Instability of Langmuir Waves in Plasma Irradiated by Directed Gamma Rays

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In this Letter we investigate the possibility of Langmuir turbulence generation by directed gamma rays penetrating a background plasma. Turbulence is generated because Compton scattering creates superthermal electrons whose distribution can be unstable to the Cherenkov radiation of Langmuir waves. Using the Klein-Nishina cross section for relativistic Compton scattering, we calculate the momentum distribution of recoil electrons and derive the increment of instability for nonmagnetized plasma. The instability appears when there is a sufficiently narrow bump, for example, annihilation line $h\nu = m_e c^2$ with a width less than $0.3\, m_e c^2$, in the gamma spectrum.

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It is well known that a directed beam of superthermal electrons generates Langmuir turbulence in a plasma [1–3]. In this Letter we investigate the possibility of Langmuir turbulence generation by directed gamma rays. We consider a nonmagnetic fully ionized hydrogen plasma at nonrelativistic temperatures.

First we find the momentum distribution of recoil electrons. We then derive the increment of a Langmuir wave with arbitrary wave vector $\vec{k}$. Then we consider in detail the case of the annihilation line and find the most unstable modes and the maximum linewidth for which the distribution of recoil electrons is unstable.

We consider recoil electrons with kinetic energy $E \gg E_T$ where $E_T = k_B T_e / m_e c^2$ is the electron thermal energy (we measure energy in units of $m_e c^2$ and momentum in units of $m_e c$). We assume that initially the electrons are at rest. The kinetic energy gained by an electron on scattering a photon of initial energy ε at an angle α is

$$E = \frac{\varepsilon^2 (1 - \cos\alpha)}{1 + \varepsilon(1 - \cos\alpha)} . \qquad (1)$$

The relativistic differential cross section of photons scattering on an electron at rest can be written in the form [4]

$$\sigma_\varepsilon(E) = \frac{3}{8} \sigma_T \frac{1}{\varepsilon^2} \Psi(\varepsilon, E) , \qquad (2)$$

where $\sigma_T = (8\pi/3)(e^2/m_e c^2)^2$ is the Thomson cross section, and

$$\Psi(\varepsilon, E) = 2 + \frac{E^2 - 2E}{\varepsilon(\varepsilon - E)} + \frac{E^2}{\varepsilon^2(\varepsilon - E)^2} . \qquad (3)$$

Photons with energy ε can create recoil electrons with kinetic energy E in the range $0 < E < E_{\max}(\varepsilon) = 2\varepsilon^2/(1 + 2\varepsilon)$. The energy distribution function of electrons produced by photons in the energy band $(\varepsilon, \varepsilon + d\varepsilon)$ during time interval Δt is

$$n(\varepsilon, E) = F_\gamma w(\varepsilon) \Delta t n_e \sigma_\varepsilon(E) , \qquad (4)$$

where n_e is the density of electrons in the plasma, F_γ is the total flux of photons, and $w(\varepsilon)$ describes the shape of the radiation spectrum. We consider a line-shaped radiation, and in our calculations we use the Gaussian profile

$$w(\varepsilon) = \frac{1}{\sqrt{\pi}\, \Delta} e^{-(\varepsilon - \varepsilon_0)^2/\Delta^2} . \qquad (5)$$

The density of recoil electrons in the plasma is given by

$$n_* = \int_0^\infty d\varepsilon \int_0^{E_{\max}(\varepsilon)} n(\varepsilon, E)\, dE = F_\gamma n_e \Delta t \bar{\sigma} , \qquad (6)$$

where $\bar{\sigma} = \int \sigma_\varepsilon w(\varepsilon) d\varepsilon$ is the average cross section, $\sigma_\varepsilon = \int_0^{E_{\max}} \sigma_\varepsilon(E)\, dE$ being the total cross section for photons with energy ε.

