Parametric X-Ray Radiation Observed in Diamond at Low Electron Energies

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Parametric x-ray radiation of type B has been produced with an electron beam of energies between 3.5 and 9.1 MeV from the superconducting accelerator S-DALINAC and diamond of thickness 55 μm. The photon intensity and its energy dependence were determined as a function of the tilt angle of the crystal. The intensity maximum varies with γ^2 and is about 3 orders of magnitude smaller than channeling radiation. Comparison with theoretical predictions exhibits very good agreement after taking into account effects caused by multiple scattering of the electrons in the crystal.

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Since the prediction about two decades ago [1–3] the interest in the origin of parametric x radiation (PXR) has particularly increased in recent years [4–20]. The highly monochromatic radiation, that can intuitively be understood as Bragg reflection of virtual photons associated with the incoming electron passing through crystal planes, is emitted into a small angular cone well separated from the electron beam and virtually free of bremsstrahlung background. The x-ray energy is nearly independent of the electron beam energy but exhibits a strong dependence from the angle between electron beam direction and crystal plane.

In commenting on the different expressions used for coherent x-ray radiation it should be noted that for the appearance of parametric x rays, in the case of the so-called *quasi-Čerenkov radiation*, two conditions, the Bragg condition and therefore the Čerenkov condition, have to be fulfilled [9,13]. This radiation is caused by electrons with energies of typically several hundred MeV. However, at electron energies of only a few MeV, strong x-ray radiation, which is not related to the Čerenkov effect, is observed [14]. This radiation might be termed "coherent x radiation" or simply PXR of type B. Recently Nitta [17] proposed to call this radiation *coherent polarization radiation* which actually describes best the physical process of the coherent emission of photons due to polarization of the crystal atoms. In this Letter specific features of coherent PXR of type B in the low energy domain are reported for the first time.

Several theoretical approaches to explain the origin and the characteristic features of the radiation have been published [1–9]. Experimentally the existence of PXR has been confirmed at various electron accelerators [10–12,16, 18–20] for electron energies between 15 and 1200 MeV. According to Baryshevskii and Feranchuk [2,9,13] PXR can be understood from the slowing down of the electron's electromagnetic wave in the crystal, which results in a shift of its phase velocity relative to the velocity of the relativistic particle with subsequent diffraction at the crystal planes. In this picture the radiation is understood as quasi-Čerenkov radiation. Recently some experiments [16, 18–20] have exhibited discrepancies from the theoretical predictions [2,13] of the angular distribution, intensity, and bandwidth of PXR. In a different explanation the slowing down of the electron polarizes the crystal atoms with subsequent emission of PXR under the Bragg condition [5,14]. This situation can be described within a perturbation theory approach with results that compare well with experimental data obtained at Kharkov [14], and it has recently been agreed upon that the perturbative approach may indeed give an adequate description for certain kinematical conditions [15]. Nitta [8] also found that this approach accounts for the experimental findings quite well. For better comparison between theory and experiment absolute data are thus needed for kinematical conditions that clearly meet the assumptions made in the various theoretical predictions.

It is the aim of the present investigation to test how far PXR(B) radiation can be described by the perturbative approach. For this purpose an electron beam energy $\gamma = E/mc^2 < 20$, with E being the total energy, and a large PXR observation angle have been selected such that only PXR type B radiation is detected and that contributions from Čerenkov and transition radiation as well as bremsstrahlung can be neglected.

For the perturbative approach the number of photons per steradian and per energy interval $d\omega$ produced by one electron on one elementary cell, in units of $\hbar = c = m = 1$, is given by [5,14]

$$\frac{d^2\sigma}{d\omega d\Omega} = 8\pi\alpha^3 \sum_{\vec{g}_\perp,\lambda} \frac{|SF|^2 f^2}{a^3} \frac{(\vec{e}_{\vec{k},\lambda}, \vec{e}_{\vec{k}_h,\lambda})^2 (\vec{e}_{\vec{k}_h,\lambda}, \vec{v})^2 \omega}{(\vec{n}, \vec{v})^2 (\vec{k}^2 - \vec{k}_h^2)^2} \delta(\xi), \tag{1}$$

