Diamagnetic Response due to Localization in Chains of Connected Mesoscopic Rings

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(Received 6 September 1994)

A dynamic response to a magnetic field of a chain of connected mesoscopic rings is considered. We show that the low frequency behavior corresponds to localization of the electrons along the chain and to diamagnetic dynamic currents inside rings. The magnetization density due to these currents does not vanish even in the limit of the infinitely long chain of strongly connected rings showing that this is a macroscopic effect. Being of a dynamic origin the currents can be destroyed by inelastic scattering but we find that their time of persistence is at low enough temperatures macroscopically large. Thus, this may yield a new contactless method of studying the localization.

PACS numbers: 72.10.Bg, 05.30.Fk, 71.25.Mg, 75.20.En

The recent measurement of the magnetization of an array of disconnected Cu rings [1] constituted the first detection of a thermodynamic mesoscopic effect and revealed its dependence on the statistical ensemble. Considerable effort was invested in a theory of the observed effect in the diffusive regime where the circumference of a ring L is larger than the elastic mean free path l. Averaging over disorder at fixed Fermi energy gives an amplitude for the persistent current which is exponentially small in L/l [2], and it was suggested that a larger amplitude can be due to the fixed number of particles in the rings [3]. However, averaging over impurities in the canonical ensemble can analytically only be done by a perturbation theory in fluctuations of the chemical potential [3].

Alternatively, it was proposed that the averaged dynamic response to the magnetic field is less sensitive to the statistical ensemble [4]. Levels in a closed ring as functions of the external magnetic field do not intersect each other and, before the averaging, the dynamic response must coincide in the limit $\omega \to 0$, where ω is the frequency, with the corresponding thermodynamic derivative. Therefore, in this case, one can hope that by calculating the dynamic response with a fixed chemical potential at $T = 0$ K, as was done in [4], one obtains a reasonable description of the thermodynamic currents. Both methods give the same order of magnitude of the persistent current in the diffusive regime [4] and the same result in the quasiballistic regime [5]. Although, possibly, not being exact, the dynamic approach can serve as a reasonable approximation scheme for the calculation of thermodynamic quantities. In Ref. [6] the possibility of a difference between the dynamic and thermodynamic responses arising from the averaging over the impurities was traced by considering a special experimental setup.

In the system considered in this Letter the situation is completely different. While level intersections are not possible in an isolated ring, they can occur in a chain of connected rings in the limit of $L_x \to \infty$, where L_x is the length of the chain. This is due to the fact that all wave functions in a disordered quasi-one-dimensional chain are localized and parts of the chain located far from each other do not interfere. This leads to an exponential suppression of the level repulsion. This is similar to the situation in a disordered one-dimensional chain [7]. The resulting level intersections in the system cause the dynamic response now to be completely different from the thermodynamic one before averaging over impurities. In fact, in the infinite system one cannot expect any finite thermodynamic magnetization density except the Landau diamagnetism which is small compared to the effect considered here.

To understand better the nature of the dynamic response we want to study in the diffusive regime, let us consider first the corresponding clean limit. The chain of N_r rings is in the ballistic regime $L < l$ an ideal diamagnet, and its magnetization due to an applied flux $\phi + \phi_\omega$ through the rings, where ϕ_ω is a small time dependent flux, is given by

$$M_{\rm dyn} = N_r V K_d \phi_\omega / 4\pi c^2, \qquad K_d = -ne^2/m. \tag{1}$$

In Eq. (1) m is the electron mass, e the electron charge, n the electron density, c the light velocity, and V the volume of a single ring. This magnetization does not correspond to the free energy minimum and due to inelastic processes decays to the smaller thermodynamic value which can exceed the Landau diamagnetism, however [8].

