Universal Scaling of the Magnetoconductance of Metallic Si:B

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Conductivity data for Si:B samples with dopant concentrations $1.01n_c < n < 1.22n_c$ at temperatures between 0.07 and 0.5 K in magnetic fields from 0 to 9.0 T collapse onto a single universal curve $\Delta\sigma = KT^{1/2}\mathcal{F}(H/T)$, the form expected for the magnetoconductance due to electron-electron interactions. This suggests that the metal-insulator transition is predominantly driven by electron correlations, and that localization, spin-flip scattering, and spin-orbit scattering are unimportant despite strong spin-orbit effects in Si:B.

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Localization and electron-electron interactions play essential roles in determining the behavior of doped semiconductors and other materials near the metal-insulator transition [1]. Despite more than a decade of theoretical and experimental work, however, their relative importance in driving the transition continues to be a subject of investigation and debate. Since a magnetic field is expected to cause different changes in the conductivity depending on whether they arise from localization or interactions, the magnetoconductance has often been used to attempt to separate these components and study them in detail.

Significant contributions due to localization have been found in many systems [2], where studies have yielded information regarding the dominant phase-breaking process. On the other hand, although a small component due to localization has been identified in the magnetoconductance of Si:P [3], early experiments at very low temperatures established that electron correlations are particularly important in this bellwether material [4].

We have made careful and extensive conductivity measurements of seven just-metallic Si:B samples at temperatures between 0.07 and 0.5 K in magnetic fields to 9 T. Our results can be interpreted within the framework of Fermi liquid theory and demonstrate that electron-electron interactions completely dominate the behavior of the magnetoconductance of this material. More specifically, we show that the magnetoconductance obeys a universal relation consistent with the general form expected for electron-electron interactions,

$$\Delta\sigma = \sigma(H,T) - \sigma(0,T) = KT^{1/2}\mathcal{F}(H/T), \quad (1)$$

where the function $\mathcal{F}(H/T)$ is approximately independent of dopant concentration n. We indicate further that the function $\mathcal{F}$ is consistent with that predicted by theory [1,5] when modified [6] to the case where total angular momentum $j = \frac{3}{2}$. Our results imply that localization, spin-orbit scattering, and spin-flip scattering all play unimportant roles in determining the behavior near the metal-insulator transition in Si:B.

Table I lists the seven nominally uncompensated Czochralski-grown metallic Si:B samples used in these studies, as well as their room temperature resistivities, resistance ratios $R_{4.2}/R_{300}$, boron dopant concentrations [7], and concentrations n/n_c relative to the (zero-field) critical concentration n_c; based on earlier studies, the critical concentration for the metal-insulator transition is 4.06×10^{18} cm^{-3} in zero field [8] and increases with magnetic field [9]. The samples were immersed directly in the ^{3}He-^{4}He mixture of an Oxford model 75 dilution refrigerator equipped with a 9-T superconducting magnet. Standard four-terminal low-frequency ac measurements were taken using a model AVS-46 resistance bridge operating at 15 Hz. A Stanford Research Systems model SR850DSP lock-in amplifier (17.5 Hz excitation frequency) was used at the lowest temperatures where very small power inputs were required. Additional details can be found in reports of earlier work [10,11].

The conductivity is generally fitted by the form

$$\sigma(0,T) = \sigma(0,0) + mT^{1/2}, \quad (2)$$

where the second term on the right hand side is due to electron-electron interactions [12]. For sample A, closest to the transition, and for the most metallic sample G, Figs. 1(a) and 1(b) show, respectively, the conductivity

TABLE I. Sample designations, room temperature resistivities ρ_{300}, resistance ratios $R_{4.2}/R_{300}$, dopant concentrations n, and dopant concentrations n/n_c relative to the zero-field critical concentration, $n_c = 4.06 \times 10^{18}$ cm^{-3}.

