

## Magnetoplasmons at Boundaries between Two-Dimensional Electron Systems

P. K. H. Sommerfeld, P. P. Steijaert, P. J. M. Peters, and R. W. van der Heijden

*Department of Physics, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands*  
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Magnetoplasmons localized at the boundary between two classical two-dimensional electron systems with different electron densities are observed and described in detail for the first time. In high magnetic fields the frequency of these interedge magnetoplasmons is proportional to the difference in electron density,  $\delta n$ , on either side of the boundary and inversely proportional to the magnetic field. The direction of propagation is determined by the sign of  $\delta n$ . A coupling between interedge modes and conventional edge modes at small spatial separation is observed.

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Although the general plasmon dispersion relation  $\omega(q)$  in two dimensions (2D) is gapless [ $\omega(q) \propto q^{1/2}$ ] [1], the finite dimensions of real samples result in a discrete spectrum of dimensional resonances [2,3] at frequencies  $\omega_p$ . When a magnetic field is applied the resonances shift to higher frequencies and eventually all approach the cyclotron frequency  $\omega_c$ , independent of dimensionality, size, or geometry. In addition to this bulk mode, a mode also exists for *bounded*, 2D samples whose frequency  $\omega_-$  is proportional to the Hall velocity and therefore *decreases* with magnetic field  $B$  as  $1/B$  [3–5]. It arises because the currents flow nearly perpendicular to the electric depolarization fields in the sample due to the Lorentz force, which leads to a rotation of the internal fields and currents [6]. In the high-field limit,  $\omega_- \ll \omega_p \ll \omega_c$ , these so-called edge magnetoplasmons (EMP's) are localized near the boundaries of the sample and are analogous to surface plasmons in three dimensions [7].

The electronic structure near the boundaries of 2D samples governs the electrical transport under quantum Hall effect conditions, which takes place in edge channels [8]. These are the lines of intersection between the Landau levels and the Fermi level. Because they are an edge phenomenon, EMP's have become widely used for investigations related to quantum transport in 2D electronic systems [9,10]. Just like an interface is a generalization of a surface in three dimensions, an interedge, i.e., a boundary between two 2D systems, is a generalization of an edge. Interedges have received considerable recent attention in connection with 2D quantum transport in high magnetic fields [11]. This interest stems partly from the fact that in real samples inevitable inhomogeneities are present, whose effects should be understood. Interedge magnetoplasmons (IEMP's) have been predicted theoretically from general and classical grounds [12]. Some features of IEMP's have already been addressed experimentally [13] and theoretically [14] in conjunction with spatially separated edge channels at quantum Hall effect plateaus. Because (I)EMP's basically are entirely classical phenomena, much insight is gained from investigations in a classical

system. In the present Letter, the first observation and detailed investigation of the classical IEMP are reported. The investigation at low frequencies and high magnetic fields for the 2D electron systems (2DES) on liquid helium allows an accurate determination of dispersion relation, dependence on density difference, and propagation direction.

The experimental cell is a cylindrical parallel plate capacitor of height  $h = 3$  mm. The helium level is typically  $d = 1.0$  mm above the bottom plate. To the top electrode and the circular guard electrode with a radius of  $R_g = 7.5$  mm potentials  $V_t$  and  $V_g$  are applied, respectively. The bottom plate is subdivided into 10 electrodes as shown in the inset of Fig. 1(a). The electric holding field determines the saturation areal density of electrons. By varying the holding field as a function of position using the bottom plate partitioning, it is possible to change the electron density locally and create density profiles [15]. These profiles can be calculated numerically from the electrostatics of the system. By keeping electrodes 1 to 8 at ground potential and changing the dc potential  $V_{M+R}$  on electrodes  $M$  and  $R$ , stepped density profiles can be created with a central circular region of density  $n_1$  and an outer annular region of density  $n_2$ . The density difference between these two regions is defined as  $\delta n = n_1 - n_2$ . At saturation the density in the homogeneous regions is proportional to the local potential difference between the top electrode and the relevant bottom electrode. Figure 1(a) shows numerical calculations of stepped density profiles. The profile of the electron layer can be changed *in situ* by changing the dc potentials of the relevant electrodes.

