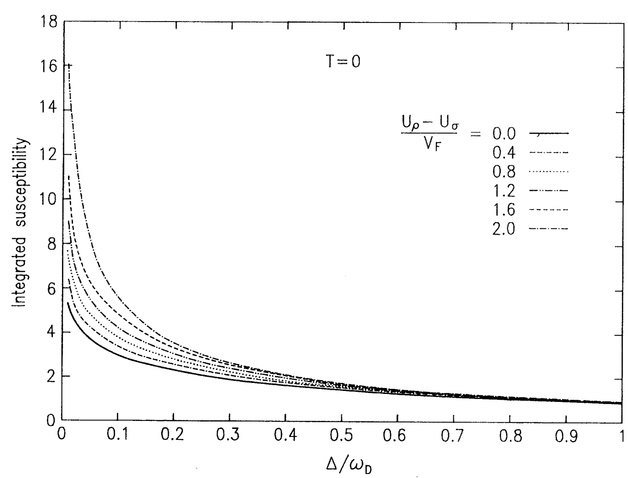

FIG. 1. The integrated pair susceptibility $2\int_0^{\omega_D} de\, \chi_0(e)$ at $T = 0$ for case (i) as a function of $\Delta_{\mathbf{k}}$, for various values of $[u_\rho - u_\sigma]/v_F$, when $u_\rho + u_\sigma = 2v_F$. The solid line is the Fermi liquid result.

$\tanh(\beta e/2)/2e$, whereas the pole contribution has vanished. On the other hand, at $u_\sigma = u_\rho$, $\lim_{T\to T_c} \chi_0^{\text{pole}}(e)$ also reduces to $\tanh(\beta e/2)/2e$, whereas the cut contribution has vanished. Hence, for the total susceptibility $\lim_{u_\rho\to u_\sigma} \lim_{T\to T_c} \chi_0(e) = \lim_{T\to T_c} \lim_{u_\sigma\to u_\rho} \chi_0(e)$, even though this is not true for either of its two contributions. The limit $u_\sigma \to u_\rho$ is thus nonsingular for the quantity T_c.

We next obtain T_c, for $u_\rho > u_\sigma$. Only the cut contribution to $\chi_0(e)$ needs to be considered. Using Eqs. (5) and (7), we get

$$\frac{1}{2\lambda} = \frac{1}{\pi} \int_0^{\beta\omega_D/2} dx\, x^{-1} H\left(x; \frac{u_\sigma}{u_\rho}\right),$$
$$H(x;\eta) \equiv \int_\eta^1 du \frac{\tanh(xu)}{\sqrt{(u^2-\eta^2)(1-u^2)}}. \tag{8}$$

For $x \ll 1$, we have $H(x;\eta) = \pi x/2$, while $\lim_{x\to\infty} H(x;\eta) = K(\sqrt{1-\eta^2})$, where $K(k)$ is a complete elliptic integral of the first kind with module k [9]. Since $H(x;\eta)$ saturates as $x \to \infty$, the integral over x diverges $\sim \ln(\beta\omega_D/2)$ as $\beta \to \infty$, as for the case $u_\rho = u_\sigma$. Because of the innocuous prefactor, the right-hand side (RHS) of Eq. (8) can be made arbitrarily large by making β large enough, and it therefore suffices to consider only the asymptotic value of $H(x;\eta)$ as $x \to \infty$. By partial integration, we obtain when $\beta\omega_D \gg 1$

$$\frac{1}{2\lambda} = \frac{K\left(\sqrt{1-u_\sigma^2/u_\rho^2}\right)}{\pi} \ln\left(\frac{\beta\omega_D}{2}\right). \tag{9}$$

We thus get $k_B T_c = [\omega_D/2] \exp(-1/\tilde{\lambda})$, where $\tilde{\lambda} = [2\lambda/\pi] K(\sqrt{1-u_\sigma^2/u_\rho^2})$, which has the same form as for the Fermi liquid case. Spin-charge separation essentially merely leads to a renormalization of the coupling constant, as anticipated above (the difference in *prefactor* is not essential). The increase of the effective coupling constant compared to the Fermi liquid case is because of the overcompensation, in the integrated susceptibility, of the loss from the pole contribution by the increase of the cut contribution as $u_\rho - u_\sigma$ increases. When $u_\sigma = u_\rho$, we have $\tilde{\lambda} = [2\lambda/\pi] K(0) = \lambda$. On the other hand, when $u_\sigma/u_\rho \ll 1$, we have $\tilde{\lambda} = [2\lambda/\pi] \ln(4u_\rho/u_\sigma)$. From the above, both $\Delta_{\mathbf{k}}(0)$ and T_c are seen to be enhanced by spin-charge separation. An enhancement of the upper critical magnetic field $H_{c2}(0)$ for this case, consistent with the above, has recently also been reported [10].