The impurity averaged response to the time dependent flux ϕ_ω is calculated using the Kubo formula in the form given in Ref. [4]. The frequency dependent magnetization M_ω is then obtained from the response function $K(\omega)$ as

$$M_\omega = N_r V \mathrm{Re}[K(\omega)\phi_\omega]/4\pi c^2. \tag{2}$$

It will be shown below that in the diffusive regime in the absence of inelastic processes the dissipative part of the magnetization vanishes in the zero frequency limit but the real part of the response can remain finite. In the limit of small frequency the diagrammatic technique leads

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to divergencies and one should use the supersymmetry method [9].

The responses both along the chain of rings with cross section $S = V/L$ as well as in the azimuthal direction of the rings can be reduced to a calculation of functional integrals over supermatrices Q. Here, we want to calculate the response in azimuthal direction of an ensemble of chains of rings whose length exceeds the localization length $L_x \gg L_c \gg L$. The dynamic response of disconnected mesoscopic rings was calculated in Ref. [4] using the zero-dimensional version of the σ model. In the general case of an arbitrary hopping amplitude between the rings one has to add in the free energy functional a term describing the coupling between the rings. Such a term is analogous to the Josephson coupling in the theory of superconductivity [10,11]. Changing the probability of tunneling from ring to ring we can describe thus the crossover from the case of isolated rings to the homogeneously weakly disordered chain of rings. Then, the free energy $F(Q)$ is

$$F[Q] = -\sum_{ij} J_{ij}\mathrm{STr}(Q_iQ_j) + \sum_i F_0(Q_i),$$
$$F_0(Q_i) = \frac{\pi\nu V}{8}\mathrm{STr}\left[-\mathcal{D}_0\left(\frac{e}{c}\mathbf{A}[Q_i,\tau_3]\right)^2 + 2i\omega\Lambda Q_i\right], \tag{3}$$

where i and j enumerate the rings in the chain, D_0 is the classical diffusion coefficient, $\mathbf{A}$ is the vector potential due to the flux through the rings, and $Q_i^2 = 1$. The notation STr stands for the supertrace. The definition of the matrices τ_3 and Λ as well as details of the supersymmetry method can be found in previous works [4,9]. The function $F_0(Q_i)$ in Eqs. (3) stands for electron motion inside the rings, while the first term in Eq. (3) describes the coupling between the rings due to electron hopping. The parameter J is related to the single electron hopping amplitude t as $J = \pi^2V^2\nu^2t^2$ if the momentum is not conserved by the hopping process. Only nearest neighbors are coupled so that $J_{ij} = J$ for the neighbors, and $J_{ij} = 0$ otherwise. The limit $J = 0$ corresponds to the chain of disconnected rings. In the limit $J \gg 1$ the model on the lattice, Eq. (3), becomes the continuous one-dimensional σ model [9]. Then, the coupling constant J is related to the classical diffusion constant D_0 as $J = \pi\nu VD_0/8L^2$.

Because of the one-dimensionality of the model Eq. (3) one can use the transfer matrix technique. Corresponding partial differential equations have been written in Refs. [9,12] for the continuous 1D σ model. For the model on the lattice, Eq. (3), the corresponding recurrence equation is an integral one [10,11]. Repeating the main steps of these works we reduce calculation of integrals over Q_i for all sites i to one integral over Q,

$$\langle\cdots\rangle_Q = \int(\cdots)\Psi^2(Q)e^{(-F_0[Q])}dQ\,. \tag{4}$$

The function $\Psi(Q)$ in Eq. (4) satisfies the equation

$$\Psi(Q) = \int \exp(2J\mathrm{STr}QQ')e^{(-F_0[Q'])}\Psi(Q')dQ'. \tag{5}$$

The solution of Eq. (5) depends on the vector potential $\mathbf{A}$ entering F_0, Eq. (3). In principle, it can be solved, at least numerically, for arbitrary magnetic fields using the parametrization proposed recently [13], but we will present results only in the limits of zero and high enough magnetic fields. These limits correspond to the orthogonal and unitary ensembles. In both cases one can omit the first term in Eq. (3) because in the unitary case the supermatrix Q commutes with the matrix τ_3 since the time reversal symmetry is broken [9]. Because of the symmetry of the free energy the function $\Psi(Q)$ depends only on one compact real integration variable λ and two noncompact integration variables $\lambda_{1,2}$ parametrizing the supermatrix Q [9]. Therefore, in the functional integral over Q one can integrate first over all other variables of the representation of Q introduced in Ref. [9].

