Low-Energy Theorem for Scalar and Vector Interactions of a Composite Spin-1/2 System

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Scalar and vector interactions, with the scalar interaction coupled to a composite spin-1/2 system so as to cause a shift of its mass, are shown to obey a low-energy theorem which guarantees that the second order interaction due to Z graphs is the same as for a point Dirac particle. Off-shell and contact interactions appropriate to the composite system cancel, and this is verified in a model of a composite fermion. The result suggests a justification for the use of the Dirac equation as it has been used in relativistic nuclear scattering and mean field theories.

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The use of the Dirac equation in nuclear physics has been a subject of interest and debate in recent years. One of its outstanding successes is in elastic scattering of protons by nuclei [1,2]. Quite large scalar and vector interactions, which almost cancel one another, characterize the proton-nucleus interaction. Solving the Dirac equation with an attractive scalar potential and a repulsive vector one, each of magnitude about 300 MeV, produces a good description of spin observables at intermediate energies [3–7]. The principal effect is due to Z graphs when the Dirac equation is used, but it may be understood also at a simpler, classical level.

Consider a classical Hamiltonian of the form

$$H = V + \sqrt{(M+S)^2 + \mathbf{p}^2}, \tag{1}$$

where V is the time component of a vector potential and S is a scalar potential, both of which are taken to be spatially uniform. Expanding in S to second order yields

$$H = \epsilon + V + \frac{M}{\epsilon}S + \frac{\mathbf{p}^2}{2\epsilon^3}S^2 + \cdots, \tag{2}$$

where $\epsilon = \sqrt{M^2 + \mathbf{p}^2}$. The momentum dependent repulsive potential term in Eq. (2) provides the main relativistic effect in proton scattering by nuclei at intermediate energies.

To obtain the same results from the Dirac equation with scalar and vector potentials, one reduces the energy expression to the form

$$E\psi = (\epsilon + \mathcal{V}^{++} + \mathcal{V}_{\rm pair})\psi, \tag{3}$$

where

$$\mathcal{V}^{++} = V + \frac{M}{\epsilon}S \tag{4}$$

and

$$\mathcal{V}_{\rm pair} = \frac{\mathbf{p}^2 S^2}{\epsilon^2[E + \epsilon - V + (M/\epsilon)S]} = \frac{\mathbf{p}^2 S^2}{2\epsilon^3} + \cdots \tag{5}$$

is the Z-graph contribution. To second order, it is the same as in Eq. (2). For a composite system such as a nucleon, general arguments have been presented that Z graphs should be suppressed strongly [8–11]. Thus the correctness of $\mathcal{V}_{\rm pair}$ as obtained from the Dirac equation for a nucleon is in doubt, notwithstanding the classical basis for the effect.

In this Letter, we show that there is a low-energy theorem for both scalar and vector interactions which guarantees the correctness of the second order result Eq. (5). The result for the vector interaction is a straightforward generalization of the low-energy theorem familiar from Compton scattering [12,13] (even though the vector interaction here is purely longitudinal), but the result for a scalar interaction is a consequence of the observations that (i) an external scalar interaction S may be defined which acts to shift the mass of any composite system from M to $M + S$, and (ii) the scalar interaction so defined satisfies Ward identities which guarantee that the second order potential for the scattering of any composite spin-1/2 bound state gives the universal result (5) at low energy.

To show explicitly how the low-energy theorem emerges for the scalar interaction, consider the following simple model Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi + \tfrac{1}{2}[\partial_\nu\phi\partial^\nu\phi - \mu^2\phi^2] - g\bar{\psi}\psi\phi^2. \tag{6}$$

Assume that this system has a bound state of mass M and spin 1/2, which is composed of the elementary fermion of mass m and the scalar boson of mass μ. By scaling all parameters in the Lagrangian with dimensions of mass to new values $m \to \lambda m$, $\mu \to \lambda\mu$, $g \to \lambda^{-1}g$, and similarly scaling all cutoff or renormalization masses associated with the theory, it is clear that the bound state mass M will be scaled to λM. To obtain a scalar interaction which satisfies the requirement (i) above, choose $\lambda = 1 + S/M$, which implies that the Lagrangian which includes interactions of the scalar field S has the form (to first order in S)

$$\mathcal{L} \to \mathcal{L} - \rho S, \tag{7}$$

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where the scale-breaking charge associated with the mass scaling is

$$\rho = (m/M)\bar{\psi}\psi + 2(\mu/M)\phi^2 - (g/M)\bar{\psi}\psi\phi^2. \quad (8)$$

In general, this scale-breaking charge ρ is proportional to the divergence of the dilatation current of the system, including any anomalous contributions generated by scaling of the cutoff masses required to regularize the model.