We consider a directed stream of photons and therefore the momentum distribution of the recoil electrons is axially symmetric. If the beam of photons propagates along the z axis then instead of ε and E we use new variables: p_z, the z component of the momentum, and p, the absolute value of the momentum. p_z is given by

$$p_z = E\left(1 + \frac{1}{\varepsilon}\right), \qquad p = \sqrt{E^2 + 2E} . \qquad (7)$$

The distribution of the recoil electrons in p and p_z is related to $n(\varepsilon, E)$ by

$$n(p, p_z) = \left| \frac{\partial(\varepsilon, E)}{\partial(p, p_z)} \right| n(\varepsilon, E) = \frac{E(E + 1)}{\varepsilon \sqrt{E(E + 2)}} n(\varepsilon, E) , \qquad (8)$$

where $|\partial(\varepsilon, E)/\partial(p, p_z)|$ is the Jacobian of the coordinate transformation. The axially symmetric distribution of the recoil electrons in momentum space is given by $n(\vec{p}) = n(p, p_z)/2\pi p$. In further calculations we use the normalized distribution function

$$f(\vec{p}) = \frac{n(\vec{p})}{n_*}, \qquad \int f(\vec{p})\, d\vec{p} = 1 . \qquad (9)$$

From Eqs. (4), (6), and (8), we get

$$f(\vec{p}) = \frac{3}{16\pi} \frac{\sigma_T}{\bar{\sigma}} \frac{\Psi(\varepsilon, E)}{E(E + 1)} w(\varepsilon) . \qquad (10)$$

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Even when the photon spectrum forms a narrow line, the momentum distribution of the recoil electrons is very different from the collimated beam case. The distribution of the recoil electrons extends from the thermal energy E_T to the maximal possible energy $E_{\max}$ that can be transferred by gamma rays, and the scattered electrons can move in a large range of angles with respect to the photon direction, $0 < \theta < \pi/2$ (see Fig. 1). The distribution produced by a narrow photon line is concentrated on the surface of an ellipsoid, in momentum space, which is axially symmetric around the direction of the photon beam. The distribution function $f(\vec{p})$ has a narrow bump along any straight line from the coordinate center which is not perpendicular to the p_z axis. This bump is narrowest and highest at small p_z, when the line is at a small angle to p_x.

An anisotropic electron distribution forming a bump in momentum space can be unstable to plasma wave generation [5]. In our case the distribution is spread over a large range of energy and a large range of angles. The growth rate of a Langmuir wave with the frequency ω and the wave vector $\vec{k}$ is (e.g., [6])

$$\gamma_{\vec{k}} = \frac{4\pi^2 \omega_{\rm pl} e^2 n_*}{k^2 m_e c} \int \delta(\vec{k} \cdot \vec{v} - \omega)\left(\vec{k} \cdot \frac{\partial f(\vec{p})}{\partial \vec{p}}\right) d\vec{p}\,, \tag{11}$$

where $\omega_{\rm pl} = \sqrt{4\pi n_e e^2/m_e}$ is the plasma frequency. In the calculations below we use Eq. (10) for $f(\vec{p})$, which is valid only for electrons with energy $E \gg E_T$. Therefore we consider only $E > 10^{-5}(k_B T_e/1\ \text{eV})$.

Langmuir waves with $k > 3\omega_{\rm pl}/v_T$, where $v_T = \sqrt{k_B T_e/m_e}$, cannot propagate because of Landau damping.

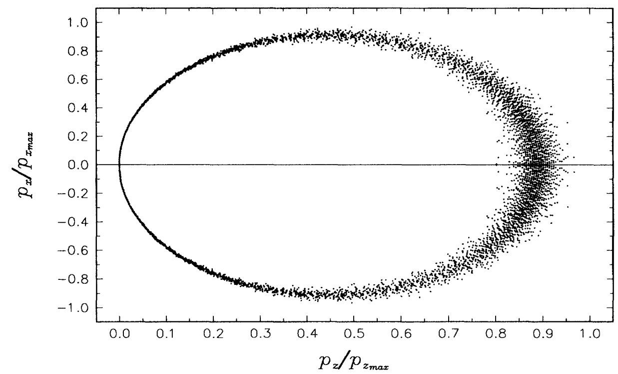

FIG. 1. An example of a distribution of recoil electrons $n(\vec{p})$ for $\varepsilon_0 = 1$, $\Delta = 0.05$. The distribution is axially symmetric around the p_z axis; therefore it is enough to present the distribution only on the (p_x, p_z) plane. There are 5000 points in this figure, representing recoil electrons with momenta within a thin layer $|p_y| < \Delta p_y$ projected on the (p_x, p_z) plane. p_z and p_x are taken in units of $p_{z\max} = E_{\max}(1 + 1/\varepsilon)$ and $p_{x\max} = \sqrt{2E(1 - E/E_{\max})}$, respectively.