Sample	ρ_{300} (Ω cm)	$R_{4.2}/R_{300}$	n (10^{18} cm^{-3})	n/n_c
A	0.0170	2.711	4.11	1.012
B	0.0167	2.466	4.20	1.034
C	0.0165	2.270	4.30	1.059
D	0.0162	2.110	4.38	1.079
E	0.0158	1.833	4.56	1.123
F	0.0154	1.609	4.75	1.170
G	0.0150	1.423	4.97	1.224

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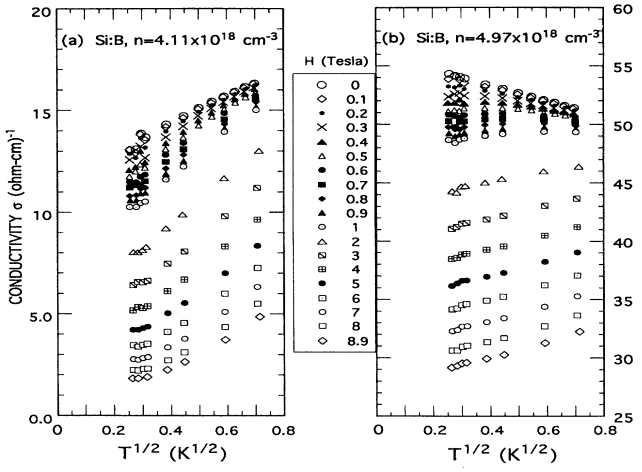

FIG. 1. Conductivity as a function of $T^{1/2}$ in zero field and 18 fixed magnetic fields between 0.1 and 8.9 T as labeled for (a) sample A closest to the metal-insulator transition ($n = 4.11 \times 10^{18}$ cm^{-3}); (b) sample G farthest from the transition ($n = 4.97 \times 10^{18}$ cm^{-3}).

as a function of $T^{1/2}$ in zero field and in 18 different fixed fields between 0.1 and 8.9 T. In the absence of a magnetic field, the slope m changes sign in silicon and germanium-based material from a positive value near the transition, as in sample A, to a negative value at higher dopant concentrations, as in sample G. A magnetic field increases the slope m in all cases. Thus, for samples far from the transition, it changes the sign of the slope m from negative to positive, as in sample G, while for samples close to the transition the field causes the positive slopes to become more positive, as in sample A. We note that sample A shown in Fig. 1(a) has a dopant concentration of 4.11×10^{18} cm^{-3}, quite close to the critical concentration for the metal-insulator transition $n_c = 4.06 \times 10^{18}$ cm^{-3}. In agreement with earlier results in Si:B [9,11], this sample enters the insulating phase at the higher magnetic fields, as evidenced here by negative zero-temperature intercepts and a distinct flattening of the curves at high fields below about 100 mK. Data at $H = 5$ T and higher were therefore not included for this sample in the scaling analysis discussed below. For the same reason, data at and above 8 T were omitted for sample B.

Earlier magnetoconductance measurements in Si:P [3,4] as well as Si:B [11] have indicated that, although localization plays a role, electron correlations account for most of the observed behavior. Electron-electron interactions are expected to reduce the conductivity in a magnetic field (positive magnetoresistance) through the Zeeman splitting of the spin-up and spin-down bands [1,5]. At a given temperature T, the change in conductivity due to a field H is given by

$$\Delta\sigma_I(H,T) = \sigma(H,T) - \sigma(0,T) = -0.77\alpha\gamma F_\sigma T^{1/2} g_3(h) \propto T^{1/2} g_3(h), \quad (3)$$

where $h = g\mu_B H/k_B T$, F_σ is an interaction parameter related to the Fermi liquid parameter F, γ depends on material properties such as valley degeneracy, mass anisotropy, and intervalley scattering [13], α is related to the diffusion constant D,

$$\alpha = (e^2/\hbar)(1.3/4\pi^2)(k_B/2\hbar D)^{1/2}, \quad (4)$$

and the function g_3 is given by an integral

$$g_3(h) = \int_0^\infty d\Omega\,[\Omega/(e^\Omega - 1)] \times \{(\Omega + h)^{1/2} + (|\Omega - h|)^{1/2} - 2\Omega^{1/2}\}. \quad (5)$$

Guided by the functional form expected for electron-electron interactions, we plotted $\Delta\sigma(H,T)/T^{1/2}$ as a function of H/T for all seven samples in Fig. 2. Despite complex field and temperature dependences, the data for each sample in *all* fields up to 9 T and temperatures between 0.07 and 0.5 K collapse onto a single universal curve. The scaled curves essentially coincide for the three samples farthest from the transition. As the dopant concentration is decreased toward the critical concentration, there is a reduction in amplitude but the shapes of the curves remain essentially unaltered. This is demonstrated in Fig. 3, where appropriate scale factors relative to the most concentrated samples have been applied to make the curves superimpose. The prefactor, shown as a function of dopant concentration in the inset to Fig. 3, increases as n increases above n_c. According to theory [1,5], this prefactor and the slope m of Eq. (2) are both linear functions of the interaction parameter F_σ, and it is thus gratifying to note that they display very similar behavior as a function of concentration (see Fig. 7 of Ref. [10]).

