Mixed-State Hall Conductivity in High-T_c Superconductors: Direct Evidence of Its Independence on Disorder

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We studied the Hall effect in a $Tl_2Ba_2CaCu_2O_8$ epitaxial film and in an $YBa_2Cu_3O_7$ single crystal in the mixed state before and after irradiation with high energy Pb ions. Pinning enhancement due to irradiation-induced columnar defects leads to the decrease in magnitude of the longitudinal and Hall resistivities, but does not modify the behavior of the Hall conductivity. This result is valid independent of the sign of the Hall effect in the pinned region (positive for $Tl_2Ba_2CaCu_2O_8$ and negative for $YBa_2Cu_3O_7$). The present work proves straightforwardly that the mixed-state Hall conductivity does not depend on the pinning strength, in agreement with theory. This result is a dual analog of the behavior of 2D electronic systems where the Hall resistivity remains unaffected by disorder.

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The Hall effect in high-T_c superconductors in the mixed state is a subject of great recent interest. The data on the Hall resistivity show an unusual sign change in the vicinity of the transition temperature T_c [1–9] which has also been observed in conventional superconductors [7,10]. Another issue that has attracted special attention is the scaling dependence between the Hall and longitudinal resistivities, ρ_{xy} and ρ_{xx}, respectively, in the pinned regime: $\rho_{xy} \sim \rho_{xx}^{\beta}$ as first observed by Luo *et al.* [11] in epitaxial $YBa_2Cu_3O_7$ films ($\beta = 1.7 \pm 0.2$) and by one of us [12] in $Bi_2Sr_2CaCu_2O_8$ single crystals ($\beta = 2 \pm 0.1$) (see also Refs. [13] and [14]). Vinokur *et al.* [15] have shown that the scaling law with $\beta = 2$ is a universal feature of any vortex state with disorder-dominated dynamics.

It is important to note that the main result of theory [15] is stronger than just to say that β should equal 2 in the pinned regime. The conclusion about the scaling law is a consequence of the more general statement that the Hall conductivity does not depend on the pinning strength. This statement has not yet been verified directly by experiment. It is, in fact, the purpose of the present Letter. We enhanced pinning in an $YBa_2Cu_3O_7$ single crystal and in a c-axis-oriented $Tl_2Ba_2CaCu_2O_8$ film by means of inclusions of columnar defects induced by high energy Pb ion irradiation. The irradiation dose (see below) was chosen to satisfy two conflicting conditions: First, the transition temperature and the normal-state ρ_{xy} and ρ_{xx} should not be substantially modified by radiation damage and second, pinning should be enhanced due to the irradiation-induced defects. While the mixed-state longitudinal and Hall resistivities drastically decreased in magnitude after the irradiation, the Hall conductivity of both compounds remained unchanged within experimental accuracy.

A good quality twinned $YBa_2Cu_3O_7$ single crystal was grown using the standard flux method. The sample was a plate of typical dimensions $1 \times 1 \times 0.03\ \mathrm{mm}^3$. Good electrical contacts (below 1 Ω) were made by sticking Pt wires with silver paste followed by annealing at 300 °C in room atmosphere for 1 h. The 300-nm-thick c-axis oriented $Tl_2Ba_2CaCu_2O_8$ film was that used in our previous work [16]. A $3.5 \times 0.75\ \mathrm{mm}^2$ strip with gold covered contacts was patterned for in-plane resistivity measurements. The Hall voltage was determined by measuring the odd contribution after exchanging the voltage and current probes as described in Refs. [12,17]. Such a technique has been shown to produce the same results as the one consisting in measuring the antisymmetric part of the transverse voltage under magnetic field reversal. The magnetic field H was directed parallel to the c axis, i.e., aligned with the tracks. The current density used for resistivity measurements was $\approx 20\ \mathrm{A/cm^2}$ for $YBa_2Cu_3O_7$ and up to $10^3\ \mathrm{A/cm^2}$ for $Tl_2Ba_2CaCu_2O_8$. Both the Hall and longitudinal resistivities were Ohmic at these currents in the range of temperatures and magnetic fields used in this study [18].