The assumption that the scalar interaction is given by the scale-breaking charge ρ allows us to obtain a Ward identity for the vertex function for the interaction of the scalar field with the composite fermion $\Lambda^S(p',p)$. Before obtaining this, recall the Ward identity for the vertex function for the vector interaction $\Lambda^0(p',p)$, which is

$$\Lambda^0(p,p) = -\frac{\partial \Sigma(p)}{\partial p_0} = -A_0\gamma^0 - 2p^0(A_0'\not{p} + B_0'), \quad (9)$$

where the self-energy $\Sigma(p)$ has the general form

$$\Sigma(p) = A(p^2)\not{p} + B(p^2), \quad (10)$$

with A and B scalar functions, and $A_0 = A(M^2)$, $A_0' = dA(p^2)/dp^2|_{p^2=M^2}$, and similarly for B_0'. Although a detailed expression for $\Sigma(p)$ is not required, it is straightforward to obtain one from the Lagrangian of Eq. (6), in which case Σ is obtained from a loop graph involving a fermion and a scalar meson propagator. Note that only the time component of the current is needed.

The Ward identity for the scalar vertex has a form similar to Eq. (9). An examination of the lowest order Feynman diagrams in our simple model shows that the vertex function corresponding to insertion of the charge ρ, in the limit where $q = p' - p \to 0$, is

$$\Lambda^S(p,p) = \frac{m}{M}\frac{\partial \Sigma(p)}{\partial m} + \frac{\mu}{M}\frac{\partial \Sigma(p)}{\partial \mu} + \frac{g}{M}\frac{\partial \Sigma(p)}{\partial g} + \frac{\Lambda}{M}\frac{\partial \Sigma(p)}{\partial \Lambda}, \quad (11)$$

where Λ is the cutoff mass. Using the fact that Σ is dimensionless, and hence invariant when all parameters with the dimensions of mass are scaled, e.g.,

$$\Sigma(m,\mu,\Lambda,g,p_\alpha) = \Sigma(\lambda m, \lambda\mu, \lambda\Lambda, \lambda^{-1}g, \lambda p_\alpha), \quad (12)$$

and expanding to first order about $\lambda = 1$, one finds

$$\Lambda^S(p,p) = -\frac{p_\alpha}{M}\frac{\partial \Sigma(p)}{\partial p_\alpha} = -A_0\frac{\not{p}}{M} - \frac{2p^2}{M}(A_0'\not{p} + B_0'). \quad (13)$$

This equation is a direct consequence of a Ward identity for the divergence of the dilatation current [14] and the low-energy theorem which we will derive depends on the existence of such an identity.

We are now ready to use the identity (13) to prove the low-energy theorem for the scalar and vector interaction. The propagator of the composite spin-1/2 system of mass M and four-momentum p may be written

$$G(p) = \frac{1}{1 - \Sigma(p)}. \quad (14)$$

By assumption, there is a pole in $G(p)$ at $\not{p} = M$. Expanding the propagator about the bound-state pole, one finds

$$G(p) = Z_2\left[\frac{1}{\not{p} - M} + \delta G(p)\right], \quad (15)$$

where

$$Z_2 \equiv -\{A_0 + 2M[MA_0' + B_0']\}^{-1} = -(\Sigma_0')^{-1} \quad (16)$$

is a wave function normalization factor with $\Sigma_0' = d\Sigma(\not{p})/d\not{p}|_{\not{p}=M}$. Nonelementary propagation due to excited states of invariant masses greater than M gives rise to

$$\delta G(p) = Z_2\frac{M + \not{p}}{4M}\Sigma_0'' + D_0(\not{p} - M) + \cdots,$$

where $\Sigma_0'' = d^2\Sigma(\not{p})/d\not{p}2|_{\not{p}=M}$ and terms omitted from the expansion are higher order in $p^2 - M^2$ and do not play a role in the low-energy limit. The detailed form of the function D_0 is not required for the proof of the low-energy theorem involving scalar interactions. The usual spectral expansion of the positive-energy pole term in $G(p)$ shows that the ground state of the composite system has the positive-energy wave function $Z_2^{1/2}u(p)$, where $u(p)$ is a Dirac spinor for an elementary fermion of mass M.

To simplify the notation, we define a vertex which combines the scalar and vector interactions and coupling strengths as follows:

$$\Lambda^{SV}(p,p) = S\Lambda^S(p,p) + V\Lambda^0(p,p). \quad (17)$$

Forward scattering of the composite fermion from the scalar and vector fields is studied in second order and in the limit $q \to 0$, where q is the momentum exchanged with the source. The diagrams of Fig. 1 yield for the potential $\mathcal{V}$ the following expression:

$$\mathcal{V} = \frac{M}{2\epsilon}Z_2^{1/2}\bar{u}(p)\{\Lambda^{SV}(p,p+q)G(p+q)\Lambda^{SV}(p+q,p) + (q \to -q) + C^{SV}(p,p)\}Z_2^{1/2}u(p), \quad (18)$$

where the first term in the curly braces is the direct pole term, Fig. 1(a), the second term with $q \to -q$ the crossed pole term, Fig. 1(b), and the third, contactlike term,

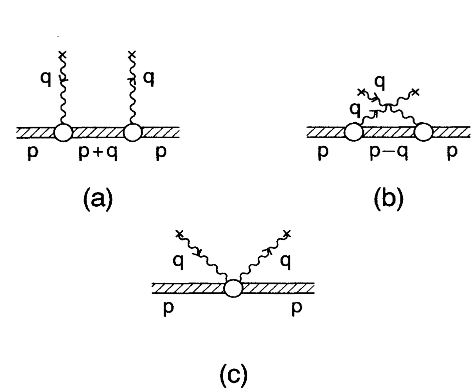

FIG. 1. Diagrammatic representation of the three contributions to Eq. (18).

