Pressure Dependence of the Superfluid Fraction in ^{3}He-A_1

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The superfluid fraction of ^{3}He-A_1 was determined in the Ginzburg-Landau (GL) region as a function of pressure between 10 and 30 bars from the measured spin-entropy wave velocity. The pressure dependence of the parameter β_{24}, proportional to the fourth order coefficients of GL free energy expansion, was measured for the first time. At low pressures the parameter approaches the weak coupling limit in agreement with the theory of Sauls and Serene. The extracted strong coupling corrections to β_{24} and β_5 at higher pressures are also consistent with the theory.

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Close to the transition temperature, the Ginzburg-Landau expression for the free energy of the superfluid ^{3}He can be written in terms of the energy gap Δ, the five fourth order invariants of the order parameter, and their associated expansion coefficients β_i [1,2]. Though the free energy expansion coefficients are very important in understanding the superfluid properties in the Ginzburg-Landau region, surprisingly very little experimental information has been available to determine or constrain their values. Based on the available experimental data, some unexpected values of expansion coefficients were obtained [3,4] in disagreement with existing theoretical predictions [5]. Since the values of the expansion coefficients restrict allowed superfluid states [6], questions have been raised on the validity of the conventional identification of the superfluid ^{3}He-A phase as the Anderson-Brinkman-Morel (or axial) state [7]. Clear importance of the question has motivated interests in the expansion coefficients. Evidence in support of the theory of β coefficients [5] and the conventional identification have been reported recently on NMR experiments in the superfluid ^{3}He-B and A phases in [8] and [9], respectively. We present here the results of spin-entropy wave experiments in the superfluid A_1 phase to measure the superfluid fraction. The measurement of the superfluid fraction provides new and additional constraints on the free energy expansion coefficients.

Consideration of the free energy expansion coefficients within the BCS weak coupling limit implies that the most stable state of the order parameter superfluid ^{3}He would be the Balian-Werthamer B state contrary to the observation of the A phase at high pressures [1]. Intense theoretical efforts have been made to calculate the "strong coupling" corrections to the weak coupling limit to account for the discrepancy [2]. A comprehensive calculation of the strong coupling corrections as well as Landau parameters and normal-state transport coefficients has been made using optimized quasiparticle scattering amplitudes [5].

While theoretical calculations of each of the expansion coefficients have been made, their measurements are difficult because only a combination of coefficients can be related to a measurable quantity. Of particular importance is the combination β_{24} ($\equiv \beta_2 + \beta_4$) which sets the magnitude of Δ in the A_1 phase and is simply related to the measured superfluid fraction (see below). Also of importance to this Letter is the measurement of the A_1 phase diagram (i.e., the transition temperatures T_{c1} and T_{c2} as a function of applied field H) which gives

$$\frac{dT_{c1}/dH}{dT_{c2}/dH} = \frac{\beta_5}{\beta_{245}}, \tag{1}$$

where $\beta_{245} \equiv \beta_2 + \beta_4 + \beta_5$ [2,10]. The measurements of the phase diagram of the A_1 phase have been carried out by the groups at USC [11] and Cornell [12]. The USC results give precision measurements over a wide pressure range. The phase diagram provides fixed temperature points extremely useful to research at ultralow temperatures in high magnetic fields. The doubt in identifying the A phase as an axial phase is in part based on the A_1 phase diagram measurement. It would, therefore, be important to provide a simple consistency check of the phase diagram in terms of the expansion coefficients. The present measurement of the superfluid fraction in the A_1 phase is combined with the phase diagram to determine β_{24} and β_5.