Only $k < 10^3 k_c (k_B T_e/1\ \text{eV})^{-1/2}$ is hereafter considered, where $k_c = \omega_{\rm pl}/c$ is the minimum wave number of a wave which can be in the Cherenkov resonance. In this domain $k \ll \omega_{\rm pl}/v_T$ and the dispersion relation is given by $\omega \approx \omega_{\rm pl}$ [5] (we neglect the thermal correction to the plasma frequency). In this approximation the Cherenkov resonance condition, which is taken into account by the δ function in Eq. (11), takes the form $u(\vec{p}) = u_0 = \omega_{\rm pl}/ck$.

Using Eq. (10) we obtain

$$\vec{k} \cdot \frac{\partial f}{\partial \vec{p}} = \frac{3}{16\pi} \frac{\sigma_T}{\bar{\sigma}} \frac{1}{E(E+1)} \left\{ \left(\vec{k} \cdot \frac{\partial \varepsilon}{\partial \vec{p}}\right)\left[\Psi \frac{dw}{d\varepsilon} + w \frac{\partial \Psi}{\partial \varepsilon}\right] + \left(\vec{k} \cdot \frac{\partial E}{\partial \vec{p}}\right)\left[w \frac{\partial \Psi}{\partial E} - \frac{2E+1}{E(E+1)} \Psi w\right] \right\}. \tag{12}$$

Let us introduce the notation $u = \vec{k} \cdot \vec{p}/k(E+1) = \vec{k} \cdot \vec{v}/k$, and let θ be the angle between $\vec{k}$ and the z axis. Then

$$\vec{k} \cdot \frac{\partial f}{\partial \vec{p}} = \frac{3}{16\pi} \frac{\sigma_T}{\bar{\sigma}} \frac{k}{E(E+1)} \left\{ \frac{\varepsilon^2}{E}\left[u\left(1 + \frac{1}{\varepsilon}\right) - \cos\theta\right]\left[w \frac{\partial \Psi}{\partial \varepsilon} + \Psi \frac{dw}{d\varepsilon}\right] + uw\left(\frac{\partial \Psi}{\partial E} - \frac{2E+1}{E(E+1)} \Psi\right) \right\}. \tag{13}$$

In order to simplify the integration in Eq. (11) we change the integration variables to u, ε, and E, instead of $\vec{p}$. To calculate the Jacobian of this transformation we choose the x axis in the $(\vec{k}, z)$ plane (this can be done when $\vec{k}$ is not parallel to z). Then we get

$$\left| \frac{\partial(\varepsilon, u, E)}{\partial(p_x, p_y, p_z)} \right| = \frac{\varepsilon^2 |p_y| \sin\theta}{E(E+1)^2}\,. \tag{14}$$

When $\theta = 0, \pi$ the Jacobian is equal to zero and the variables ε, u, and E are then not independent. We will consider only the case where $\theta \neq 0, \pi$.

To get the explicit form of the Jacobian we express p_y as a function of ε, u, and E,

$$p_y^2 \sin^2\theta = (E^2 + 2E)\sin^2\theta + 2uE(E+1)\left(1 + \frac{1}{\varepsilon}\right)\cos\theta - E^2\left(1 + \frac{1}{\varepsilon}\right)^2 - u^2(E+1)^2. \tag{15}$$

