CP Violation in $B^{\pm} \to \gamma\pi^{\pm}\pi^{+}\pi^{-}$

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We consider CP-violating effects in decays of the type $B^{\pm} \to \gamma a_{1,2}^{\pm}$, where $a_{1,2}$ are the $J^P = 1^+$ and 2^+ resonances, each decaying to the common final state via $a_{1,2}^{\pm} \to \pi^{\pm}\rho^0$. The resonances enhance the CP asymmetries and also knowledge of their masses and widths facilitates calculations of the effects. Several types of CP asymmetries are sizable ($\sim$10%–30%); these large asymmetries together with a model calculation of the ratio between the penguin and the annihilation amplitudes, can therefore provide a possible method for measuring the angle α in the unitarity triangle.

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Intense experimental activity is in progress for CP-violation searches in the B meson system. Indeed, at the Stanford Linear Accelerator Center (SLAC) and at KEK, new accelerator facilities, i.e., asymmetric B meson factories, are under construction. The primary focus of these machines, as well as numerous other B-physics experiments in progress, and being proposed at many facilities is CP violation in the *neutral* B. This is due to the fact that, at the moment, reliable predictions can only be made for B^0 as the needed strong phase difference is given in terms of the experimentally measured B^0-$\bar{B}^0$ mixing parameter [1].

Unlike the neutral B system, in charged B decays, it has, so far, not been possible to make reliable quantitative predictions, as the strong final state phase difference that is required cannot be calculated reliably. This is due to the traditional difficulties in handling hadronic matrix elements which involve complicated dynamics of QCD and bound states. To alleviate this outstanding problem we propose to consider decay modes of $B^{\pm}$ that are dominated by at least two neighboring resonances [2]. This has the advantage that the known widths and masses of the resonances allow one to calculate the required strong final state phase difference in terms of the masses and widths of the interfering resonances. Furthermore, dominance of the channels by the resonances and coherent superposition of the contributing amplitudes from the resonances can lead to significant enhancements in the asymmetries close to the resonance region [2]. Let us also briefly recall, in passing, that the charged B meson system has the advantage that (1) all CP violation is unambiguously of the "direct" type, (2) no tagging of "the other" B is necessary, and (3) experiments can be performed at the conventional machines (e.g., Cornell Electron Storage Ring) as well as at the aforementioned asymmetric B-factories that are under construction at SLAC and at the KEK.

We are thus led to investigate the prospects for CP violation in radiative decays of $B^{\pm}$ mesons to pionic final states, i.e., $B^{\pm} \to \gamma\pi^{\pm}\pi^{+}\pi^{-}$. The key feature of this reaction we wish to exploit is in the region where it is dominated by two overlapping resonances, namely, the $J^P = 1^+$, a_1 ($M_{a_1} = 1260$ MeV, $\Gamma_{a_1} \sim 400$ MeV) and $J^P = 2^+$, a_2 ($M_{a_2} = 1318$ MeV, $\Gamma_{a_2} = 110$ MeV). So the reactions of interest are

$$B^{\pm} \to \gamma a_1^{\pm}, \quad a_1^{\pm} \to \rho^0\pi^{\pm}, \quad \rho^0 \to \pi^+\pi^-, \tag{1}$$

$$B^{\pm} \to \gamma a_2^{\pm}, \quad a_2^{\pm} \to \rho^0\pi^{\pm}, \quad \rho^0 \to \pi^+\pi^-. \tag{2}$$

The formalism for assessing CP-violation effects in presence of interfering resonances was given in Ref. [2] where, as an illustration, it was used for radiative decays of B mesons to final states that are dominated by kaonic resonances, i.e., $B \to \gamma K^*(892)$, $\gamma K_1(1270)$, $\gamma K_1(1400)$, $\gamma K^*(1410)$, and $\gamma K_2(1430)$. This class of reactions is, of course, driven largely by the $b \to s$ penguin transition, whereas what we will report in the present study are purely pionic final states which therefore result from $b \to d$ quark transitions. Since in the standard model (SM) all CP violation has to proceed via a single, unique, invariant quantity [3], and since $b \to d$ transitions are relatively suppressed compared to $b \to s$, it is therefore clear that CP-violating asymmetries should be larger in reactions of the type (1) and (2), compared to our previous study involving $B \to \gamma K^*$-like resonances.