Case (ii): $\alpha > 0$, $u_\rho = u_\sigma$.—The pole singularities in the Green's function below T_c are now determined from the condition $[(i\omega_n)^2 - e_k^2]^{1-\alpha}\omega_c^{2\alpha} - |\Delta_{\mathbf{k}}|^2 = 0$, with the general solution [5]

$$(i\omega_n)^2 = e_k^2 + \exp\left(\frac{2i\pi n}{1-\alpha}\right)|\tilde{\Delta}_{\mathbf{k}}|^2, \tag{10}$$

where $n = 0, \pm 1, \ldots$, and $|\tilde{\Delta}_{\mathbf{k}}| = |\Delta_{\mathbf{k}}|(|\Delta_{\mathbf{k}}|/\omega_c)^{\alpha/(1-\alpha)}$. In the extreme limit $\alpha = 1$, the *effective* gap $\tilde{\Delta}_{\mathbf{k}}$ collapses even if the *bare* gap $\Delta_{\mathbf{k}} > 0$. As will be shown below, it turns out that the bare gap $\Delta_{\mathbf{k}}(0)$ may vanish for $\alpha < 1$. This collapse reflects the smeared momentum distribution at the Fermi surface as α increases; equivalently it is a manifestation of the reactive nature of the coupling $V_{\mathbf{k},\mathbf{k}'}$ to a quantum liquid with non-Fermi-liquid scaling.

The superconducting propagator in this case has an infinite number of poles located in the plane z^2 on a circle of radius $|\tilde{\Delta}_{\mathbf{k}}|^2$ around the point $e^2(\mathbf{k})$. However, as emphasized in Ref. [5], the only physically relevant solution is the one on the principal Riemann sheet $z = |z| \exp(i\theta)$; $\theta \in [0, 2\pi\rangle$, which implies that $i\omega_n = \pm[e_{\mathbf{k}}^2 + |\tilde{\Delta}_{\mathbf{k}}|^2]^{1/2} \equiv \pm E_{\mathbf{k}}$. In addition to poles, the propagator in the superconducting state is seen to have branch points at $\pm e_{\mathbf{k}}$. As in the previous case, we thus get a Fermi-liquid-like pole contribution to the susceptibility, but with a modified residue, as well as a non-Fermi-liquid-like cut contribution to the pair susceptibility. We obtain

$$\chi_0(\mathbf{k}) = \left(\frac{|\tilde{\Delta}_{\mathbf{k}}|}{\omega_c}\right)^{2\alpha} \frac{1}{1-\alpha} \frac{\tanh(\beta E_{\mathbf{k}}/2)}{2E_{\mathbf{k}}} + \chi_0^{\text{cut}}(\mathbf{k}),$$
$$\chi_0^{\text{cut}}(\mathbf{k}) = \frac{1}{\pi} \int_0^{e_{\mathbf{k}}} dx \tanh\left(\frac{\beta x}{2}\right) \frac{g_{s\alpha}}{\omega_c^{2\alpha}(\tilde{g}_{c\alpha}^2 + g_{s\alpha}^2)}, \tag{11}$$

where $g_{s\alpha} = g_\alpha \sin(\pi\alpha)$, $g_{c\alpha} = g_\alpha \cos(\pi\alpha)$, $\tilde{g}_{c\alpha} = g_{c\alpha} + |\tilde{\Delta}_{\mathbf{k}}|^{2(1-\alpha)}$, and $g_\alpha \equiv |x^2 - e_{\mathbf{k}}^2|^{1-\alpha}$. Note that the cut contribution is nonsingular at $\tilde{\Delta}_{\mathbf{k}} = 0$ for all α and $0 \le T \le T_c$. The $T = 0$ integrated pair susceptibility $2\int_0^{\omega_D} de\, \chi_0(e)$ is shown in Fig. 2 as a function of $\tilde{\Delta}_{\mathbf{k}}$, for various values of α. The Fermi liquid result $\alpha = 0$ is given by the solid line. Note the finite value at $\Delta_{\mathbf{k}} = 0$ for $\alpha > 0$. The logarithmic divergence for $\alpha = 0$ as $\Delta_{\mathbf{k}} \to 0$ is thus replaced by a finite value due to the modified residue of the pole contribution and the nonsingular cut contribution. Comparing with Eq. (5), we conclude that *a critical value of the coupling constant λ is required for a solution to the gap equation for all $0 < T < T_c$.*