That the data collapses onto a single, universal curve suggests that the measured magnetoconductance derives entirely from electron-electron interactions. The

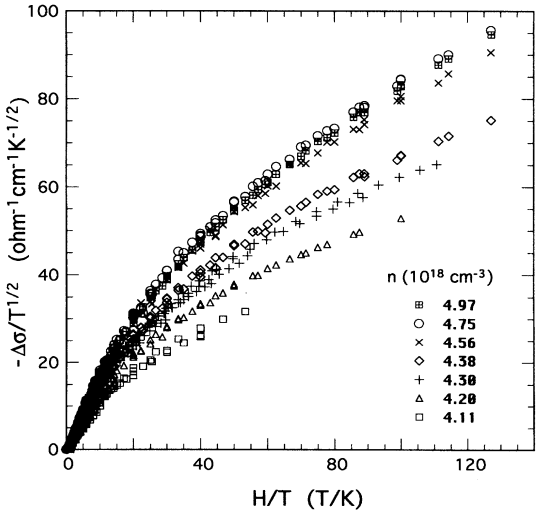

FIG. 2. $-\Delta\sigma/T^{1/2}$ vs H/T for samples A through G, as labeled.

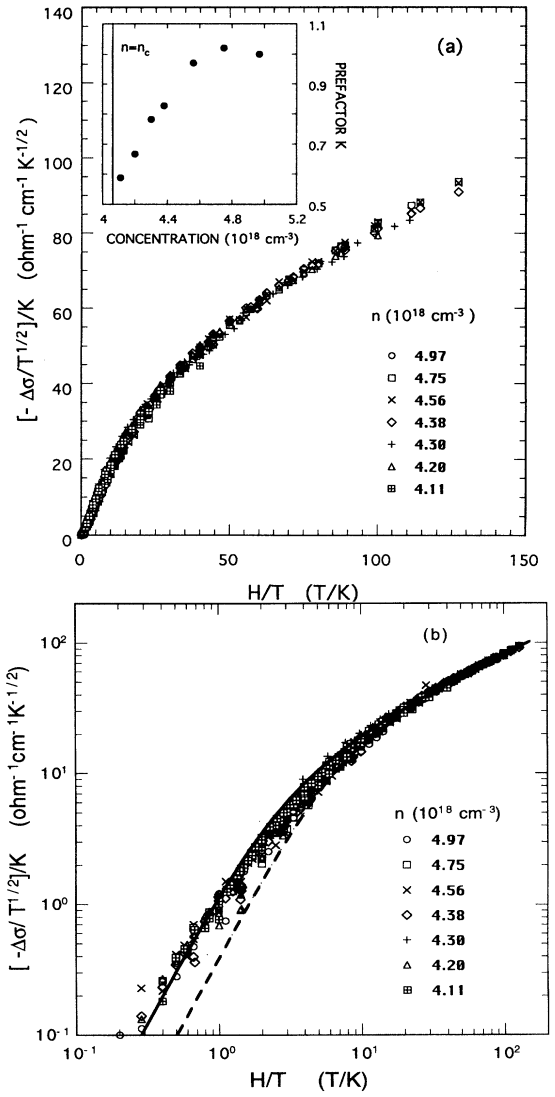

FIG. 3. $[-\Delta\sigma/T^{1/2}]/K$ vs H/T for samples A through G: (a) on a linear scale; (b) on a log-log scale. Here K is chosen to shift each curve relative to sample G. The theoretical function $g_3(g\mu_B H/k_B T)$ of Eq. (5) is shown for $j = \frac{1}{2}$ and $j = \frac{3}{2}$ by the dashed and solid lines, respectively. The inset shows the prefactor K as a function of dopant concentration; the vertical line denotes the critical concentration $n_c = 4.06 \times 10^{18}$ cm^{-3}.