The expression (11) for the increment can now be rewritten in the form

$$\frac{\gamma_{\vec{k}}}{\omega_{\rm pl}} = 2\pi\left(\frac{\omega_{\rm pl}}{kc}\right)^2 \frac{n_*}{\varepsilon_0 n_e} I(k, \theta)\,, \tag{16}$$

where an additional factor of 2 appears because the transformation from the variables (ε, u, E) to (p_x, p_y, p_z) is in the ratio 2:1. The dimensionless quantity $I(k,\theta)$ is given by

$$I(k,\theta) = \int\int\int \varepsilon_0 \delta\left(u - \frac{\omega}{kc}\right)\left(\frac{\vec{k}}{k} \cdot \frac{\partial f}{\partial \vec{p}}\right) \frac{E(E+1)^2 du\, d\varepsilon\, dE}{\varepsilon^2 |p_y| \sin\theta}. \tag{17}$$

The δ function can be integrated out by the simple substitution $u = u_0$ in the integral and by an appropriate choice of the integration limits. The minimum photon energy ε_1 for which a recoil electron can be in resonance with a given plasma wave, and the range of energies of the resonant electrons, $E_1(\varepsilon) < E < E_2(\varepsilon)$, are given in the Appendix. Using this information we have

$$I(k,\theta) = \varepsilon_0 \int_{\varepsilon_1}^{\infty} d\varepsilon \int_{E_1(\varepsilon)}^{E_2(\varepsilon)} \left[\left(\frac{\vec{k}}{k} \cdot \frac{\partial f}{\partial \vec{p}}\right) \frac{E(E+1)^2}{\varepsilon^2 |p_y| \sin\theta}\right]_{u=u_0} dE. \tag{18}$$

Taking into account (A8) and substituting Eq. (13) for $\vec{k} \cdot \partial f / \partial \vec{p}$, we rewrite Eq. (18) in the form

$$I(k,\theta) = \frac{3}{16\pi} \frac{\sigma_T}{\bar{\sigma}} \varepsilon_0 \int_{\varepsilon_1}^{\infty} \frac{d\varepsilon}{\sqrt{A}} \int_{E_1(\varepsilon)}^{E_2(\varepsilon)} \frac{F(\varepsilon, E) dE}{\sqrt{(E_2 - E)(E - E_1)}}, \tag{19}$$

where

$$F(\varepsilon, E) = \frac{E+1}{E}\left[u_0\left(1 + \frac{1}{\varepsilon}\right) - \cos\theta\right]\left[w \frac{\partial \Psi}{\partial \varepsilon} + \Psi \frac{dw}{d\varepsilon}\right] + \frac{u_0 w (E+1)}{\varepsilon^2}\left[\frac{\partial \Psi}{\partial E} - \frac{2E+1}{E(E+1)} \Psi\right] \tag{20}$$

and $E_{1,2}$ and A are given by (A9) and (A10).

In the calculations it is convenient to use a new variable s which is related to ε and E by

$$E(\varepsilon, s) = E_1(\varepsilon)\cos^2 s + E_2(\varepsilon)\sin^2 s. \tag{21}$$

Denoting $\Phi(\varepsilon, s) = F[\varepsilon, E(\varepsilon, s)]$, we obtain

$$I(k,\theta) = \frac{3}{8\pi} \frac{\sigma_T}{\bar{\sigma}} \varepsilon_0 \int_{\varepsilon_1}^{\infty} \frac{d\varepsilon}{\sqrt{A}} \int_0^{\pi/2} \Phi(\varepsilon, s) ds. \tag{22}$$

Unstable plasma waves have positive $\gamma_{\vec{k}}$. The occurrence of instability is illustrated in Fig. 2 for an annihilation line ($\varepsilon_0 = 1$). The most unstable waves propagate along the z axis ($\theta = 0$) and have wave numbers near the minimum possible value $k_{\min} = \omega_{\mathrm{pl}}/v_{\max} = k_c(v_{\max}/c)^{-1}$, where $v_{\max} \approx 0.8c$ is the maximum electron velocity. Waves with $k < k_{\min}$ cannot be in the Cherenkov resonance with recoil electrons and therefore the positive increment falls off steeply towards small k and becomes zero at $k = k_{\min}$. The instability exists only if the annihilation line is rather narrow, $\Delta < 0.3$. The largest increment $\gamma_{\max}/\omega_{\mathrm{pl}} \sim 25 n_*/n_e$ is attained for an extremely narrow line, $\Delta \approx 0$.