For the unitary ensemble, the time reversal symmetry is broken and there is only one noncompact integration variable λ_1. Corresponding calculations are not very different from those performed in Ref. [4] and we obtain

$$K^u(\omega) = -i\omega\sigma_0\left\{1 + \frac{1}{2}\int_{-1}^{1}\int_{1}^{\infty} e^{(i\pi\omega/\Delta-\delta)(\lambda_1-\lambda)} \times \Psi^2(\lambda,\lambda_1)d\lambda\, d\lambda_1\right\}, \tag{6}$$

where $K^u(\omega)$ is the response along the circumference of the rings in the unitary ensemble, $\Delta = (\nu V)^{-1}$ is the mean level spacing in one ring in the chain, $\sigma_0 = e^2\tau n/m$ is the Drude conductivity, and $\delta \to 0$. There remains a finite response after ensemble averaging, if at low frequencies ω the second term in the brackets in Eq. (6) is proportional to $1/\omega$. In the limit $\omega \to 0$ solving Eq. (5) becomes more simple because the main contribution to the integral over λ_1 comes from $\lambda_1 \sim 1/\omega$. Then, the function $\Psi(Q)$ depends only on one variable $z = 2\omega\lambda_1$ and we obtain for the response $K(\omega)$

$$K^u(\omega \to 0) = -\sigma_0\Delta_{\mathrm{eff}}(J)/\pi\,, \tag{7}$$

where the effective mean level spacing Δ_{eff} is a nontrivial function of the coupling J between the rings,

$$P(J) = \Delta_{\mathrm{eff}}(J)/\Delta = \int_0^\infty \exp(-z)\Psi_J^2(z)dz\,. \tag{8}$$

This function gives the probability that an electron does not leave a ring forever and is known numerically for arbitrary J [11]. Here, we have to remark that the order in which the limits were taken, first taking the length of the chain L_x to infinity and then performing the zero-frequency limit, means that our results apply for chains of finite length only if the frequency ω is not smaller than the mean level spacing of the chain $\Delta_0 = 1/\nu SL_x$.

At high frequencies the second term in Eq. (6) is small and one obtains the classical response function $K(\omega) = -i\omega\sigma_0$. We see from Eqs. (6) and (7) that the characteristic frequency of the crossover from the quantum to the classical regime is of the order of $\Delta_{\rm eff}$.

In the limit $J = 0$ corresponding to disconnected rings, $\Psi = 1$ and $\Delta_{\rm eff}(0) = \Delta$. The magnetization of the chain of disconnected rings is then, using Eq. (2),

$$M^u_\omega(J = 0) = N_r V K_d \tau \Delta \phi_\omega / 4\pi^2 c^2. \quad (9)$$

In the opposite limit $J \gg 1$ the function $\Psi(z)$ is the solution of the differential equation $z d^2\Psi/dz^2 - 16J\Psi = 0$. Solving this equation and calculating the integral, Eq. (8), we find that there is still a finite probability $P(J) = 1/96J$ that an electron stays in one ring. This is a direct consequence of the localization of the electrons along the chain. In the unitary ensemble the localization length L_c was calculated in Ref. [9] and can be related to the coupling J as $L_c = 32JL$, where we included an additional factor 2 since the effective cross section of the chain of rings is $2S$. Note that with an effective length of a ring $L/2$ in the chain the inverse participation ratio becomes $\Gamma = 2/3L_c$ which is the well known result in the unitary ensemble. Here we obtain for the effective level spacing $\Delta_{\rm eff}(J \gg 1) = (3\nu S L_c)^{-1}$. Thus, we see that in the limit of large J the response $K^u(0)$, Eq. (7), looks as if the chain of N_r rings consisted effectively of $L_x/3L_c(J)$ rings. In this limit we obtain with Eq. (2) that the magnetization does not depend on the disorder:

$$M^u_\omega(J \gg 1) = [N_r V/(8\pi M_T)^2] K_d \phi_\omega / 4\pi c^2, \quad (10)$$

where $M_T = k_F^2 S/4\pi^2$ is the number of transverse channels in a single ring.