The samples were irradiated at room temperature by 5.6 GeV Pb ions for the $YBa_2Cu_3O_7$ crystal and by 1 GeV Pb ions for the $Tl_2Ba_2CaCu_2O_8$ film. The irradiation resulted in continuous cylinders of amorphous matter, about 7 nm in diameter through the thickness of the samples (see Refs. [17,19], and references therein). The irradiation fluences were $10^{11}\ \mathrm{ions/cm^2}$ for $YBa_2Cu_3O_7$ and $5 \times 10^{10}\ \mathrm{ions/cm^2}$ for $Tl_2Ba_2CaCu_2O_8$ which in terms of the matching field B^* (i.e., the field at which the number of vortex lines matches the number of columnar defects) correspond to ≈ 2 T and ≈ 1 T, respectively.

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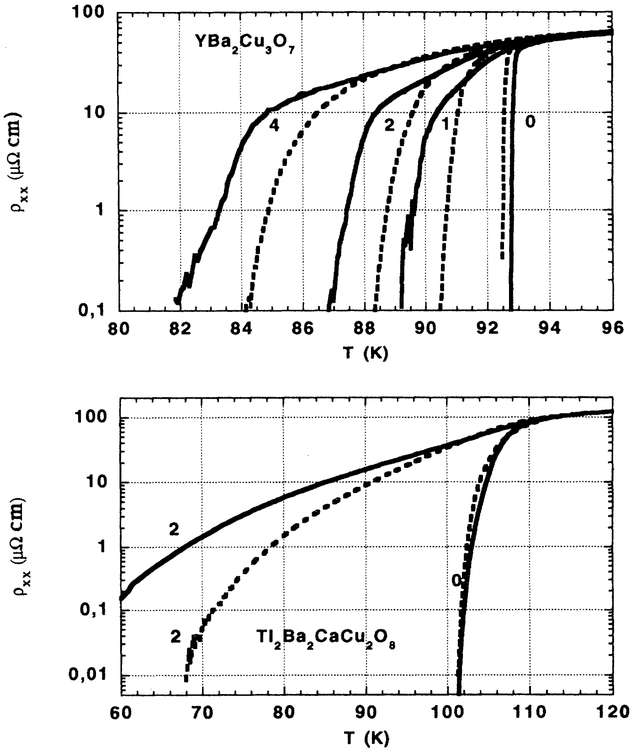

FIG. 1. Longitudinal resistivity in YBa$_2$Cu$_3$O$_7$ (top) and in Tl$_2$Ba$_2$CaCu$_2$O$_8$ (bottom) before (solid lines) and after (dashed lines) irradiation.

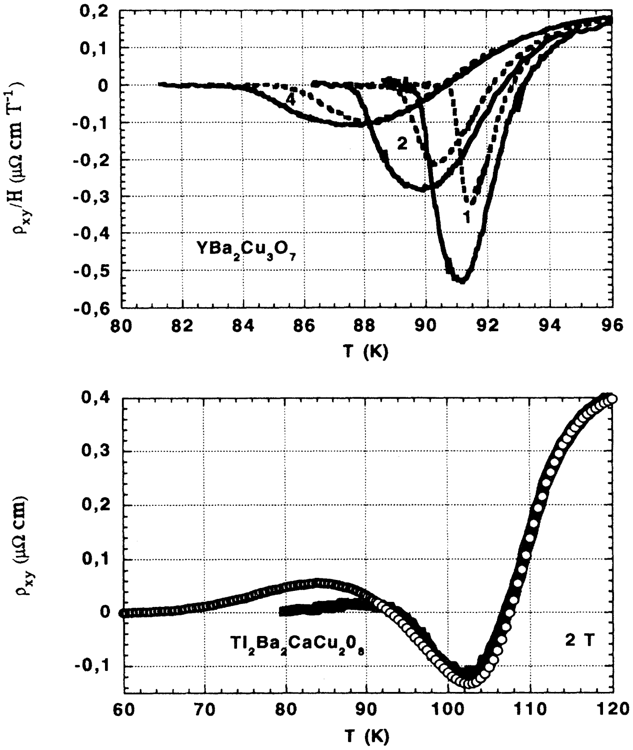

FIG. 2. Top: Hall resistivity in YBa$_2$Cu$_3$O$_7$ before (solid lines) and after (dashed lines) irradiation. Magnetic fields are 1, 2, and 4 T as indicated. Bottom: Hall resistivity in Tl$_2$Ba$_2$CaCu$_2$O$_8$ at $H = 2$ T before (open symbols) and after (solid symbols) irradiation.