Plasma turbulence generated by recoil electrons has an interesting special feature which is illustrated in Fig. 3: the increment can be positive for waves propagating in the opposite direction (at angle $\theta > \pi/2$). Unstable waves with $\theta > \pi/2$ have higher wave numbers and are generated by electrons with lower energy.

The instability develops in a wide range of parameters k, θ; however, everywhere outside of the narrow interval of the highest increment (near $\theta = 0$) the growth rate of unstable waves is much smaller than $\gamma_{\max}$.

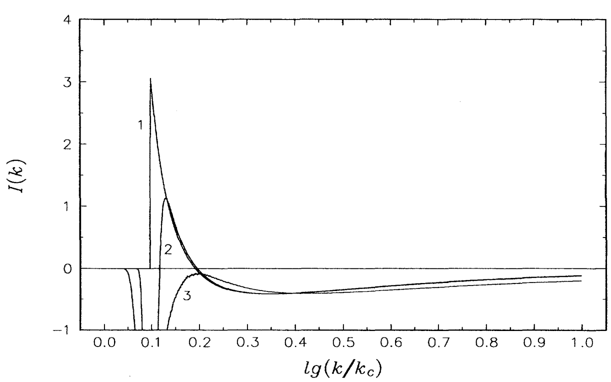

FIG. 2. The increment of Langmuir waves propagating along the z axis ($\theta = 0$) as a function of wave number k for the case of the annihilation line ($\varepsilon_0 = 1$). Several curves are presented for different linewidth Δ: (1), $\Delta = 0$; (2), $\Delta = 0.1$; and (3), $\Delta = 0.3$. Wave numbers are taken in units of $k_c = \omega_{\mathrm{pl}}/c$.

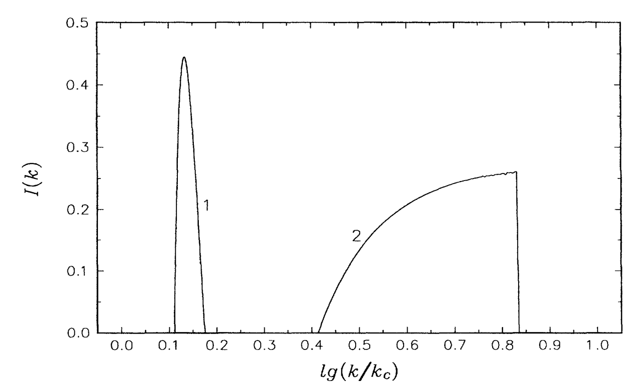

FIG. 3. Unstable plasma waves at angles $\theta = \pi/4$ (1) and $\theta = 3\pi/4$ (2) for extremely narrow annihilation line ($\varepsilon_0 = 1$, $\Delta = 0$).

In summary, the only condition for the instability to occur is that the gamma line should be sufficiently narrow, $\Delta\nu/\nu \ll 1$, and the characteristic energy of the recoil electrons should be larger than the thermal electron energy. This requires $h\nu > \sqrt{E_T m_e c^2}$.

The amplitude of unstable plasma waves increases, and this leads to development of turbulence in the plasma. The problem of relaxation of the turbulence will be considered later. The basic picture should be the same as in the case of beam instability [6–9]. In our case it could be important that the recoil electrons occupy a large region in the momentum space and they initiate instability at small ($k \sim k_c = \omega_{\rm pl}/ck$) and high ($k \sim \omega/3v_T$) wave numbers.

Because of nonlinear interactions, turbulence of transverse plasma waves appears [6]. This process can be responsible for creating a radio emission in a plasma. When the scale of the plasma object is smaller than $1/\mu$, where $\mu \sim \omega_{\rm pl}^4/n_e v_T^3 c$ is the coefficient of collisional absorption [6], radio waves leave the object and can be observed. The corresponding value of the maximum scale is $R_{\max} \sim (2 \times 10^{14}\ {\rm cm})(n_e/1\ {\rm cm}^{-3})^{-1}(k_B T_e/1\ {\rm eV})^{3/2}$.