To see the relation between the spin-entropy wave velocity and β coefficients, recall that the velocity is given to a very good approximation by $\mathbf{C}_{se}^2 = (\rho_\mathbf{s}/\rho_\mathbf{n})(\rho/\chi) \times (\hbar\gamma/2m)^2$, where $\rho_\mathbf{s}/\rho_\mathbf{n}$ is a tensor quantity of the superfluid to normal component density ratio, ρ the total density, χ the magnetic susceptibility, γ the gyromagnetic ratio, and m the mass of ^{3}He. In the Ginzburg-Landau regime of the A_1 phase, the superfluid fraction [($\perp$ $\|$) refers to the sound propagation vector $\mathbf{q}$ being perpendicular or parallel to the anisotropy vector $\hat{\mathbf{l}}$] is given by [2]

$$\frac{\rho^0_{s\perp\|}}{\rho} = \frac{\rho_{s\perp\|}}{\rho}\,\frac{1 + (F_1^s/3)}{1 + (F_1^s/3)\rho_{s\perp\|}/\rho} = \frac{R_{\perp\|}}{\beta_{24}}\left(1 - \frac{T}{T_{c1}}\right), \tag{2}$$

where $\rho^0_{s\perp\|}/\rho$ is the "bare superfluid fraction" stripped of Fermi liquid corrections, $R_\perp = 4$, and $R_\| = 2$. The quantity $1 + F_1^s/3$ is the ratio of effective mass to atomic mass

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of ^{3}He, and it accounts for Fermi liquid backflow effects by including the Landau phenomenological parameter F_1^s [13]. The factor $R_{\perp\parallel}/\beta_{24}$ is referred to as the "bare slope." The spin-entropy wave velocity thus gives a simple and direct measure of β_{24}.

Since the spin-entropy wave propagation occurs in bulk liquid, the present superfluid density measurement is free of size effects and tortuosity corrections often associated with fourth sound techniques. No extrapolation of data close to T_c is needed as the A_1 phase occupies the Ginzburg-Landau territory near T_c. The value of $1 - T/T_{c1}$ in the present experiment is at most 0.045.

The spin-entropy wave propagation measurement apparatus is identical to the one described in our previous report [14]. Oscillating superleak membrane transducers are used to drive and detect the spin-entropy plane wave resonance ($\mathbf{q} \parallel$ axis) in a cylindrical chamber (radius = 3.75 mm, length = 12.5 mm). The frequency response of a particular mode is monitored as the cell cools down or warms up slowly. Least-squares fits to the frequency response are carried out to determine the peak amplitude and the resonance frequency ($\lesssim$500 Hz). The external magnetic field $\mathbf{H}$ ($\parallel \mathbf{q}$) for producing the A_1 phase is fixed at 2 T in this Letter. The liquid pressure is varied between 10 and 29 bars. A melting curve thermometer (attached to the demagnetization stage) and a vibrating wire (located within the resonator chamber) are used as temperature sensors.

To see in which direction $\hat{\mathbf{l}}$ points relative to $\mathbf{q}$ in our cell, recall that the order parameter of the superfluid A_1 phase may be defined by two sets of real orthogonal unit vectors in spin space ($\hat{\mathbf{d}}$, $\hat{\mathbf{e}}$), and in orbital space ($\hat{\mathbf{m}}$, $\hat{\mathbf{n}}$), where one usually writes $\hat{\mathbf{f}} = \hat{\mathbf{d}} \times \hat{\mathbf{e}}$ and $\hat{\mathbf{l}} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$. The free energy terms which depend on the orientation of $\hat{\mathbf{l}}$ arise from the magnetic, dipolar, bending, and flow energies. Consideration of minimizing the magnetic energy taking the anisotropy in magnetic susceptibility into account shows that $\hat{\mathbf{f}}$ is strongly polarized along the direction of the applied magnetic field $\mathbf{H}$ [15]. Now the orientation dependent part of the dipolar interaction energy between Cooper pairs may be written as $g(\hat{\mathbf{f}} \cdot \hat{\mathbf{l}})^2$, where g is a positive constant. The dipolar energy is minimized when $\hat{\mathbf{f}} \perp \hat{\mathbf{l}}$ and the $\hat{\mathbf{l}}$ vector is forced into the plane perpendicular to $\mathbf{H}$. Since $\mathbf{H} \parallel \mathbf{q}$ in our cell, the sound propagation vector $\mathbf{q}$ is almost certainly perpendicular to the $\hat{\mathbf{l}}$ vector [16]. The bending and flow energies are much smaller and do not contribute significantly to the orientation. It is concluded that the superfluid tensor component measured in our experiment is the perpendicular component $\rho_{s\perp}$ in Eq. (2), and the corresponding bare slope is $R_\perp/\beta_{24}$.