where $\alpha = 1/137$, S is the structure factor, a is the lattice constant, $f^2 = \exp(-g^2u^2)$ the Debye Waller factor for a vibrational amplitude u of the crystal atoms perpendicular

to the crystal plane and a reciprocal lattice vector $\vec{g}$. The atomic form factor for isolated atoms is denoted by F. The wave vector of the real photon is $\vec{k}$, which is the wave vector of the detected radiation, $\vec{k}_h = \vec{k} + \vec{g}$ is the wave vector of the virtual photon, $\vec{e}_{\vec{k},\lambda}$ and $\vec{e}_{\vec{k}_h,\lambda}$ denote the polarization vector of the real and the virtual photons, respectively, and λ characterizes the polarization of the radiation. Furthermore, $\vec{v}$ stands for the velocity of the electrons passing through the crystal, $\vec{n}$ is the unit vector perpendicular to the crystal surface, and ω is the energy of the radiation. The argument of the delta function is defined by $\xi = [\omega - (\vec{k}_h, \vec{v})]/(\vec{n}, \vec{v})$, and $\vec{g}_\perp$ means that only reciprocal lattice vectors which are not parallel to $\vec{n}$ contribute.

In order to be directly comparable with experiment, (1) has to be integrated over the energy $d\omega$ to get the number of photons of energy $\omega_k = (\vec{g}\vec{v})/(1 - \hat{k}\vec{v})$ per steradian. The vector $\hat{k}$ is the unit vector in the direction of $\vec{k}$. The dielectric susceptibility $|\chi_0| = |4\pi\alpha FS/\omega| \ll 1/\gamma^2$ can be neglected at low energies. For diamond and the observed x-ray energies the factor $L_a[1 - \exp(-L/L_a)]$ used in Ref. [13] to take into account the absorption in the crystal simplifies to L since $L \ll L_a$, where L is the target thickness and L_a the absorption length. In our experiment the vectors $\vec{v}$, $\vec{g}$, and $\vec{k}$ are all lying in one plane. The scattering angles β_1 and β_2 of the electrons in the crystal are small compared to the observation angle $\theta = \angle(\vec{v}, \vec{k})$ and experimental tilt angle $\phi = \angle(\vec{v}, \vec{n})$. Here β_1 is the angle between the direction of the incident electron beam and the scattered electron velocity vector in the plane of $\vec{g}$ and $\vec{k}$, while β_2 is the angle perpendicular to this plane. After multiplication of the integrated formula (1) by the number L/a^3 of elementary cells per $\lambda\!\!\!^{-2}_C$, where $\lambda\!\!\!^{-}_C$ is the Compton wavelength of the electron, one gets the number of photons per solid angle produced by one electron,

$$\frac{d\sigma}{d\Omega} = 8\pi\alpha^3 \frac{L}{a^6} \frac{|SF|^2 f^2}{\cos\phi'} \frac{\omega_k v}{1 - v\cos\theta'} \left\{ \beta_2^2 + \left[\frac{(\omega_k \sin\theta' - g\cos\phi')[\omega_k + g\sin(\phi - \theta)]}{(\omega_k \sin\theta' - g\cos\phi')^2 + \omega_k^2 v^{-2}} \right]^2 \right\} \times \frac{1}{[(\omega_k \sin\theta' - g\cos\phi')^2 + \omega_k^2 (v\gamma)^{-2}]^2}, \tag{2}$$

where $\phi' = \phi + \beta_1$, $\theta = \theta + \beta_1$, and $1/\gamma^2 = 1 - v^2$. For the $(\bar{1}11)$ plane of diamond the squared structure factor is $|S|^2 = 32$. The square of the vibrational amplitude was assumed to be $129\lambda\!\!\!^{-2}_C$, which corresponds to a Debye temperature of 1800 K and a temperature of 293 K for the target. The atomic form factor was set to $F = 3.282$. We emphasize that this experimental value [21] is larger than theoretical predictions [22,23] yielding $F \approx 3.04-3.14$.