For such doses, the T_c reduction was ~0.3 K for both compounds (see Fig. 1).

In what follows, we will discuss simultaneously results for both compounds: Top panels in Figs. 1–3 represent data for YBa$_2$Cu$_3$O$_7$, while data for Tl$_2$Ba$_2$CaCu$_2$O$_8$ are depicted in bottom panels. Figures 1 and 2 are plots of the temperature dependence of ρ_{xx} and ρ_{xy}, respectively, before and after irradiation. As seen from Fig. 1, the radiation damage shifts the onset of dissipation to higher temperatures. The resistivity transitions of YBa$_2$Cu$_3$O$_7$ crystals containing columnar defects have been analyzed previously by Legris, Rullier-Albenque, and Lejay [19] allowing one to determine quantitatively the pinning enhancement. Similar analysis for the YBa$_2$Cu$_3$O$_7$ sample under study revealed [17] that the potential energy well increased by a factor of ~3.5 for $H = 1$ T and by a factor of ~2.5 for $H = 4$ T. For the Tl$_2$Ba$_2$CaCu$_2$O$_8$ film, values of the activation energy U at some temperatures and fields as determined from the Arrhenius plots of the longitudinal resistivity before and after irradiation are given in [20].

The Hall resistivity (Fig. 2) of the unirradiated samples in the normal state is positive and depends linearly on H. As the temperature is lowered to the superconducting region, ρ_{xy} becomes negative. On further cooling, disorder occurring in pristine materials freezes out the vortex motion. In highly anisotropic Tl$_2$Ba$_2$CaCu$_2$O$_8$ the pinning is weaker than in YBa$_2$Cu$_3$O$_7$; correspondingly, the temperature range available for the transport measurements in Tl$_2$Ba$_2$CaCu$_2$O$_8$ is wider than in YBa$_2$Cu$_3$O$_7$. In the former compound, the Hall resistivity first exhibits a second sign change (from a negative value to a positive one) and then drops below resolution limit.

For the rather small doses of irradiation we used in this work, the loci for the first sign reversal of the Hall effect occurring in the vicinity of the transition temperature did not shift significantly. In fact, this shift to lower temperatures did not exceed the reduction in T_c (~0.3 K for both compounds). In the mixed state, the effect of irradiation-induced defects on the Hall resistivity is to decrease its magnitude. The lower the temperature is, the bigger is the ratio of ρ_{xy} before and after irradiation.

We will discuss the results within the framework of a recent work by Vinokur *et al.* [15] who calculated the effect of flux pinning on the longitudinal and Hall resistivities. Within their model, the flux lines are supposed to be driven by the Lorentz force produced by the transport current. This force is balanced by a drag force which consists of a friction term $-\gamma^*\mathbf{v}$ and a Hall term $\alpha\mathbf{v} \times \mathbf{n}$

