"Granular" Convection in a Vibrated Fluid

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Numerical simulation of the Navier-Stokes equation in a continuously vibrated box demonstrates the formation of convection rolls. These rolls circulate with the opposite orientation to those found in granular flows. By introducing a negative-slip boundary condition, we recover the roll orientation and roll reversal with increasing side wall angle that have been obtained in experiments on granular convection.

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When a rigid vessel containing a granular material, such as glass beads or sand, undergoes vertical shaking that is periodic in time, steady convection rolls emerge. These rolls may be readily observed in a box with one very short side (~3–20 particle diameters) perpendicular to the driving, so that the particle motion is essentially two dimensional, in a plane with sides many tens to hundreds of particle diameters. Experiments on this system described by Knight, Jaeger, and Nagel exhibit a rich behavior, depending not only upon the frequency and amplitude of the driving but also on the shape of the container [1].

For a *rectangular* container, two rolls are observed, each of a characteristic length of order the box size. They are roughly symmetric about the vertical bisecting the box. In a layer adjacent to its respective exterior side wall, each roll carries the grains downward. Away from the side wall, the material flows, on average, upward toward the surface of the bed. When the container has a *circular* shape in the direction parallel to the driving, the granular flow is inverted: The grains climb the vessel walls and circulate downward in the interior. The orientation of the rolls in a trapezoidal container, wherein the angle of the side wall with respect to the vertical may be readily adjusted, has also been examined, with some attention to how the direction of circulation changes as a function of driving and side wall angle [2].

A number of theoretical proposals have been made for the origin of convection in this kind of system [3–5]. Some recent arguments have relied on particle dynamics simulation, typically in two dimensions, which follows the time-averaged motion of a large number of disks with elastic and frictional interactions [6]. These simulations have yielded, for some parameter values, flows resembling those found experimentally in the rectangular container. On the basis of these simulations, qualitative explanations of the phenomenon have been put forward. These explanations are largely descriptive in character, and, consequently, we find their merits difficult to evaluate. In particular, net particle displacement over a period often involves the relatively small residual of large vertical oscillations. It is not evident to us that these arguments are sufficient to reveal the sign of the residual.

Somewhat earlier, Savage adopted a complementary approach to the problem, with the aim of constructing continuum model for granular convection [7]. In the situation he examined, the flow is driven by the container bottom, which undergoes periodic and sinusoidal *spatial* deformation. He suggests that the underlying mechanism of convection is an analog of "acoustic streaming." In his picture, acoustic waves propagating through the bed catalyze the circulation; the bed is modeled as a compressible fluid. The boundaries of the container play no apparent role within his model (although it is not obvious that the geometry of his experiment justifies their neglect), nor is any elastic contribution manifest.

In this paper, we model granular flow by the Navier-Stokes equation, supplemented by a boundary condition that may restore the physics of granular flow essential to roll formation and orientation. Our model shares certain features with each of the mechanisms we discussed above, but also differs from them in crucial ways. The importance of the frictional interaction with the vessel walls suggests that we cannot ignore them, as Savage does in his work. On the other hand, we believe that the apparent fluidity of the convecting granular phase suggests that a continuum description is natural, and, like Savage, we find that streaming underlies the observed dynamics.

Our strategy involves a numerical examination of the behavior of a simple Newtonian fluid, subject to boundary conditions appropriate for the experimental geometry [2]. Whereas it has been claimed [5] that convection in a vibrating container is a phenomenon peculiar to granular materials, we find that a Navier-Stokes fluid yields convective behavior [8] whose origin is analogous to that in the granular system; however, we find that the orientation of the rolls in the Navier-Stokes fluid is not what is observed in the granular convection experiments. It is only when we perform the phenomenological introduction of a negative slip at the vertical side walls that we recover the roll direction

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characteristic of the experiment—in particular, we obtain roll reversal as a function of side wall angle.

We remark that the choice of appropriate boundary conditions at vessel walls, and whether one should think of the frictional coupling at the walls as removing, or adding, energy or momentum to the bulk flow, have been recurring issues in granular systems, and have not to our knowledge been satisfactorily resolved [9,10].