Fig. 1(c) describes processes involving scattering from the constituents within a single self-energy bubble. Note that the iteration of the first order potential is contained in Eq. (18). It must be subtracted to avoid double counting. The appropriate subtraction is based on the second-order scattering by the equivalent potential, $S + \gamma^0 V$, using the positive-energy pole part of the Dirac propagator.

For timelike vector interactions at zero momentum transfer, the contact term is $V^2 C^{00}(p,p) = V^2 \partial^2 \Sigma(p)/\partial p^0 \partial p^0$. For scalar interactions the contact term corresponds to two insertions of Eq. (8). By expanding Eq. (12) to second order about $\lambda = 1$, it is possible to show that the interaction vertex to second order in ρ is

$$C^{SS}(p,p) = (p_\mu p_\nu / M^2) \partial^2 \Sigma(p)/\partial p_\mu \partial p_\nu + (2/M^2) p_\mu \partial \Sigma(p)/\partial p_\mu . \qquad (19)$$

There are also cross terms involving one ρ and one vector insertion. Collecting the various contact terms and coupling strengths, we have

$$C^{SV}(p,p) = S^2 C^{SS}(p,p) + 2SVC^{0S}(p,p) + V^2 C^{00}(p,p),$$

where $C^{0S}(p,p) = -\partial \Lambda^S(p,p)/\partial p_0$.

Because a denominator in $G(p \pm q)$ vanishes with q, it is necessary to evaluate numerator factors correct to first order in q before going to the limit $q \rightarrow 0$. This involves expanding the vertex $\Lambda^{SV}(p',p)$ about p or p' in a Taylor's series,

$$\Lambda^{SV}(p, p+q) = \Lambda^{SV}(p,p) + q_\mu \left[\frac{\partial \Lambda^{SV}(p',p)}{\partial p_\mu} \right]_{p'=p} + \cdots , \qquad (20)$$

and a similar expansion of $\Lambda^{SV}(p-q,p)$. Because of symmetries in the expansion, these contributions can be expressed also as second derivatives of $\Sigma(p)$ with respect to momenta. Thus there are cancellations with the contact terms.

Finally, the SS contribution to the potential (18) follows from substituting (13), (15), (19), and (20). Keeping only terms which contribute as $q \rightarrow 0$, we find

$$\mathcal{V} = \frac{\mathbf{p}^2 S^2}{2\epsilon^3} + \frac{S^2}{\epsilon}[\xi + (\xi - 1) + (1 - 2\xi)] = \frac{\mathbf{p}^2 S^2}{2\epsilon^3}, \qquad (21)$$

where $\xi = Z_2 M \Sigma_0''/2$. The contributions to Eq. (21) arise as follows: the $\mathbf{p}^2$ term from the composite particle Z graphs, ξ from the off-shell propagation $\delta G(p)$, $\xi - 1$ from the contact terms, and $(1 - 2\xi)$ from the off-shell expansion of the vertex functions, Eq. (20). Cancellations render the overall result independent of the factor ξ. This demonstrates the low-energy theorem for the scalar interaction given in Eq. (8), and shows that a scalar interaction capable of shifting the mass generates a repulsive potential of the same form as that obtained classically from a mass shift in Eq. (1) or from the Dirac equation from the Z-graph contribution, $\mathcal{V}_{\rm pair}$.

We have carried out a similar analysis for the VV and SV terms and have found that the second order potential is zero, in agreement with Eq. (5). This result emerges from the cancellation of five terms: the four which arose in the scalar case plus a new term. This new term appears because of the subtraction mentioned below Eq. (18).

The low-energy theorem establishes an equivalence between second order scattering of a composite spin-1/2 system by scalar plus vector interactions of arbitrary strength and the second-order scattering of a Dirac particle by similar potentials. The key condition is that the scalar interaction be coupled to the scale-breaking charge so as to cause a mass shift. The composite particle Z graph contributions due to such a scalar interaction are not suppressed. The demonstration that the second-order potential is unaffected by compositeness is based on the $q \rightarrow 0$ limit, implying constant interactions. This should be sufficient to ensure a corresponding result for slowly varying interactions, such as those describing elastic scattering of protons by nuclei.

The analysis suggests conditions under which the relativistic effect in proton-nucleus scattering may be unaffected by compositeness of the nucleon. Whether these conditions apply to the scalar interaction implied by QCD remains an open question.

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