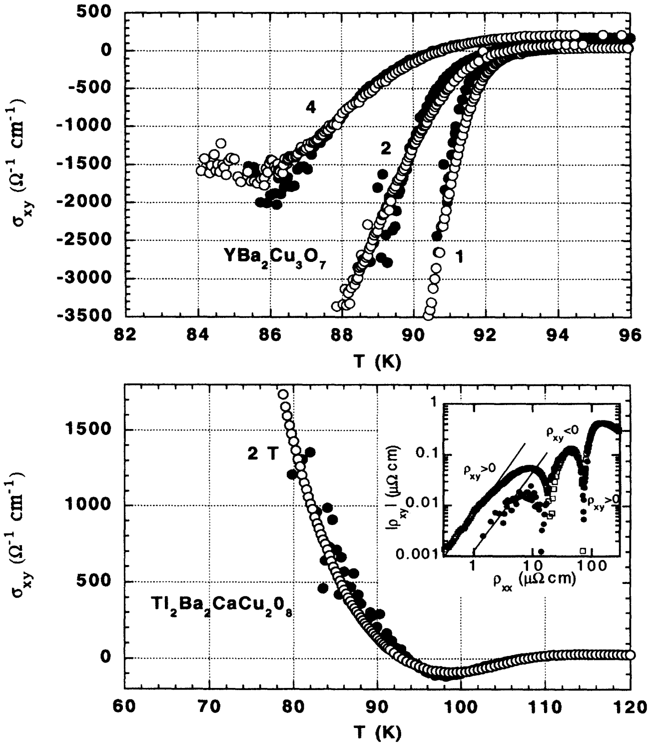

FIG. 3. Top: Hall conductivity in $YBa_2Cu_3O_7$ before (open symbols) and after (solid symbols) irradiation. Magnetic fields are 1, 2, and 4 T as indicated. Bottom: Hall conductivity in $Tl_2Ba_2CaCu_2O_8$ at $H = 2$ T before (open symbols) and after (solid symbols) irradiation. Inset: $\log_{10}|\rho_{xy}|$ vs $\log_{10}\rho_{xx}$ plot for $Tl_2Ba_2CaCu_2O_8$ at $H = 2$ T before (open squares) and after (solid circles) irradiation. Solid lines represent a fit to the power-law dependence.

(**v** is the average velocity of the vortices, **n** is the unit vector along the vortex lines). The coefficient γ^* takes into account contributions coming from the viscosity and from the average pinning force: $\gamma^* = \eta + \gamma$. The longitudinal and Hall resistivities are simply related to the coefficients of the momentum balance equation α and γ^*,

$$\rho_{xx} = B\Phi_0/\gamma^*, \tag{1}$$

$$\rho_{xy} = \alpha B\Phi_0/\gamma^{*2}, \tag{2}$$

where B is the magnetic induction and Φ_0 is the flux quantum. The only effect of pinning is to renormalize the friction leaving the Hall coefficient α unchanged [15]. Thus, we relate the decrease in magnitude of ρ_{xx} and ρ_{xy} after irradiation to the friction enhancement due to pinning by the columnar defects.

As follows from Eqs. (1) and (2), the coefficient α is related to the Hall conductivity $\sigma_{xy} = \rho_{xy}/\rho_{xx}^2$ (we neglect ρ_{xy}^2 in comparison with ρ_{xx}^2) as

$$\alpha = B\Phi_0\sigma_{xy}. \tag{3}$$

To examine the independence of the Hall coefficient α on pinning, we calculated σ_{xy} using data of Figs. 1 and 2 and plotted the Hall conductivity as a function of temperature in Fig. 3. The data before irradiation are depicted by the open symbols, while the solid symbols denote the data after irradiation. For $YBa_2Cu_3O_7$ (Fig. 3, top), we observe a negative increase of the Hall conductivity on cooling at fields 1 and 2 T, while at $H = 4$ T a minimum in $\sigma_{xy}(T)$ dependence becomes visible. The behavior of σ_{xy} in the $Tl_2Ba_2CaCu_2O_8$ film before irradiation has been discussed elsewhere [16]. The Hall conductivity in this compound also demonstrates a negative minimum which is followed by the second sign change on further cooling (Fig. 3, bottom). As we have already seen in Figs. 1 and 2, the temperature range available for the transport measurements shrinks after irradiation because pinning sets in at higher temperatures. This is the reason why the data for the Hall conductivity of the irradiated samples are not continued down to lower T (Fig. 3). But wherever it is possible to compare the data for the mixed-state Hall conductivity before and after irradiation, one can see that the incorporation of the columnar defects has not modified, within experimental accuracy, the behavior of σ_{xy} [21]. This result, which is valid both for $YBa_2Cu_3O_7$ (having the negative sign of the Hall effect in the pinned region for magnetic fields 1–4 T, see Fig. 3, top) and $Tl_2Ba_2CaCu_2O_8$ (for which σ_{xy} is positive at low temperatures, see Fig. 3, bottom, and Ref. [16]) is the main finding of the work.