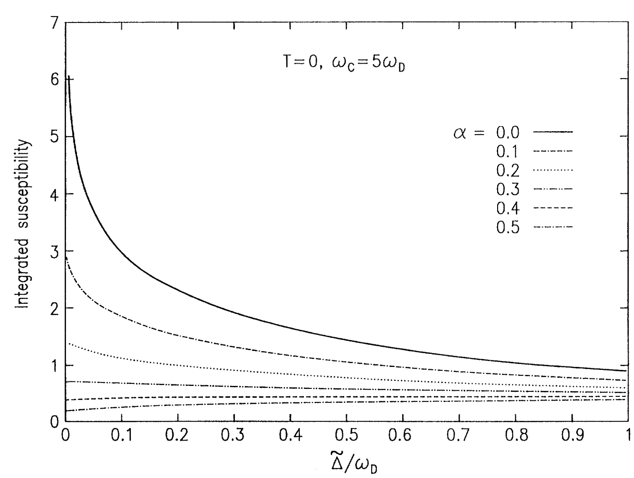

FIG. 2. The integrated pair susceptibility $2\int_0^{\omega_D} de\, \chi_0(e)$ at $T = 0$ for case (ii) as a function of $\tilde{\Delta}_{\mathbf{k}}$, for various values of α, with $\omega_c = 5\omega_D$. The solid line is the Fermi liquid result.

As $\alpha \rightarrow 0$, the cut contribution vanishes, whereas the pole contribution to the pairing susceptibility reduces to the Fermi liquid result. The pole contribution is reduced as α increases, whereas the cut contribution increases up to $\alpha = \frac{1}{2}$. However, at small values of $\tilde{\Delta}_{\mathbf{k}}$, the increase in the cut contribution is not sufficient to compensate the reduction of the pole contribution as α increases. At $T = 0$, $\alpha = 0$, the pair susceptibility diverges as $\Delta_{\mathbf{k}}^{-1}$ on the Fermi surface, leading to a logarithmic divergence of the integrated susceptibility as $\Delta_{\mathbf{k}} \rightarrow 0$. For $\alpha > 0$, the cut contribution vanishes on the Fermi surface, and the pair susceptibility diverges as $\tilde{\Delta}_{\mathbf{k}}^{2\alpha-1}$. At $T = 0$, the Fermi liquid limit is approached smoothly as $\alpha \rightarrow 0$. At $T = T_c$, $\alpha > 0$, only the cut contribution survives. There is an important difference here compared to the previous case: $\lim_{\alpha\rightarrow 0} \lim_{T\rightarrow T_c} \chi_0(e) \neq \lim_{T\rightarrow T_c} \lim_{\alpha\rightarrow 0} \chi_0(e)$. The limit $\alpha \rightarrow 0$ is thus singular for the quantity T_c.

We next obtain T_c, for $\alpha > 0$. This has previously been treated less explicitly by a scaling approach [11]. Again, only the cut contribution to $\chi_0(e)$ needs to be considered. Using Eqs. (5) and (11), we get

$$\frac{1}{2\tilde{\lambda}} = \left(\frac{2}{\beta\omega_c}\right)^{2\alpha} \int_0^{\beta\omega_D/2} dx\, x^{2\alpha-1} F(x;\alpha),$$

$$F(x;\alpha) \equiv \int_0^1 du \frac{\tanh(xu)}{(1-u^2)^{1-\alpha}}. \qquad (12)$$

Here $\tilde{\lambda} \equiv \lambda \sin(\pi\alpha)/\pi$. For $x \ll 1$, we have $F(x;\alpha) = x/2\alpha$, while $\lim_{x\rightarrow\infty} F(x;\alpha) = B(\frac{1}{2},\alpha)/2 - 1/x$, where $B(x,y)$ is the beta function [9]. Since $F(x;\alpha)$ saturates as $x \rightarrow \infty$, the integral over x diverges $\sim(\beta\omega_D)^{2\alpha}$ as $\beta \rightarrow \infty$. The prefactor however implies that the RHS of Eq. (12) cannot be made arbitrarily large by making β large enough. A critical coupling constant λ is thus required for a solution, as for $\Delta_{\mathbf{k}}(0)$. By partial integration, we obtain when $\beta\omega_D \gg 1$

$$\frac{1}{\tilde{\lambda}} = \frac{1}{\alpha}\left(\frac{\omega_D}{\omega_c}\right)^{2\alpha} F(\beta\omega_D/2;\alpha) = \frac{1}{\tilde{\lambda}_c} - \frac{g(\alpha)}{\beta\omega_D}, \qquad (13)$$

where the critical coupling now is found explicitly $\tilde{\lambda}_c = 2\alpha(\omega_c/\omega_D)^{2\alpha}/B(\frac{1}{2};\alpha)$, and $g(\alpha) = 2(\omega_D/\omega_c)^{2\alpha}/\alpha$. The RHS of Eq. (13) is a monotonically increasing function of β. When $\tilde{\lambda} > \tilde{\lambda}_c$, we have $k_B T_c = [\omega_D/g(\alpha)][1/\tilde{\lambda}_c - 1/\tilde{\lambda}]$, which should be compared to $T_c \sim \exp(-1/\lambda)$ for $\alpha = 0$. Note that $\lim_{\alpha\rightarrow 0} \tilde{\lambda}_c = 0$.