These reactions receive contributions from the penguin and the annihilation graph as well. However, since, due to the Cabibbo angle, the annihilation graph for $b \to d$ reactions is larger than it is for the reactions $b \to s$, the two contributing graphs (namely, the penguin and the annihilation) tend to be of comparable strength, and that too enhances the prospects for larger CP asymmetries for reactions (1) and (2). Indeed, asymmetries are typically several tens of percents so that effects at the 3σ level should be observable with about 5×10^8 $B^{\pm}$ mesons. Furthermore, such a final state is expected to reveal CP-conserving asymmetries as well which depend on the CP-conserving "interaction" phase(s) originating from strong interactions, thus giving a better handle on deducing the underlying CP-violating Cabibbo-Kobayashi-Maskawa (CKM) phase.

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Since resonances a_1 and a_2 have different quantum numbers, the amplitudes for reactions (1) and (2) can be simply written as

$$M_j = A_j \Pi_j b_j \,, \tag{3}$$

with $j = 1, 2$. Here A_j describes the weak decay $B \to \gamma a_j$ and therefore contains the CP-violating CKM phase. Π_j is the Breit-Wigner propagator:

$$\Pi_j^{-1} = s - m_j^2 + i\Gamma_j m_j \,,$$

and thus is one source for the CP-conserving "interaction phase." In Eq. (3) b_j describes the strong decay of the resonance a_j to the final state $\rho_0 \pi^{\pm}$. Because of its width the decay of the ρ_0 via $\rho^0 \to \pi\pi$ introduces an additional source of an interaction phase that has to be included.

As in Ref. [2] we use a bound state model [2,4] to describe the conversion from the quark level weak amplitudes to the formation of resonances in the exclusive channels via $B \to \gamma a_{(1,2)}$. In the leading approximation in which we are working, the virtual W boson emerging from the annihilation of B cannot make the $(J = 2)$ a_2 resonance. Therefore we set the corresponding amplitude for the formation of a_2 via the annihilation graph to be 0. In addition, using [6,7] $B(b \to s\gamma) = 2.5 \times 10^{-4}$ (corresponding to $m_t \sim 170$ GeV), and the constraints from experiment and theory on $b \to u$ and $b \to c$ transitions, K-$\bar{K}$ and B-$\bar{B}$ mixing [8,9] we find

$$B_1^{\mathrm{pen}} \simeq (1.3\text{–}2.0) \times 10^{-7} \,, \tag{5a}$$

$$B_1^{\mathrm{ann}} \simeq (1.5\text{–}4.6) \times 10^{-7} \,, \tag{5b}$$

$$B_2^{\mathrm{pen}} \simeq (1.0\text{–}1.7) \times 10^{-7} \,, \tag{5c}$$

where, e.g., B_1^{pen} is the branching ratio for $B \to \gamma a_1$ via the penguin graph. The CP-violating phase δ_{cp} is then given by

$$\delta_{cp} = \mathrm{Arg}\Big[A_2^{\mathrm{pen}}\Big(A_1^{\mathrm{ann}*} + A_1^{\mathrm{pen}*}\Big)\Big]. \tag{6}$$

Using the standard Wolfenstein parametrization [8,10,11] of the CKM matrix one gets

$$\mathrm{Arg}(A_2^{\mathrm{pen}} A_1^{\mathrm{ann}*}) = \mathrm{Arg}[(\rho + \mathrm{i}\eta)(1 - \rho + \mathrm{i}\eta)] \tag{7}$$

$$= \gamma + \beta = \pi - \alpha \,, \tag{8}$$

where ρ, η are the usual parameters of that matrix and α, β, and γ are the angles in the unitarity triangle [8,11]. Thus

$$\delta_{cp} = \mathrm{Arg}\left[\sqrt{B_1^{\mathrm{pen}}} - \sqrt{B_1^{\mathrm{ann}}}\, e^{-i\alpha}\right], \tag{9}$$

and therefore it follows that the charged B mesons via modes under discussion, namely (1,2), should allow a determination of one of the angles (namely α) in the unitarity triangle. Note also that as the $B(B \to \gamma a_1)$ and $B(B \to \gamma a_2)$ get experimentally measured (which should happen well before the CP asymmetries become observable), the uncertainties in Eq. (9) due to the model dependence of Eq. (5) should get reduced. However, some model dependence still survives as one needs to know the fraction of the rate for $B \to \gamma a_1$ that proceeds through the annihilation graph.