Similar measurements were made by Budhani, Liou, and Cai on an epitaxial $Tl_2Ba_2Ca_2Cu_3O_{10}$ film containing columnar defects [14]. As their conclusions differ from ours, a few comments are in place. Based on the fact that the coefficient A in the power law $\rho_{xy} = A\rho_{xx}^{\beta}$ with $\beta = 1.85 \pm 0.1$ decreases after irradiation (Fig. 4 in [14]), it has been concluded that irradiation leads to the decrease of $\alpha(T)$. We believe that such a conclusion is not correct. Let us consider the $\log_{10}|\rho_{xy}|$ vs $\log_{10}\rho_{xx}$ dependence for the $Tl_2Ba_2CaCu_2O_8$ film (inset of Fig. 3, bottom). The coefficient A is given by the intercept of the solid lines in the inset of Fig. 3 on the $\log_{10}|\rho_{xy}|$ axis. The presence of columnar pins does decrease the constant A by a factor of ~4–5 for $Tl_2Ba_2CaCu_2O_8$ (and by a factor of ~2 for $YBa_2Cu_3O_7$). In other words, in the pinned region (at low ρ_{xx}), the Hall resistivity ρ_{xy} as a function of ρ_{xx} decreases after irradiation by a factor of ~4–5. Correspondingly, the Hall conductivity $\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2) \approx \rho_{xy}/\rho_{xx}^2$ decreases by the same factor as a function of the *longitudinal resistivity*. To illustrate this, let us choose $\rho_{xx} = 4\ \mu\Omega$ cm, for instance. For our $Tl_2Ba_2CaCu_2O_8$ sample before irradiation this resistivity level corresponds to $T \approx 76$ K. At $T = 76$ K, before irradiation, $\rho_{xy} \approx 0.04\ \mu\Omega$ cm and $\sigma_{xy} \approx 2500\ \Omega^{-1}\,\mathrm{cm}^{-1}$. After irradiation, the resistivity level $4\ \mu\Omega$ cm corresponds to $T \approx 84.7$ K; the Hall resistivity ρ_{xy} and the Hall conductivity σ_{xy} are equal to $0.01\ \mu\Omega$ cm and $625\ \Omega^{-1}\,\mathrm{cm}^{-1}$, respectively. The latter value is 4 times smaller than that of the Hall conductivity before irradiation for the same ρ_{xx}. However, as we have already seen in the main frame of Fig. 3, σ_{xy} as a function of *temperature* remains unchanged. Discussing the negative Hall effect, Budhani, Liou, and Cai state that "sign anomaly" (meaning "Hall resistivity") diminishes with increasing pinning strength. We modify this statement to the following: ρ_{xy} does decrease in magnitude with increasing disorder, but independent of the sign of the Hall effect, the intrinsic parameter α does not change (until radiation damage alters the transition temperature and the normal-state resistivities).

In summary, we have studied the Hall effect in a $Tl_2Ba_2CaCu_2O_8$ epitaxial film and in an $YBa_2Cu_3O_7$ single crystal before and after irradiation with high energy Pb ions. We have shown that in both compounds the incorporation of pinning centers does not change the mixed-state Hall conductivity. This finding has essential consequences for the understanding of the vortex dynamics in superconductors. Our work proves experimentally that data on the Hall conductivity can be analyzed within the framework of a theory such as the one based on the time-dependent Ginzburg-Landau (TDGL) equations (see, e.g., Ref. [22]), which does not include the effect of pinning but reveals intrinsic properties of the dynamics of the order parameter. An example of such an analysis for $Tl_2Ba_2CaCu_2O_8$ can be found in Ref. [16]. For $YBa_2Cu_3O_7$, a similar analysis has been done in a recent work by Kunchur *et al.* [23]. Also, it would be interesting to carry out thermomagnetic transport measurements in a superconducting sample before and after irradiation. In the pinned regime, the ratios N/ρ_{xx} and S/ρ_{xx} (where N and S are the Nernst and Seebeck coefficients, respectively) have been predicted not to change by the incorporation of defects [24], similarly to the behavior of the Hall conductivity. Another important issue to be pointed out is the analogy between the behavior of the mixed-state Hall *conductivity* as reported here and the behavior of 2D electronic systems where the conductivities σ_{xx} and σ_{xy} vanish in such a way that the Hall *resistivity* $\sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ remains finite and disorder independent ("Hall insulator," Ref. [25]). Indeed, in the case of type-II superconductivity, the dissipation is determined by the vortex motion, and the roles of conductivity and resistivity are exchanged.

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