The discussion has so far been carried out for a pairing interaction which is nonlocal (nondiagonal) in $\mathbf{k}$ space; cf. our choice of $V_{\mathbf{k},\mathbf{k}'}$ below Eq. (4). We next consider the effect of including a local, in $\mathbf{k}$ space, pairing interaction. Such a local pairing interaction has recently been considered in the context of an interlayer tunneling mechanism of high-T_c superconductivity [4]. Specifically, we consider the modified gap equation for close superconducting bilayers coupled by a coherent pair-tunneling term with matrix element T_J. It is readily shown that this leads to an effective renormalization of the pair susceptibility χ_0 because of the local contribution to the pairing kernel [4]. Equation (5) is now replaced by [4]

$$\frac{1}{\lambda} = 2\int_0^{\omega_D} de \frac{\chi_0}{1 - T_J\chi_0}. \qquad (14)$$

For dominant T_J, the instability at $T = 0$ may be determined from the condition $1 = T_J\chi_0(e)$, with χ_0 *evaluated on the Fermi surface* [4]. For the Fermi liquid case, this gives $\Delta_{\mathbf{k}}(0) = T_J/2$.

For case (i), we find, using Eq. (7), $\Delta_{\mathbf{k}}(0) = T_J/2$, i.e., a result precisely as for the Fermi liquid case, since spin-charge separation does not affect the spectrum on the Fermi surface. For case (ii) we find at $T = 0$, $1 = [T_J/2\omega_c(1-\alpha)]x^{2\alpha-1}$; $x \equiv \tilde{\Delta}_{\mathbf{k}}/\omega_c$. As long as $\alpha < \frac{1}{2}$, one can obtain a solution for arbitrarily small T_J. This is no longer guaranteed if $\alpha > \frac{1}{2}$. For $\alpha < \frac{1}{2}$, using $\tilde{\Delta}_{\mathbf{k}} \equiv \Delta_{\mathbf{k}}(\Delta_{\mathbf{k}}/\omega_c)^{\alpha/(1-\alpha)}$ and Eq. (11), we find $\Delta_{\mathbf{k}}(0) = (1/\omega_c)^{\alpha/(1-2\alpha)}[T_J/2(1-\alpha)]^{(1-\alpha)/(1-2\alpha)}$, which approaches $T_J/2$ as $\alpha \rightarrow 0$. The local contribution to the pairing kernel thus to a certain extent helps to preserve the structure of the solution to the gap equation obtained from using a Fermi liquid form of χ_0.

This work was supported by the Norges Forsknings Råd, Grant No. 101172-432. The author thanks Professor S. Chakravarty for informative communications, and Dr. M. P. Das for the invitation to the High-T_c Workshop at the ANU, Canberra, where this work was completed.

[1] P. W. Anderson, Science **235**, 1196 (1987); Phys. Rev. Lett. **64**, 1839 (1990); **65**, 2306 (1990).
[2] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. **63**, 1996 (1989); **64**, 497(E) (1990).
[3] J. M. Wheatley, T. C. Hsu, and P. W. Anderson, Phys. Rev. B **37**, 5897 (1988).
[4] S. Chakravarty, A. Sudbø, P. W. Anderson, and S. Strong, Science **261**, 337 (1993).
[5] S. Chakravarty and P. W. Anderson, Phys. Rev. Lett. **72**, 3859 (1994).
[6] D. G. Clarke, S. P. Strong, and P. W. Anderson, Phys. Rev. Lett. **72**, 3218 (1994).
[7] V. Meden and K. Schönhammer, Phys. Rev. B **46**, 15 753 (1992); J. Voit, Phys. Rev. B **47**, 6740 (1993); M. Fabrizio and A. Parola, Phys. Rev. Lett. **70**, 226 (1993).
[8] For extensive work on the spectral properties of the 1D case, see also J. Voit, J. Phys. Condens. Matter **5**, 8305 (1993).
[9] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products,* (Academic, London, 1980).
[10] R. G. Dias and J. M. Wheatley, Phys. Rev. B **50**, 13 887 (1994).
[11] A. V. Balatskiĭ, Philos. Mag. Lett. **68**, 251 (1993).