For the strong decay $a_1 \to 3\pi$ the amplitude is given by

$$b_1 = c_1 m_1 a_1^{\mu}\Big[(p_0 - p_1)_{\mu}\, \pi_{01} + (p_0 - p_2)_{\mu}\, \pi_{02}\Big], \tag{10}$$

where m_1 is the mass of a_1, p_1, p_2 are the momenta of the two identical pions and p_0 that of the third pion, $\pi_{ij} = [(p_i + p_j)^2 - m_{\rho}^2 + i\Gamma_{\rho} m_{\rho}]^{-1}$, and $i, j = 0, 1, 2$. Similarly, for $a_2 \to 3\pi$ the strong amplitude is

$$b_2 = 2c_2 a_2^{\mu\nu}[(p_0 - p_1)_{\mu}\, p_{2\nu} \pi_{01} + (p_0 - p_2)_{\mu}\, p_{1\nu} \pi_{02}]. \tag{11}$$

The constants c_1 and c_2 are determined by the measured total widths [12] to be 22.75 and 28.20, respectively.

Contributions to CP-violating observables require interference between the CP-violating phase δ_{CP} with the strong rescattering phase(s). In our formulation, encapsulated in Eq. (3), the strong phases originate from the widths of $a_{1,2}$ as well as from the width of ρ_0. Thus the importance of the resonances idea lies in allowing a clean determination of the necessary strong final state (CP-conserving) phases in terms of the known [12] masses and the widths of the resonances. It is, therefore, not necessary to calculate these phases by using QCD and bound state models, and as a result the uncertainty in predicting CP-violation asymmetries, due to these sources, can get significantly reduced. However, the method does not allow for the calculation of the magnitudes of the amplitudes, and, consequently, these are still left to a theoretical model-dependent calculation.

To understand the various asymmetries that arise we rewrite the propagators for $a_{1,2}$ so that the relevant rescattering phases are explicitly exhibited. Thus for the $a_{1,2}$ we write

$$\Pi_j = \hat{\Pi}_j \exp(-i\alpha_j) \,. \tag{12}$$

Furthermore, since there are two pions with the same charge in the final state [e.g., $B^+ \to \gamma\pi^+(p_1) + \pi^+(p_2) + \pi^-(p_0)$], therefore there are two ways in which the ρ propagator enters. For convenience, we decompose this in a symmetric (Σ) and an antisymmetric (Δ) combination:

$$\Sigma = \pi_{02} + \pi_{01}, \quad \Delta = \pi_{02} - \pi_{01} \,.$$

Once again we factor out the phases

$$\Sigma = \hat{\Sigma} \exp(-i\rho_1), \quad \Delta = \hat{\Delta} \exp(-i\rho_2) \,.$$

The resulting phases that determine the asymmetries are then the differences:

$$\Delta\alpha = \alpha_1 - \alpha_2 \quad \text{and} \quad \Delta\rho = \rho_1 - \rho_2 \,.$$

Altogether there are six types of CP-violating asymmetries that arise. All of the CP-odd quantities, of course, have to be proportional to $\sin\delta_{CP}$. But, in addition, those observables that are odd under "naive time reversal" (denoted by T_N and meaning time $\to$ –time without interchange of initial and final states) will also have to be proportional to $\cos\Delta\alpha$ or $\cos(\Delta\alpha \pm \Delta\rho)$, whereas the T_N-even ones are proportional to $\sin\Delta\alpha$ or $\sin(\Delta\alpha \pm \Delta\rho)$. Thus the square of the invariant amplitude can be expressed as

$$|M_1 + M_2|^2 = P + \sin\delta_{CP}R\,, \tag{13}$$

where P is the CP-conserving part, and $R = R_o + R_e$ is the CP-violating part. Here R_o (i.e., the C-even, P-odd, T_N-odd part) contains terms proportional to $\cos\Delta\alpha$ or $\cos(\Delta\alpha \pm \Delta\rho)$. R_e (i.e., C-odd, P-even, T_N-even part) contains terms proportional to $\sin\Delta\alpha$ or $\sin(\Delta\alpha \pm \Delta\rho)$.