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Appendix.—To determine the integration limits in Eq. (18) for given k, θ, let us find the range of u accessible for recoil electrons. By definition, $u = v_x \sin\theta + v_z \cos\theta$ (electron velocity v is in units of c). A recoil electron with energy E gained by scattering of a photon with energy ε has velocity components parallel and perpendicular to the z axis

$$v_z = \frac{E}{a(E+1)}, \quad v_\perp = \sqrt{2av_z - (a^2+1)v_z^2}, \quad \text{(A1)}$$

where $a = \varepsilon/(\varepsilon+1)$. Recoil electrons with fixed ε and E have u in the range

$$u_1(\varepsilon, E) \le u \le u_2(\varepsilon, E), \quad \text{(A2)}$$

where

$$u_{1,2}(\varepsilon, E) = \pm v_\perp(\varepsilon, E)\sin\theta + v_z(\varepsilon, E)\cos\theta. \quad \text{(A3)}$$

When E is changing in the range $0 < E < E_{\max}$, v_z is changing monotonically in the range $0 < v_z < v_{z\max}(\varepsilon) = 2a/(1+a^2)$. Considering $u_{1,2}$ as a function of ε, E, we find that u_1 attains its minimum at $v_z = v_{z1}$ and u_2 attains its maximum at $v_z = v_{z2}$, where

$$v_{z1,2} = \frac{a}{1+a^2}\left(1 \pm \frac{\cos\theta}{\sqrt{1+a^2\sin^2\theta}}\right). \quad \text{(A4)}$$

The allowed minimal and maximal values of u for given ε are

$$u_{\min,\max}(\varepsilon) = \frac{a}{1+a^2}(\cos\theta \pm \sqrt{1+a^2\sin^2\theta}). \quad \text{(A5)}$$

Now we can find the domain on the (ε, E) plane covered by the resonant electrons. Let us note that $u_{\min}(\varepsilon) < 0$ while $u_{\max}(\varepsilon) > 0$. The resonant value $u = u_0$ is positive; therefore the condition $u_0 \ge u_{\min}(\varepsilon)$ is always satisfied. The essential constraint on ε comes from the condition $u_0 \le u_{\max}(\varepsilon)$. $u_{\max}(\varepsilon)$ is monotonically growing and therefore this condition requires that $\varepsilon \ge \varepsilon_1$, where ε_1 is uniquely determined by the equation $u_{\max}(\varepsilon_1) = u_0$.

The allowed range of E is determined by the double inequality

$$u_1(\varepsilon, E) \le u_0 \le u_2(\varepsilon, E), \quad \text{(A6)}$$

where ε is now considered as a parameter. This inequality is not satisfied for $E = E_{\max}(\varepsilon)$ unless u_0 equals exactly $v_{z\max}\cos\theta$. However, some intermediate values of $E < E_{\max}$ have to meet (A6), since the condition $\varepsilon > \varepsilon_1$ guarantees the existence of recoil electrons in resonance with a given plasma wave.

Taking into account Eq. (A3) we write the double inequality (A6) in the equivalent form

$$v_\perp^2(E)\sin^2\theta \ge [u_0 - v_z(E)\cos\theta]^2. \quad \text{(A7)}$$

This condition coincides with the condition that the right hand side of Eq. (15) is positive and it can be rewritten as

$$p_y^2 \sin^2\theta = A(E_2 - E)(E - E_1) > 0, \quad \text{(A8)}$$

where

$$E_{1,2}(\varepsilon, u_0) = \frac{1}{A}(B \pm \sqrt{B^2 - Au_0^2}), \quad \text{(A9)}$$

$$A = -\sin^2\theta - \frac{2u_0\cos\theta}{a} + \frac{1}{a^2} + u_0^2 > 0,$$
$$B = \sin^2\theta + \frac{u_0\cos\theta}{a} - u_0^2. \quad \text{(A10)}$$

The condition $\varepsilon > \varepsilon_1$ guarantees that $E_{1,2}$ exist and $0 < E_1 < E_2 < E_{\max}$. The resonant recoil electrons produced by photons with given ε have energies in the range $E_1(\varepsilon) < E < E_2(\varepsilon)$.

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