Decreasing the coupling J makes the localization length shorter which leads to a larger response. For intermediate values J, the localization is an amalgam of weak localization and the effect of the barriers between the rings and the dependence on disorder is nontrivial.

The response K, Eq. (7), describes the dynamic magnetization per unit length along the chain and therefore the total magnetization is proportional to the length of the chain. It has been mentioned that the corresponding thermodynamic response is small as well as the difference between the canonical and grand canonical ensembles. To make a more explicit estimate, one can use the expansion in terms of fluctuations of the chemical potential [3,14] and calculate the correlation functions entering the expansion with the supersymmetry technique. It turns out that the thermodynamic magnetization density of the chains calculated in this approximation vanishes for macroscopically long chains [15].

As we found above, the dynamic magnetization is due to the localization along the chain. As soon as the localization is lifted and electrons can travel through the chain, the magnetization should vanish. At temperatures $T > \Delta_{\rm eff}$ and at frequencies ω smaller than $t^{-1}(J)$, where $t(J)$ is a time due to inelastic processes, the dynamic magnetization decays due to a dc conductivity proportional to $t^{-1}(J)$, whereas at higher frequencies $t(J)$ has no influence on this magnetization. The simplest way to include the time $t(J)$ into formulas obtained above is to substitute $\omega \to \omega + it^{-1}(J)$ in all expressions for K/ω, where K can stand for the responses in both the longitudinal and azimuthal directions. (K/ω is proportional to the product of two Green functions, and it is this quantity that is calculated in the linear response theory.) With this substitution the magnetization would decay with time $t(J)$. Of course, this is a hypothesis and we suggest it to make a rough estimate only. Concerning the longitudinal conductivity in the regime of the localization, it can be different from zero due to hopping between the localization centers [16]. Explicit calculations for disordered chains [17] at not too low temperatures corresponding to $T > \Delta_{\rm eff}$ lead to the result $t(J) = \tau_{\rm ph}$, where $\tau_{\rm ph}$ is the electron-phonon scattering time. To the best of our knowledge nobody has performed analogous calculations for thick wires which are equivalent to the chain of the rings, but we hope that this estimate can give a reasonable estimate also for the case considered here. Then, the inverse decay time is proportional to T^3. Thus, at temperatures much below the Debye temperature ω_D but above the effective level spacing, $\omega_D \gg T > \Delta_{\rm eff}$, one can have $1/t(J) < \Delta_{\rm eff}$, and it should be possible to observe the large diamagnetic response.

At lower temperatures $T \ll \Delta_{\rm eff}$ the exponential Mott law, $t(J) \sim (1/\omega_D)\exp[4(3\Delta_{\rm eff}/T)^{1/2}]$, might serve as a lower limit for the decay time, since the time of equilibration of the whole system should greatly exceed this effective hopping time of an electron. Thus, it increases exponentially by decreasing the localization length.

Analogous calculations can be performed also for the orthogonal ensemble corresponding to the zero static component ϕ of the flux through a ring. In this case we obtain as in Ref. [4]

$$M^0_{\omega\to 0}(0) = 0. \quad (11)$$

Changing the static component ϕ of the flux we can have a crossover between Eqs. (7) and (11). The characteristic flux ϕ_c of this crossover is of the order of $\phi_0(\Delta_{\rm eff}/E_c)^{1/2}$, where $\phi_0 = 2\pi c/e$ and $E_c = \pi^2 D_0/L^2$ is the Thouless energy. The dependence of the response on the flux ϕ is periodic with the period $\phi_0/2$. We note that its flux average is finite and we expect a large diamagnetic response even in a quasi-one-dimensional conductor.