An ac potential of about  $10$  mV<sub>rms</sub> and frequency  $f \leq 120$  kHz is applied to one of the outer bottom electrodes, and the current induced on another electrode is measured using a preamplifier and a phase sensitive detector (typically electrodes 1 and 5 are used). The frequency is swept and the in- and out-of-phase currents are recorded at constant magnetic field and temperature  $T < 1$  K. At certain frequencies the wavelengths of the EMP's become equal to the circumference  $P$  of the sample leading to resonances. Figure 1(b) shows the frequency response of stepped density 2DES's with profiles as shown in Fig. 1(a). Starting

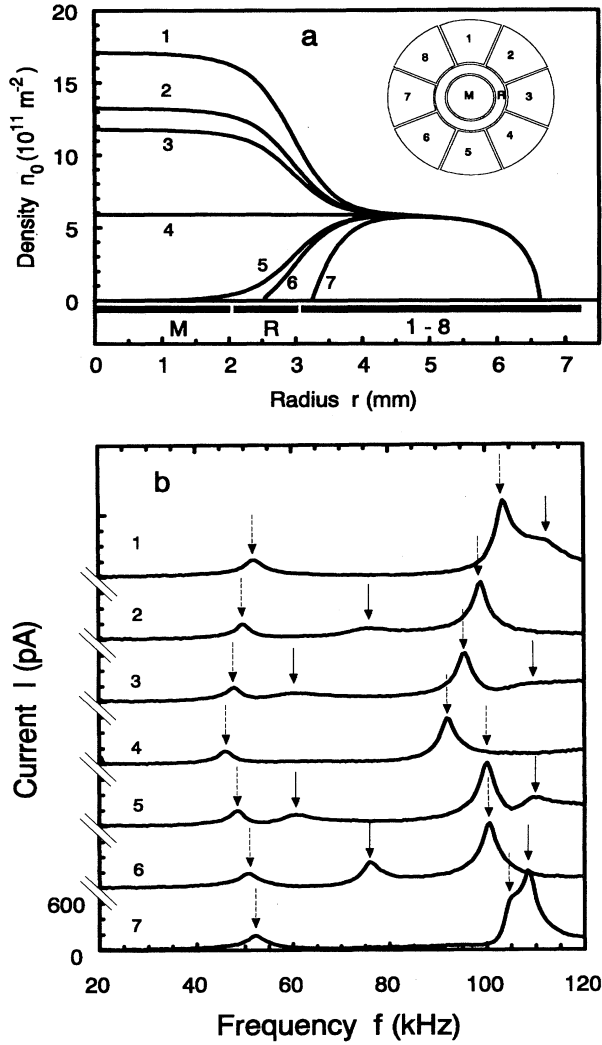


FIG. 1. (a) Density profiles for  $V_t = -10$  V,  $V_g = -15$  V,  $V_{1-8} = 0$  V, and  $V_{M+R} = 19$  V (1), 12.5 V (2), 10 V (3), 0 V (4), -10 V (5), -12.5 V (6), -35 V (7). The inset shows the bottom plate. (b) Amplitude of the measured current as a function of frequency for the density profiles shown in (a) using the same labeling at  $T = 0.5$  K and  $B = 2.3$  T.

with large positive  $\delta n = 2n_2$  (large positive  $V_{M+R}$ ) and keeping  $n_2$  constant,  $n_1$  was decreased to go from a positive step to a homogeneous profile,  $\delta n = 0$ , and finally to a ring,  $\delta n = -n_2$  [see Fig. 1(a)]. This procedure ensures that the system is always at saturation density since the total charge in the system is decreasing successively. The spectra show one set of resonances, indicated by dashed arrows, whose position depends only weakly on  $V_{M+R}$ . These are attributed to the normal EMP's at the outer edge of the sample [5,16]. In addition to the EMP's there is another set of resonances of smaller amplitude, indicated by solid arrows, whose position depends strongly on  $V_{M+R}$  and therefore on the density difference  $\delta n =$

$n_1 - n_2$ . These are assigned to fundamental and harmonic IEMP resonances [17].