Numerical results for the asymmetries are given in Table I [13]. A simple observable that exhibits a sizable asymmetry is

$$\epsilon_{fb} = \langle Q_B\sigma(\cos\theta)\sigma(s - s_0)\rangle\,, \tag{14}$$

where $\sigma(x) = +1$ if $x > 0$ and -1 if $x < 0$, $\cos\theta \equiv \hat{\mathbf{p}}_0 \cdot \hat{\mathbf{q}}$ where, $\mathbf{q}$ is the momentum of the photon, and $\mathbf{p}_0$ is the momentum of the π^- (in B^+ decay) in the rest frame of $a_{1,2}$. Q_B is the charge of the $B^\pm$ meson. The quantity s is the invariant mass of the three pions, and

$$s_0 = \frac{\Gamma_1 m_1 m_2^2 - \Gamma_2 m_2 m_1^2}{\Gamma_1 m_1 - \Gamma_2 m_2} \tag{15}$$

is the point at which $\sin\Delta\alpha$ switches sign. Thus ϵ_{fb} is a CP-violating forward-backward asymmetry, and from Table I we see that it ranges from 7%–11%.

In the Table we also show a simple triple product correlation asymmetry,

$$\epsilon_t \equiv \langle\sigma(\sin 2\phi)\rangle\,, \tag{16}$$

where $\sin\phi = [(\mathbf{p}_2 \times \mathbf{p}_1)\cdot\mathbf{q}]/|\mathbf{p}_1 \times \mathbf{p}_2||\mathbf{q}|$, $\cos\phi = (\mathbf{p}_2 - \mathbf{p}_1)\cdot\mathbf{q}/|\mathbf{p}_2 - \mathbf{p}_1||\mathbf{q}|$. For the purpose of this observable the momentum of the identical pions ($p_{1,2}$) is ordered by energy. The resulting CP-violating asymmetry ranges from 7% to 10%.

From Eq. (11), following Ref. [14], the optimal observable for CP violation is

$$\epsilon_{\rm opt} \equiv \langle R/P\rangle\,. \tag{17}$$

We find $\epsilon_{\rm opt}$ to be about 20%–35%. This CP-violating observable can be separated into T_N-odd and T_N-even pieces. The corresponding observables, $\epsilon_o \equiv \langle R_o/P\rangle$ and $\epsilon_e = \langle R_e/P\rangle$, are about 15%–20% and 20%–30%, respectively.

In addition to such CP-violating asymmetries, the final state also exhibits rather interesting CP-conserving asymmetries. As an example of this class of asymmetries we show, in Table I,

$$\zeta_{fb} \equiv \langle\sigma(\cos\theta)\rangle\,,$$

which is about 20%-25%. Measurements of such CP-conserving asymmetries would yield information on the CP-conserving interaction phase(s).

It is useful to note that the measurement of the momenta of the four particles in the final state allows one to construct all of the CP-violating and CP-conserving asymmetries we are discussing. Of course, demonstration of CP violation as well as relating the observable to the basic parameter (namely α) can be done through any one of the observables.

In Fig. 1 we show the differential asymmetries as a function of s for the three cases mentioned above. We have assumed typical values for the CKM parameters. These expectations (in Table I and Fig. 1) are based on the two resonances dominating the continuum of the three pions in the interval $1 \lesssim s \approx 2.5$ GeV^2. The experimental data on the invariant mass distribution of the three pion would of course be a very reliable indicator of the extent to which the resonances dominate over the continuum. The stronger the dominance holds the better the approximation used here will work. The CP-conserving distribution given in Fig. 1 will also help in testing the resonance hypothesis and in unravelling the CP-conserving strong phase.

In calculating the numbers given in Table I and in Fig. 1 we used the bound state model of Isgur *et al.* [4] with modifications given in Ref. [2]. The ranges in

TABLE I. Observables and their transformation properties. The ranges of the expected asymmetries are obtained by varying over the allowed region of the CKM parameters. (See Ref. [9].) $N_B^{3\sigma}$ is the number of $B^\pm$ needed for detection at the 3σ level [15].