It should be pointed out that formula (2) differs from the formulas of Fiorito *et al.* [16] and of Asano *et al.* [19]. We can neglect the susceptibility χ_0 and the absorption of photons in the crystal. Both expressions [16,19] hold for high energies and extremely small solid angles in the vicinity of the Bragg angle, PXR(A), while ours is valid for low energies, PXR(B).

PXR spectra were obtained by bombarding a diamond crystal type Ia and of 55 μm thickness with electrons of 3.5, 8.3, and 9.1 MeV at the low energy channeling site [24–26] of the superconducting Darmstadt electron linear accelerator (S-DALINAC), which provides a continuous wave (cw) electron beam. The beam divergence was kept below 0.3 mrad and the electron current at a few nA. Si(Li) detectors were placed under $\theta = 44°$ with respect to the electron beam axis for the PXR counting and under 0° for simultaneous recording of channeling radiation. The electrons and the photons leave the crystal through the same surface.

A typical PXR(B) spectrum at $\theta = 44°$, i.e., twice the Bragg angle $\phi_B = 22°$, for the scattering at the $(\bar{1}11)$ plane is shown in Fig. 1. Electrons of only 8.3 MeV, i.e., $\gamma = 17.24$, a strong monochromatic PXR line at about 7 keV is detected above a low background level. Since the energy of the line varies as $\omega_k = vg\sin\phi/(1 - v\cos\theta)$, already small electron beam energies ($\gamma < 20$) indeed suffice to produce x rays of this energy. The derived width of the PXR line, including broadening effects like multiple scattering and beam divergence, was found to be $\Delta\omega = 160 \pm 80$ eV where the fairly large uncertainty results from the deconvolution of the detector resolution (~250 eV). This corresponds to a percent bandwidth of the PXR spectral line, averaged over the observed

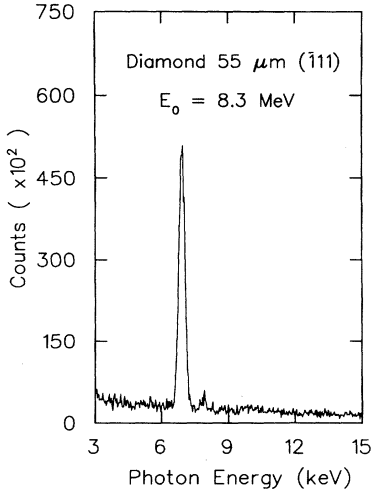

FIG. 1. Typical PXR(B) spectrum obtained of 8.3 MeV electrons, recorded by a Si(Li) detector placed under 44° with respect to the electron beam axis.

energy range and with respect to the Bragg energy $\omega_B = g/(2\sin\phi_B) = 8.1$ keV, of $\Delta\omega/\omega_B = (3.3 \pm 0.2)\%$.

The variation of the PXR photon energy ω as a function of the tilt angle ϕ is shown in Fig. 2. For a change of ϕ between 17° and 32° ω varies from 4 to 12 keV. The solid line represents a fit of a function $\omega = a\sin\phi$ to the data points. The easily achievable variation makes the PXR a potential candidate for a tunable x-ray photon source.

The spectral intensity of the PXR(B) has been obtained by fitting a Gaussian to the line and a polynomial to the background. The results for the spectra taken at different tilt angles for the electron bombarding energy of $E_0 = 8.3$ MeV are displayed in Fig. 3 in absolute units. The angular distribution exhibits a distinct minimum near the Bragg angle and two maxima of different intensity at $\phi = 20°$ and 25°, i.e., below and above ϕ_B. The angular distribution calculated from Eq. (2) describes the experimental data fairly well (dashed curve). Considering multiple scattering of electrons inside the crystal [27] results in a filling of the minimum at $\phi = 22°$ that is expected from Eq. (2) and leads to an improvement in the comparison between experiment and theory. Note, however, that the theoretical angular distribution had to be multiplied by a factor of 1.12 in order to fit the experimental one (solid curve in Fig. 3), i.e., we have an agreement between theory and experiment within 12%. The small deviations at large tilt angles come presumably from the particular distribution of core and valence electrons in a diamond lattice. For PXR wavelengths smaller than the spatial extension of the valence electron (sp^3) distribution, the radiation can originate from various points. Therefore, the angular distribution may especially be affected at the high energy tail (large tilt angles) pulling the theoretical curve down to the experimental values.