The resonant frequencies are plotted in Fig. 2 as a function of  $V_{M+R}$ . For  $V_{M+R} \geq V_t$  ( $V_t = -10$  V in Fig. 2),  $\delta n$  is proportional to  $V_{M+R}$ . In this regime, the frequencies of the IEMP's are proportional to  $|\delta n|$ . The magnetic field dependence of the IEMP resonant frequency for a positive and a negative density step  $\delta n$  is shown in Fig. 3. In both cases the resonant frequency of the IEMP is inversely proportional to the magnetic field. By exciting on electrode 1 and measuring the phase of the signal at resonance on electrodes 3 to 7 the direction of propagation of the IEMP is obtained. The insets of Fig. 3 indicate that for positive  $\delta n$  the direction of propagation for IEMP's and EMP's is the same whereas for negative  $\delta n$  the travel in opposite directions.

The theoretical dispersion relation for IEMP's is similar to that of the EMP's in the high-field limit. The density  $n$ , however, has to be replaced by the density difference  $\delta n$  between densities of the adjacent regions so that it becomes [12]

$$\omega_- = \frac{\alpha |\delta n| e}{\epsilon_r \epsilon_0 B} q. \quad (1)$$

Here,  $\epsilon_r \epsilon_0$  is the dielectric permittivity of liquid helium,  $e$  is the electronic charge, and  $\alpha$  is a parameter which depends on the shape of the density profile, screening of electrons by the electrodes, and possibly logarithmically on the conductivity [6]. The wave vector  $q$  for the resonances is equal to an integer times  $2\pi/P$ ,  $P$  now being the circumference taken at the location where the density average  $(n_1 + n_2)/2$ . Data for EMP's for electrons on helium in the high-field limit are in good agreement with Eq. (1) when  $\delta n$  is replaced by  $n$  [5,16] and the experimental value of  $\alpha$  is about 0.4. The data for IEMP's shown in Figs. 2 and 3, at least for not too negative  $V_{M+R}$ , are in good agreement with Eq. (1) assuming a constant  $\alpha$ .

When  $V_{M+R} < V_t$ ,  $n_1 = 0$  so that  $\delta n = -n_2$ , independent of  $V_{M+R}$  a ring is formed. In this special case, magnetoplasmons propagate at the outer and inner edges [18]. The increase in frequency with  $V_{M+R}$  decreasing below -10 down to -20 V of the inner edge resonance, which is not caused by a change in  $\delta n$ , cannot be explained by the change of the inner radius of the ring [see Fig. 1(a)], which would even lead to a decrease in frequency. Assuming Eq. (1) still to be valid, the increase therefore must be attributed to the change in  $\alpha$ .

Recently, there has been a lot of theoretical [19] and experimental [20] interest in multiply connected 2DES's like antidots or rings. It has been shown for the 2DES in semiconductor structures at frequencies of the order of  $\omega_p$  and  $\omega_c$  that magnetoplasmons also propagate at an inner edge [20]. In the high-field limit the behavior of these magnetoplasmons is qualitatively the same as of those at the outer edge [19]. This is confirmed by the present experiments.

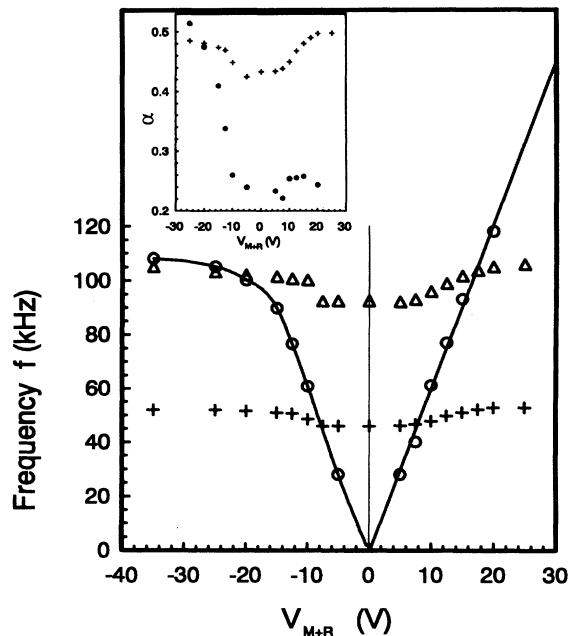


FIG. 2. Frequencies of the EMP and IEMP resonances as a function of  $V_{M+R}$  at  $T = 0.5$  K and  $B = 2.3$  T: (+) fundamental EMP resonance (outer edge), ( $\Delta$ ) second harmonic EMP resonance (outer edge), ( $\circ$ ) fundamental IEMP resonance (inner edge). Inset:  $\alpha$  as a function of  $V_{M+R}$ : (+) outer edge; ( $\bullet$ ) inner edge.