Adding magnetic or spin-orbit impurities changes Eqs. (7) and (11). The system with the magnetic impurities corresponds to the unitary ensemble. One can use as before Eq. (7) provided $\Delta_{\rm eff}(J)$ is substituted by $\Delta_{\rm eff}(2J)/2$. If the magnetic impurities are absent, spin-orbit ones lead to a different function $\tilde{\Delta}_{\rm eff}(J)$. However,

this difference is only numerical and does not change the sign of the response which is in all cases diamagnetic.

We remind one that for the isolated rings the possibility of the averaging with the fixed chemical potential is a certain assumption while for $J \neq 0$ one should consider the whole chain, and in the limit $L_x \to \infty$ the difference between the canonical and grand canonical ensembles vanishes. Taking into account charging effects one can have effectively isolated rings at finite J. Thus, the Coulomb repulsion could increase both the amplitude and the decay time of the dynamic magnetization. A detailed analysis of these effects will be the subject of further research.

In conclusion, we showed that a magnetic field applied perpendicular to a chain of disordered connected rings causes a macroscopic diamagnetic current. In the absence of inelastic scattering which can correspond to low temperatures this current can live for a very long time. The corresponding magnetization density remains finite even in the limit of a macroscopically long chain of perfectly connected rings. At the same time the longitudinal response along the chains shows a dielectric behavior usual for disordered quasi-one-dimensional wires. In fact, the shorter the localization length along the chain, the larger the current in the rings and the larger the relaxation time to its vanishing thermodynamic value. We suggest checking our theoretical results by measuring the dynamic magnetic response of connected mesoscopic rings in the diffusive regime or, alternatively, of linear antidot lattices where the mean free path due to elastic scattering is smaller than the distance between neighboring antidots. In order to get the full information about the interplay between localization and persisting currents in these samples one should do the measurements as a function of the following.

(a) Flux: In order to check the predicted flux periodicity $\phi_0/2$, and to get $\Delta_{\text{eff}}(J)\tau$ from the maximum response, Eqs. (2), (7), (9), and (10). Furthermore, one can check if the crossover is approximately at $\phi_0(\Delta_{\text{eff}}/E_c)^{1/2}$.

(b) Frequency: One should see a departure from linear decay with frequency at $\omega \approx \Delta_{\text{eff}}$, and one can thus check the value obtained from (a). Below this frequency one should see for temperatures $T < \Delta_{\text{eff}}$ a clear plateau and only at very low frequencies $\omega < 1/t(J)$ a further decay of the response. At higher temperatures $T > \Delta_{\text{eff}}$ instead of a clear plateau one may see only a bump whose onset gives again Δ_{eff} and whose vanishing at lower frequencies may provide information about $t(J)$ and thus about the relaxation mechanism.

(c) Temperature: This gives another estimate of Δ_{eff} and may provide information about the temperature dependence of the relaxation mechanism. Thus, if one knows the material parameters $n, \tau, \tau_\phi, M_T, S$ from independent measurements one should be able to unambiguously check the validity of our simple model and stimulate further numerical evaluation of the theory. In summary, the conditions necessary to observe the effect are $L_x \gg L_c(J) > L$, and the dephasing rate should not exceed the effective level spacing which gives for the dephasing length $l_\phi = \sqrt{D(J)\tau_\phi}$ the condition $l_\phi > \sqrt{L_c(J)L}$. For $J < 1$ this is the usual mesoscopic regime met in Ref. [1]. For $J \gg 1$ one may need to have phase coherence over several rings, since then $L_c = M_T l$. Thus we may hope that this yields a new contactless method of studying the localization.

Both of us are grateful to V. Falko for a critical reading of the manuscript and useful advice. S. K. thanks the Weizmann Institute, where this work was completed, for hospitality, and Alex Kamenev, Nathan Argaman, Peter Kopietz, Eugene Altshuler, Yoseph Imry, Alexander M. Finkel'stein, Yuval Gefen, and Daniel Braun for most useful discussions. S. K. acknowledges support from the Minerva Foundation.

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