The results obtained at 3.5 MeV electron impact energy show a very similar behavior. The agreement between

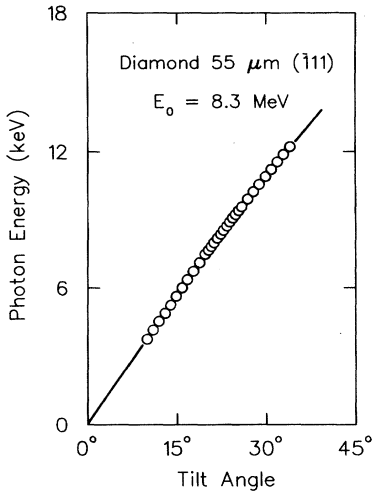

FIG. 2. Variation of the PXR(B) photon line energy as a function of the crystal tilt angle.

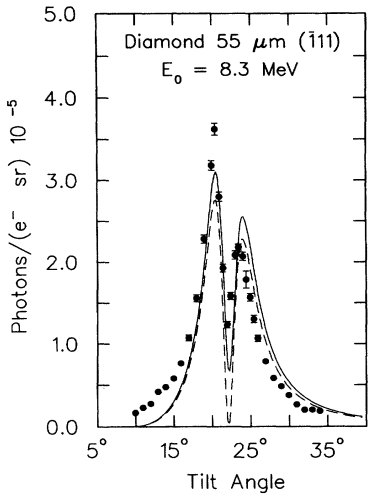

FIG. 3. The intensity of the PXR(B) line as a function of the tilt angle. The results obtained from Eq. (2) are represented by the dashed curve. The solid line was obtained by including multiple scattering and a factor of 1.12.

experiment and theory is of the same quality (12%). The number of photons, however, is clearly reduced (Fig. 4). Here the number of photons of the maximum below the Bragg angle is plotted for the three energies investigated. The uncertainty of the data point at 9.1 MeV is larger than for the points at the other energies because at this energy only one data point at $\phi = 21°$ was measured. After some manipulation of Eq. (2) it follows that the number of photons at the tilt angle where it is maximal should vary according to $I \sim \gamma(\theta\gamma - 1) \approx \gamma^2$, as is indicated by the solid line in Fig. 4, in very good agreement with the experiment.

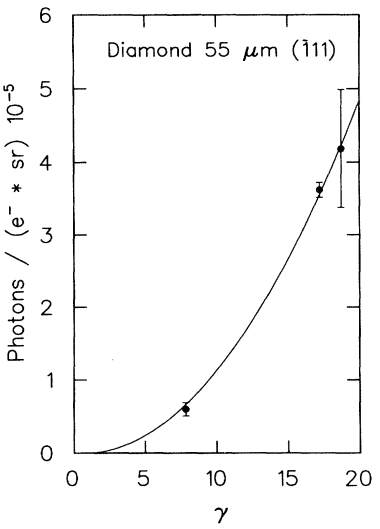

FIG. 4. Intensity of the first maximum of the PXR(B) line at different electron energies.

In conclusion, for the first time PXR(B) radiation has been observed at low electron bombarding energies ($8 < \gamma < 19$). The PXR lines of diamond are fairly sharp, they sit on a low background, and their energy is easily changeable by tilting the crystal. Their maximum intensity is about 3 orders of magnitude smaller compared to channeling radiation [26] in diamond. Comparison with a theoretical approach based on perturbative methods reveals that the latter describes the data at present within about 12%. This clearly can be viewed as a success. Finally, the predicted γ^2 behavior of the radiation could also be demonstrated for the first time.

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