In addition to the above strong argument for $\mathbf{q} \perp \hat{\mathbf{l}}$ alignment based on energetics, there is also some experimental evidence to support the argument. The velocity and attenuation studies of ultrasound [17] in the A_1 phase in a cell similar to ours showed that the same $\hat{\mathbf{l}}$ vector texture was reproduced from run to run when $\mathbf{H} \parallel \mathbf{q}$. In our cell the measured superfluid fraction is reproducible upon not only warming and cooling through both T_{c1} and T_{c2} but up to room temperature. Introducing externally applied heat suddenly (inadvertently at times) into the resonator chamber (and thereby creating some irreproducible large supercurrents) does not change the measured superfluid fraction upon recovery close to the original temperature. All of these observations indicate that the $\hat{\mathbf{l}}$ texture in the cell is in the stable equilibrium state.

At the phase transition from the A_1 to the A_2 phase, the spin-entropy wave would be expected to disappear abruptly as the governing hydrodynamics changes from one phase to the other. The observations of rather gradual disappearance and anomalous damping of the spin-entropy wave in the vicinity of T_{c2} have been reported [14]. Similar anomalous behavior is observed at all pressures of our study. The pressure dependence of this effect will be reported elsewhere. While the nature of the increased damping is not yet understood, the temperature at which the wave propagation vanishes can be measured precisely. To illustrate this, the fitted peak amplitude A of the $n = 2$ mode at 16 bars near T_{c2} is shown as a function of the square of the resonant frequency (normalized to $n = 1$ mode) in the inset to Fig. 1. The temperature at which the amplitude extrapolates to zero is defined as T_{c2} [11]. The extrapolated resonant frequency of the nth mode [$\equiv f_n(T_{c2})$] at that temperature determines spin-entropy wave velocity at T_{c2} and hence the superfluid fraction via $(\rho_s/\rho_n)_{\perp T_{c2}} = \{[2f_n(T_{c2})L/n]/(\rho/\chi)\hbar\gamma/2m\}^2$, where L is

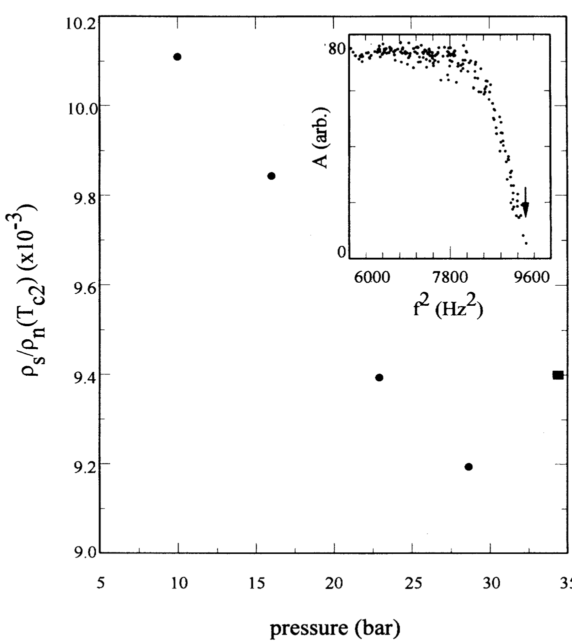

FIG. 1. Measured ratio of the superfluid to normal component densities at T_{c2} vs pressure. The dots are our data and the square at the melting pressure is from Ref. [19]. The inset shows the fitted resonance peak amplitude vs square of the resonance frequency. The arrow indicates the point where the spin-entropy wave propagation amplitude extrapolates to zero at T_{c2}.

the length of the resonator. The tabulation of ρ and χ compiled by Wheatley [18] was used.