It must be stressed that we do not expect the present fluid model to apply to all, or even most, granular phases. For example, we have nothing to say about "fluidization," rather we only address phenomena within the fluidized phase. We also remind the reader that many material properties of a *static* sandpile, such as sound speeds or shear moduli, do not necessarily apply to sand vibrated continuously at high frequencies.

In the absence of time-resolved information on convection driven at small amplitudes and large frequencies, our physical intuition has been guided by observations of convection in discretely "tapped" beds, in which a large and sudden acceleration is applied to a quiescent bed, at intervals spaced sufficiently to allow for the complete relaxation of the sandpile. Frozen video frames reveal the sand distribution displayed schematically in Fig. 1; neither the profile of the driving cycle nor phase of the frame is known. We observe that while the interior of the bed is suspended, the side walls have retarded the displacement of a parallel layer near the container bottom, on the order of 10 particles in thickness. Before the rest of the bed descends, the material in the retarded boundary layer relaxes to its static angle of repose. A similar process occurs, at a distinct part of the driving cycle, at the top of the bed. We infer that two mechanisms couple to generate the global flow: (1) frictional attenuation at the side wall causes a *phase* delay, between bulk and boundary, in the local density fluctuations; (2) these density fluctuations relax through inertial transport. Our model for continuous driving, when the bed has insufficient time to relax in one cycle, will incorporate these density fluctuations as acoustic waves propagating through the bed. Our work addresses the continuum form of boundary-modulated attenuation that might yield the experimentally observed roll pattern, in a bed otherwise modeled as a simple Newtonian fluid.

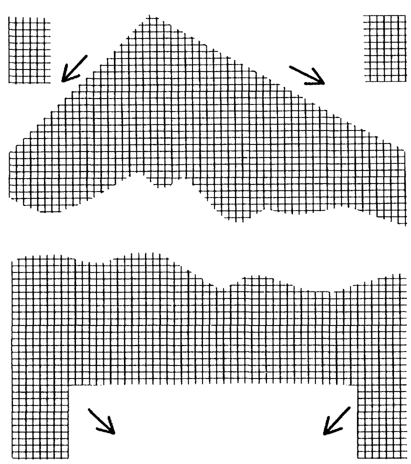

FIG. 1. Schematic of frozen video frames from tapped granular beds, where the bed completely relaxes within a period of the driving. Sand is in the shaded regions. The lower and upper levels of the figure correspond to distinct (and unknown) instants in the driving cycle. Side wall friction generates density fluctuations that are relaxed by inertial transport (arrows) before the bulk collapses, yielding net convective motion.

We made a number of simplifications to obtain a tractable model. In a variety of regimes, particularly at high frequencies ($\approx$30 Hz) and small amplitudes (at most a few particle diameters), experiments suggest very small density fluctuations within the granular bed, on the order of at most a few percent [11]. The upper interface of the granular bed appears to be quite well defined in a variety of experimental regimes, with the density decaying from approximately the bulk value to zero over a distance of less than a particle diameter. We eliminate the interface entirely from our model, since we anticipate that the importance of the interface in determining the bulk roll orientation is minor. We remove the interface by reflecting the container around a horizontal axis, and filling the entire cavity with fluid. The presence of density fluctuations, whether generated near the bed surface, or by cavitation in the interior, is subsumed in the compressibility of our fluid model.

In the calculations we discuss here, we set g, the gravitational acceleration, to zero. We confirmed that finite g does not alter the morphology and orientation of the rolls appreciably, even if we choose an equation of state such that most of the fluid remains in the bottom half of the vessel. Gravity might be expected to induce a critical driving below which the net circulation vanishes; in its absence we find no such threshold [12].

For our shaking frequencies and amplitudes, the density fluctuations turn out to be small; we use a linear equation of state $p(\rho) = p_0 + c^2\rho$, where p, ρ are, respectively, pressure and density, and we set the mean density to unity. While $\nabla \cdot \mathbf{u}$ ($\mathbf{u}$ the fluid velocity) may acquire finite values in our simulation, we anticipate that stratified or Boussinesq ($\nabla \cdot \mathbf{u} = 0$) flow will share the qualitative features discussed below. We assume constant uniform temperature and no independent dynamics for energy transport and dissipation; our model does not incorporate any granular temperature [13].