crossover from the low-field behavior $\mathcal{F}(H/T) \propto (H/T)^2$ to the high-field behavior $\mathcal{F}(H/T) \propto (H/T)^{1/2}$ occurs at $k_B T \approx g\mu_B H$, indicating that Zeeman splitting indeed determines the magnetoconductance. Independent of any detailed theory, this strongly supports the contention that interactions are dominant. We note that localization is expected to contribute a term that depends on DH/T_p, with a diffusion constant D that should decrease rapidly as the transition is approached [1]. In a similar way, Cooper (particle-particle) channel contributions (which arise also from orbital effects) are expected to depend on DH/T [14]. No admixture of either term is apparent, suggesting that they both play negligible roles. We note, however, that the form of the function found experimentally and shown in Fig. 3 is not consistent with the simple theory for electron-electron interactions of Lee and Ramakrishnan [1,5]. The dashed line of Fig. 3 gives the result of a numerical integration of the function g_3 of Eq. (5) using a g value of ≈ 1.2 appropriate for Si:B; it is clear that the data do not agree with this prediction [15]. Varying the g value causes a simple horizontal shift in the theoretical curve. Since the *shapes* of the theoretical and experimental functions are different, no choice of g value can yield agreement.

We suggest that these discrepancies reflect the limitations of the existing theory, which applies to spin-$\frac{1}{2}$ electrons in the absence of spin-orbit coupling. It is well known that in the case of p-type materials such as Si:B, the acceptor states are characterized by a total angular momentum $j = \frac{3}{2}$ due to strong spin-orbit coupling associated with the valence band. In the range of dopant concentrations of our samples, transport takes place via acceptors in the impurity band, and it is thus plausible to consider the carriers to be particles with $j = \frac{3}{2}$. One can show [6] that the total angular momentum is conserved in the transport process at these relatively low dopant densities. This could explain the absence of spin-orbit *scattering* in the presence of strong spin-orbit *coupling*.

With this in mind, we have generalized [6] the standard theory for interactions to the case of $j = \frac{3}{2}$ particles. Most of the qualitative predictions as well as the general form of Eqs. (3)–(5) remain valid. We find, however, that the scaling function $g_3(h)$ of Lee and Ramakrishnan [5] is replaced by

$$g'(h) = a_1 g_3(h) + a_2 g_3(2h) + a_3 g_3(3h), \qquad (6)$$

where a_1, a_2, and a_3 are three independent parameters that depend on the interaction amplitudes in the three magnetic interaction channels. More precisely, the total angular momentum of the particle and the hole $J = j_1 + j_2$ is conserved, corresponding to the three channels $M_J = \pm 1, \pm 2, \pm 3$. Making even the simplest assumption $a_1 = a_2 = a_3$ yields an excellent fit, shown by the solid line in Fig. 3(b); clearly, a better fit can be obtained by allowing [6] independent variation of the a's.

The contribution due to interactions is expected only in the absence of spin-flip and/or spin-orbit scattering. If either is present one must replace $g\mu_B H$ in Eqs. (3)–(5) by $g\mu_B H + \hbar/\tau_s$, where τ_s is the appropriate scattering time. This would weaken the magnetic field dependence and, more importantly, would violate the scaling behavior expected in the absence of such perturbations [1]. Thus, the very existence of H/T scaling suggests that spin-orbit scattering has little effect in the range of temperatures and magnetic fields of our experiments, despite the large spin-orbit coupling that characterizes the valence band of silicon. We note further that, although

Eqs. (3)–(6) were obtained in the limit of weak disorder (corresponding to high dopant concentrations), one can argue on quite general grounds within Fermi liquid theory [16] that they should have the same form at low temperatures throughout the metallic phase, including very near the transition. The universal behavior found in our experiments over an appreciable range of concentrations indicates that Fermi liquid theory gives a good description of the transport behavior of these systems.

In summary, the conductivity of Si:B measured at temperatures between 0.07 and 4.2 K in magnetic fields from 0 to 9.0 T collapses onto a single universal curve given by Eq. (1), as demonstrated in Fig. 3. Interpreted within Fermi liquid theory, this suggests that the metal-insulator transition is predominantly driven by electron correlations, and that localization, spin-flip scattering, and spin-orbit scattering are unimportant despite strong spin-orbit effects in Si:B. It is important to note that, whether or not the scaling found experimentally agrees with current theory, its simplicity as well as its validity over a broad range of temperature and magnetic field merits theoretical attention and investigation.

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