Observable	Transformation Property CP	P	T_N	Expected size (%)	$N_B^{3\sigma}/10^8$
ϵ_{fb}	$-$	$+$	$+$	7 – 11	30 – 40
ϵ_t	$-$	$-$	$-$	7 – 10	40 – 50
$\epsilon_{\rm opt}$	$-$	Mixed	Mixed	20 – 35	3 – 5
ϵ_e	$-$	$+$	$+$	20 – 30	5 – 6
ϵ_o	$-$	$-$	$-$	15 – 20	8 – 12
ζ_{fb}	$+$	$+$	$+$	20 – 25	4 – 10

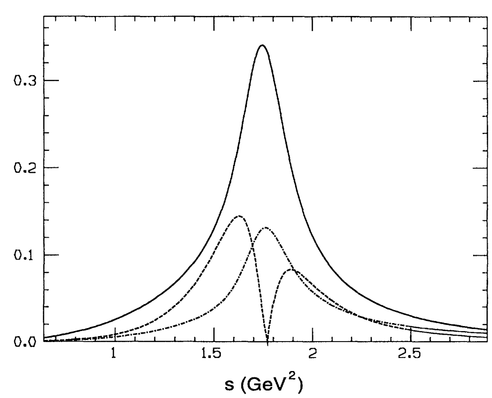

FIG. 1. Asymmetries as a function of s for the Wolfenstein parameters $\{A = 0.86,\ \rho = 0.10,\ \eta = 0.45\}$. The solid line is for $|m_1^2\, d\zeta_{fb}/ds|$, the dashed line is for $|m_1^2\, d\epsilon_{fb}/ds|$, and the dot-dashed line is for $|m_1^2\, d\epsilon_t/ds|$. The solid line is therefore for a CP-conserving asymmetry; the other two are CP violating.

Table I are obtained by varying over the allowed 90% CL limits of the CKM parameters [9]. We note, in passing, that the asymmetries, being ratios of rates, tend to be less dependent on the bound state model as compared to the rates. Also, as we mentioned earlier, the model dependence should be further reduced as data on the branching fractions become available.

As the numbers in the Table indicate, these effects should be observable with about 10^8–10^9 $B^{\pm}$ mesons [15]. This is especially notable given that we are dealing here with radiative transitions. The basic idea of interfering resonances when used in the context of purely hadronic modes should need significantly fewer B mesons. We shall discuss some of these applications in forthcoming publications.

Summarizing, the theoretical ideas presented in this Letter allow the extraction of the CP-violating phase (δ_{CP}) from the CP-violating asymmetries in a model independent fashion. However, the extraction of the CP-violating angle α of the unitarity triangle from δ_{CP} is still model dependent as it requires a calculation of the ratio between the penguin and the annihilation amplitudes. The resulting uncertainties are hard to estimate and could be rather large.

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[7] For related experimental literature, see CLEO Collaboration, R. Ammar *et al.*, Phys. Rev. Lett. **71**, 674 (1993); Y. Rozen, Ph.D. thesis, Syracuse University, 1993 (unpublished).

[8] For a recent update on the CKM parameters and the unitarity triangle, see M. Witherell, Report No. UCSB-HEP-94-02 (unpublished).

[9] Our constraints are a little different form those in Ref. [8]. The difference is primarily due to the fact that we use results from the lattice for the relevant hadronic parameters. Thus we use (in standard notation), $\hat{B}_K = 0.8 \pm 0.1$, $f_B = 187 \pm 50$ MeV, and $\hat{B}_B = 1 \pm 0.2$, where the errors are our best estimates at 90% C.L. As a result we find $0.15 \lesssim |V_{td}/V_{ts}| \lesssim 0.30$, $10° \lesssim |\mathrm{Arg}\, V_{td}| \lesssim 40°$; $0.035 \lesssim |V_{ub}/V_{cb}| \lesssim 0.130$ and $30° \lesssim |\mathrm{Arg}\, V_{ub}| \lesssim 135°$.

[10] See Particle Data Group, Phys. Rev. D **11**, III.65–67 (1992); L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983); see also L. L. Chau and W. Y. Keung, *ibid.*

[11] See, e.g., Y. Nir and H. Quinn, Annu. Rev. Nucl. Part. Sci. **42**, 211 (1992).

[12] See Particle Data Group (Ref. [10]), in particular, p. II.7.

[13] Since the interfering resonances do not have identical quantum numbers, partial rate asymmetries cannot arise. See Ref. [2].

[14] D. Atwood and A. Soni, Phys. Rev. D **45**, 2405 (1992).

[15] Note that detector and particle identification efficiencies are not included in the estimates that are given here for the needed number of B's.