It has been established [11] that the temperature width of the A_1 phase is linearly proportional to the externally applied field in the range of field and pressure of interest in this Letter. The measured superfluid fraction at T_{c2} in our cell is also linearly proportional to the applied field as demonstrated in Ref. [14]. The measured superfluid fraction at T_{c2} can then be written as $\{\rho_s/\rho_n\}_{\perp T_{c2}} = a(1 - T_{c2}/T_{c1})$, where a is a pressure dependent slope. The temperature dependence is in accord with Eq. (2). The measured density ratio, ρ_s/ρ_n at T_{c2}, is shown as a function of pressure (at 10.0, 16.0, 22.9, and 28.6 bars) in Fig. 1. Uncertainty arising from the extrapolation near T_{c2} is estimated to be less than 1.5%. The only other spin-entropy wave measurement was carried out at the melting pressure in 0.846 T [19]. Their data at T_{c2}, corrected for the difference in applied field, is shown in Fig. 1. Extrapolation of our data to the melting pressure is reasonably close (within 4%) to theirs.

Equation (2) is used to compute the bare slope from the measured superfluid fraction at T_{c2} and the reduced temperature $1 - T_{c2}/T_{c1}$ [11]. The result is shown in Fig. 2. As expected, there is a systematic increase in the slope as the strong coupling correction increases at higher pressures. The slope tends towards unity at low pressures as expected in the weak coupling limit with $R_\perp = 4$. The data of [19] are not included since the phase diagram measurement of [11] does not extend up to the melting pressure.

The pressure dependence of β_{24} derived from the bare slope in Fig. 2 is shown in Fig. 3. To our knowledge, this

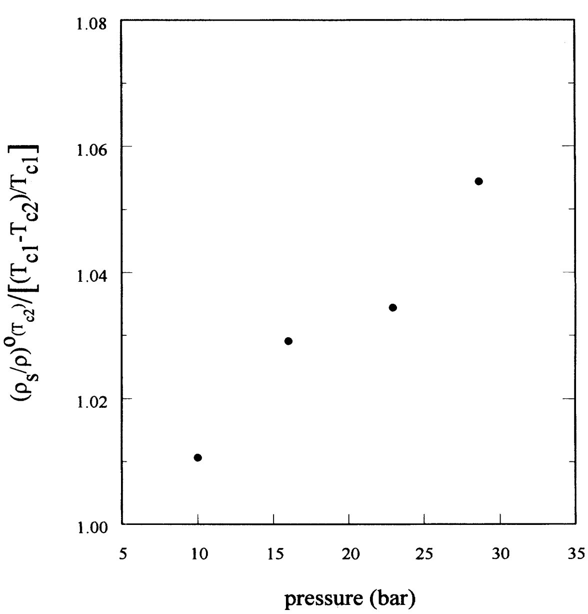

FIG. 2. The bare slope of the superfluid fraction vs pressure. The weak coupling limit of unity is approached at low pressures.