The inset of Fig. 2 shows the values of  $\alpha$  for the fundamental EMP and IEMP resonances as a function of  $V_{M+R}$  assuming that the dispersion relation is given by Eq. (1), with  $n$  and  $\delta n$ , respectively. For the IEMP resonance with  $V_{M+R} > V_t$ ,  $\alpha$  is roughly constant and about a factor of 2 smaller than  $\alpha$  for the EMP, which is probably associated with the shape of the profile. This confirms that the resonant frequency is insensitive to the sign of  $\delta n$ . When a ring is formed for  $V_{M+R} < V_t$ , the inner edge becomes sharper resulting in a rapid increase of  $\alpha$ , which, for  $V_{M+R} \ll V_t$ , exceeds even the value for the EMP resonance. Note that the linewidth of the IEMP decreases with decreasing  $\delta n$ . Whether this is caused by the changes in the profile is not yet clear.

It is surprising that the frequencies of the fundamental and second harmonic of the outer EMP resonance increase at large  $|V_{M+R}|$  as shown in Fig. 2, since neither  $n_2$  nor the shape of the profile or the radius change for the outer edge. Apparently, the presence of the IEMP has an influence on the EMP. Possibly, this is because the spatial extent of the EMP and/or the IEMP (expected to be in the order of  $d$ ) is comparable to or larger than their separation so that they interact. A dramatic change in character of an EMP propagating around a single narrow wire was observed when its width becomes comparable to the width of the EMP [10].

The influence of Coulomb crystallization [21] on EMP's has not been observed so far, in agreement with

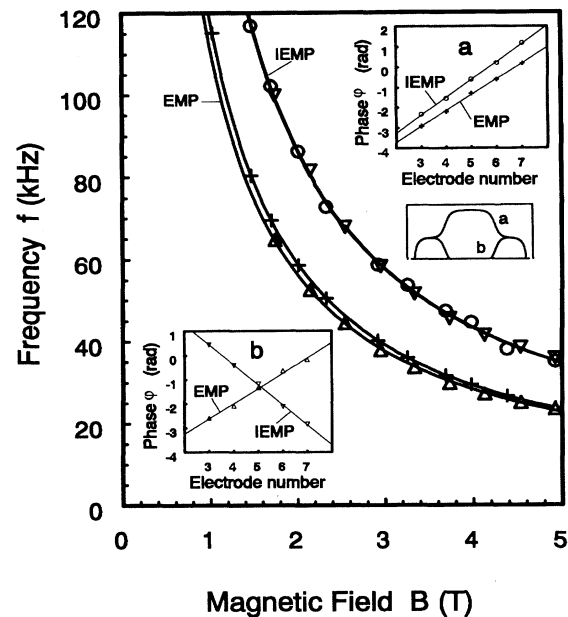


FIG. 3. Frequencies of the EMP (outer edge) and IEMP (inner edge) resonances for  $V_{M+R} = -12.5$  V ( $\Delta$ ,  $\nabla$ ) and for  $V_{M+R} = 12.5$  V (+,  $\circ$ ) as a function of magnetic field at  $T = 0.5$  K. The solid lines are  $1/B$  fits to the data. Insets: phase of the EMP and IEMP resonances as a function of electrode number for (a)  $V_{M+R} = 12.5$  V (+ EMP,  $\circ$  IEMP) and (b)  $V_{M+R} = -12.5$  V ( $\Delta$  EMP,  $\nabla$  IEMP),  $B = 1.8$  T,  $T = 0.5$  K. Third inset shows the corresponding density profiles [see also Fig. 1(a)].

earlier reports [4]. A complication is that the density involved is not well determined since the EMP propagates in the edge region of vanishing density. The IEMP could be sensitive to crystallization since both sides of the boundary can crystallize. Preliminary measurements suggest that there is a marked increase in the IEMP linewidth on cooling below the melting temperature. IEMP's therefore might become an interesting tool for studying Wigner crystallization in a magnetic field.

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