The calculations are carried out entirely in the (moving) frame of the vessel. The reason for transforming to the moving frame is that the boundary conditions, which apply to the velocity of the fluid relative to that of the walls, are most readily implemented in that frame. Since the vessel is rigid, the box velocity is independent of position. One readily finds that in the moving frame, the convective derivative of velocity in the y direction gives rise to an effective body force that appears on the right-hand side of our equation. The Navier-Stokes equations [14] may then be written as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p + \nu \nabla^2 \mathbf{u} = 4\pi^2 \rho \Gamma \hat{\mathbf{y}} \sin(2\pi t), \tag{2}$$

where we have scaled length by box side, and time by the period of the driving ω^{-1}. Γ denotes the amplitude of driving; $\hat{\mathbf{y}}$ is the upward pointing unit vector. The boundary conditions at the walls are

$$u_n = 0, \tag{3}$$

$$u_t + \sigma \frac{\partial u_t}{\partial n} = 0. \tag{4}$$

u_n, u_t denote, respectively, the components of $\mathbf{u}$ normal (directed outward) and tangential to the wall; σ is the *slip*. When σ is positive, the tangential component of the velocity crosses zero outside the boundary; negative σ corresponds to a zero crossing internal to the fluid. The vessel was taken to be a trapezoid of approximately unit aspect ratio, reflected about its upper horizontal boundary, with angular deviation α of the side walls outward from vertical with respect to the top and bottom.

We employed two distinct explicit algorithms to solve these equations in their conservation form. For rectangular boundaries, we used a two-step Mac-Cormack algorithm, second order in space and time [15]. In addition, we studied the equations with a second-order centered-difference scheme, first order in time. For finite angles, we used exclusively the latter method. The trapezoidal coordinates were mapped onto a rectangular grid, upon which a uniform discretization was carried out. The boundary conditions at the walls were enforced implicitly, to second order in grid spacing.

Our parameters were restricted to the domain of validity obtained by linear stability analysis in the absence of boundaries [15]. We established that our results converged as the square of the grid spacing, and as the first power of the time step. Our smallest grid spacing was $\frac{1}{64}$. Reasonable initial conditions do not affect the periodic steady state achieved by the flow after many cycles; parameters may be varied by an order of magnitude with the same qualitative results.

Figure 2 depicts temporal averages of $\rho\mathbf{u}$ over a full period; initial transients have been discarded. We have shown only the lower half of the container, the remainder obtainable by symmetry. The negative slip boundary condition applies. Three features of the experiments are conspicuous in this figure: (1) A roll emerges at small $\alpha \geq 0$, in which fluid descends at the side boundary; (2) the fluid velocity has a local *maximum* at the boundary, for α near 0; (3) for large enough α, the roll orientation reverses. Note the route taken by the transition from one roll orientation to the other: A new steady roll grows from the interior of the vessel, eventually taking over for sufficiently large α. Other possibilities exist *a priori;* the new roll might grow from the outside, or the net motion of fluid might cease in the crossover regime. Experiments to examine how the roll reversal actually occurs are under way.