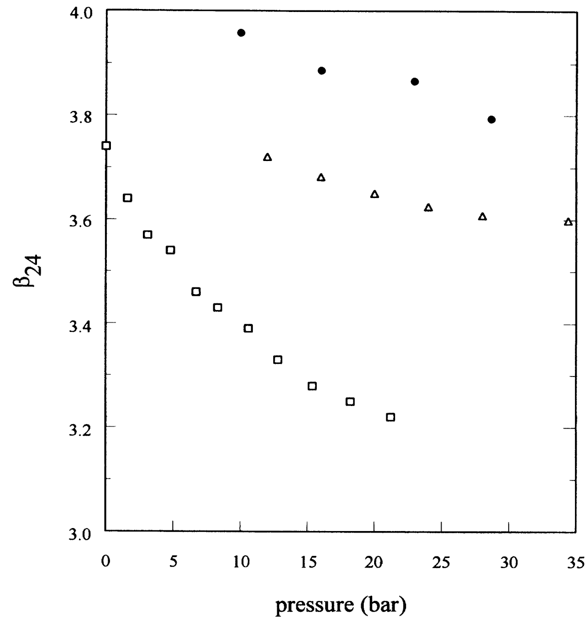

FIG. 3. Pressure dependence of a combination of Ginzburg-Landau expansion coefficients β_{24}. The dots are the present results. The triangles are the theoretical predictions of Sauls and Serene [5]. The squares are taken from Tang *et al.* [3].

is the first "direct" measurement of β_{24} in the pressure range. The theoretical calculation [5] is fairly close in magnitude and pressure dependence to the present result. The weak coupling limit of β_{24} is 4, and our results approach that limit at low pressures. There is significant departure from the data of [3] at high pressures.

Using the values of β_{24} in Fig. 3, the phase diagram measurement [11], and Eq. (1) we compute β_5. Its pressure dependence is shown in Fig. 4. The strong coupling correction to β_5 is found to be negative at all pressures in agreement with theory [5] but in disagreement with [3]. Since the strong coupling correction to β_4 is expected to be negative and the value of β_5 is found to be less than -2 in our analysis, the sign of β_{45} would then be negative. As discussed in [6], the negative sign of β_{45} implies that the order parameter of the A phase would be that of the axial state. This is consistent with the recent confirmation of the conventional axial state identification [9].

The measured jump in specific heat at the normal to B transition [13] is in agreement with the theoretical calculation on strong coupling corrections to β_{12} and β_{345} [5]. The measured A_1 phase diagram [11] is in reasonable agreement with the theoretical ratio β_5/β_{245}. Our present work on the superfluid fraction in the A_1 phase, when analyzed together with the A_1 phase diagram, gives values of β_{24} and β_5 which are also in reasonable agreement with the theory. The recent NMR measurement [8] is in qualitative agreement with β_{345} derived by the theory. Contrary to this general agreement, however, the analysis of the measured depression of the B to A transition

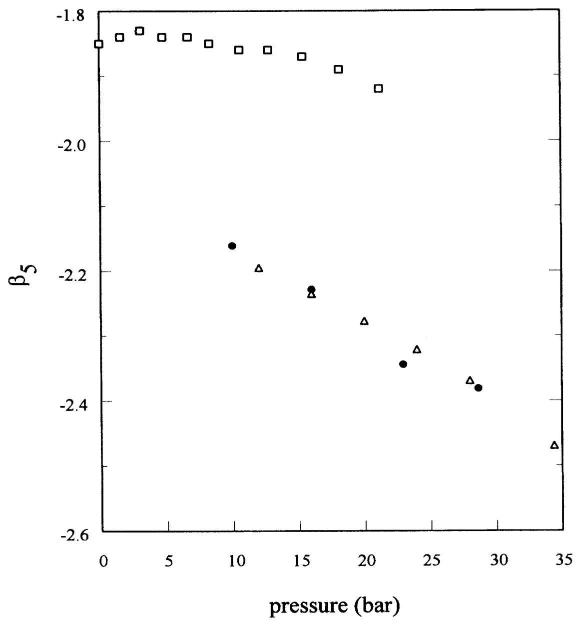

FIG. 4. Pressure dependence of β_5. The legend is the same as in Fig. 3.

temperature in magnetic field gives values of β coefficients which deviate significantly from the theory [3]. We do not have an explanation for this deviation. Our work tends to show that the deviation does not arise from the measured A_1 phase diagram.

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