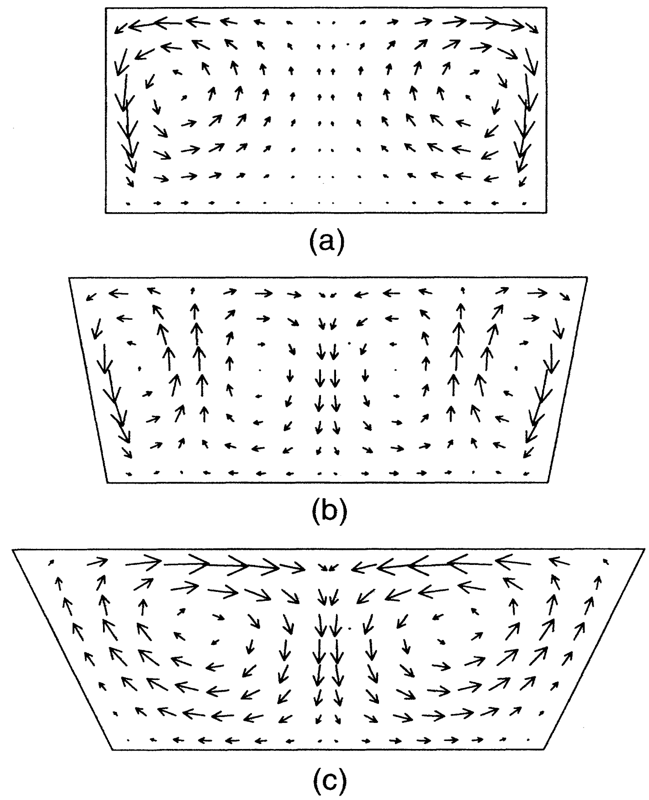

FIG. 2. Steady-state velocity profiles, averaged over one period of the driving, of a simulation of a vertically shaken box. The gravitational acceleration g has been set to 0. The driving is sinusoidal with unit frequency and amplitude $\Gamma = 0.01$. The fluid obeys a linear equation of state with sound speed $c \equiv (dp/d\rho)^{1/2} = 1$, where p and ρ denote, respectively, pressure and density. The viscosity ν is set to 0.05; the slip σ is equal to -0.9. The largest arrow length is scaled to the maximum (averaged) velocity v_m. The vertical deviation of the side walls α is the only parameter that varies among the three simulations: (a) $\alpha = 0°$, $v_m = 2.4 \times 10^{-4}$; (b) $\alpha = 8°$, $v_m = 1.1 \times 10^{-4}$; (c) $\alpha = 16°$, $v_m = 3.5 \times 10^{-4}$.

The critical angle for roll reversal depends monotonically on the slipping length σ. For the no-slip condition applicable to most fluids, $\sigma = 0$, flow is downward at the side walls, independent of angle. With positive σ, we do not observe roll reversal for $\alpha \geq 0$. As $\sigma \to 0^-$, the flow is upward at the side walls; the critical σ decreases in inverse proportion to σ (data not shown).

Emphasizing that we make no claim to explain it, much less suggest a microscopic derivation, we conclude with a discussion of our negative slip condition. A variety of granular flows display a high mobility layer, a few particles in thickness, localized in the vicinity of a boundary [16]. Simulations of dilute continuously sheared granular flows have shown that density gradients can arise near a wall [9]. When the inelasticity of the particle-wall interaction exceeds that of the interparticle interaction, a parallel layer of increased particle density arises directly adjacent to the boundary. Such density gradients also occur in simulations with transversely oscillating side walls; indeed, particle simulation reveals that changing solely the dissipative interaction of particle with wall can cause roll reversal [3,17]. It seems natural to expect that a continuum description of granular flow would require a special correction to the viscous fluid equations, at the boundary. Negative "apparent slip" velocities have been proposed for the continuum description of certain polymeric shear flows, where (material) phase separation occurs near the boundary, resulting in the storage and delayed release of elastic energy [18].

A common feature of particle simulation [17], the present fluid computations, and experiment [2] is wave propagation through the granular bed. Net transverse motion appears to be localized in the neighborhood of these excitations. This behavior is characteristic of acoustic streaming, a phenomenon known to Rayleigh [19]. By that mechanism, spatial inhomogeneity in wave amplitude leads to net flow at second order in the vibration amplitude. Savage [7] proposed that streaming modulates granular flow excited by a bottom plate undergoing *spatial* deformations. He claims that any effect of the vertical boundaries of his container may be ignored; consequently, he does not obtain the anti-Newtonain toll orientation we derive here [20]. In our case, wave attenuation through interaction with the side walls plays an essential role in determining the orientation of the net flow.

In summary, our calculations suggest that convection in a vertically oscillating box should be observed for a Navier-Stokes fluid, as well as for granular materials. The orientation of the rolls depends on the coupling of the fluid to the walls, and, in particular, on the slip condition at the boundary [21]. We expect that many of our results can be reproduced within perturbation theory; these calculations are in progress.

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[1] J. B. Knight, H. M. Jaeger, and S. R. Nagel, Phys. Rev. Lett. **70**, 3728 (1993).
[2] J. B. Knight (private communication).
[3] Y-h. Taguchi, Phys. Rev. Lett. **69**, 1367 (1992).
[4] J. A. C. Gallas, H. J. Hermann, and S. Sokolowski, Phys. Rev. Lett. **69**, 1371 (1992); H. J. Hermann, Physica (Amsterdam) **191A**, 263 (1992); J. A. C. Gallas, H. J. Hermann, and S. Sokolowski, *ibid.* **189A**, 437 (1992).
[5] J. Rajchenbach, Europhys. Lett. **16**, 149 (1991).
[6] P. A. Cundall and O. D. L. Strack, Geotechnique **29**, 47 (1979).
[7] S. B. Savage, J. Fluid Mech. **194**, 457 (1988).
[8] To our knowledge, studies of the Faraday instability have not revealed convection in the bulk; the instability is thought to involve only the surface of the fluid. We expect that the parameter range that applies to the Faraday instability would be inappropriate for the phenomenon described here.
[9] P. K. Haff, J. Fluid Mech. **134**, 401 (1983); K. Hui, P. K. Haff, J. E. Ungar, and R. Jackson, J. Fluid Mech. **145**, 223 (1984); G. M. Gutt and P. K. Haff, Int. J. Multiphase Flow **17**, 621 (1991); C. S. Campbell, Annu. Rev. Fluid Mech. **22**, 57 (1990).
[10] J. T. Jenkins and M. W. Richman, J. Fluid Mech. **171**, 53 (1986); J. T. Jenkins and S. B. Savage, *ibid.* **130**, 187 (1983); J. T. Jenkins, J. Appl. Mech. **59**, 120 (1992).
[11] With the possible exception of a region near the lower horizontal wall (bottom) of the container, whose height is of the order of the driving amplitude. As it turns out, our results are largely independent of the boundary condition at top and bottom walls. So we take this lower region to be part of the bulk granular bed.
[12] To study the *onset* of convection in a granular system on Earth, we would need to include gravity and incorporate a strain dependence into our stress tensor.
[13] S. Ogawa, A. Umemura, and N. Oshima, J. Appl. Math. Phys. **31**, 483 (1980).
[14] L. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1987). We have taken the viscosity ν independent of position. For simplicity we have omitted the grad div $\mathbf{u}$ term from the dissipation in the simulations shown here; restoring this term causes only insignificant departures from the results we describe.
[15] R. Peyret and T. D. Taylor, *Computational Methods for Fluid Flow* (Springer-Verlag, New York, 1983).
[16] H. M. Jaeger and S. R. Nagel, Science **255**, 1523 (1992), see, especially Fig. 1. We are grateful to T. Witten for his comments on this subject.
[17] J. Miller (unpublished). That the boundary conditions at the side wall can create these density gradients as if they were effectively cooling an ideal gas is reminiscent of convection in a vertical slot [see, e.g., D. J. Tritton, *Physical Fluid Dynamics* (Oxford University Press, London, 1988)].
[18] W. Kozicki, C. J. Hsu, and C. Tiu, Chem. Eng. Sci. **22**, 487 (1967).
[19] J. Lighthill, *Waves in Fluids* (Cambridge University Press, London, 1978).
[20] Anti-Newtonian flow originating from non-Newtonian character of the *bulk* fluid has been studied in polymeric flows; see R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymer Liquids* (John Wiley and Sons, New York, 1987); S. Rosenblatt, J. Fluid Mech. **85**, 387 (1978); C. F. Chang and W. R. Schowalter, J. Non-Newtonian Fluid Mech. **6**, 47 (1979).
[21] A behavior similar in character to our negative-slip condition has been described by H. S. Caram and D. C. Hong, Mod. Phys. Lett. **B6**, 761 (1992).