Alpha Clustering and Structure of ^{94}Mo and ^{212}Po

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The α-particle clustering structure of ^{94}Mo and ^{212}Po, which is of interest for investigating the persistence of α clustering in heavy nuclei, is studied within the framework of a local potential approach using a double folding model. It is shown that the model, which describes α scattering from ^{90}Zr well, locates the ground state of ^{94}Mo at the energy corresponding to experiment. A similar result was obtained for the $\alpha + ^{208}$Pb system. It is found that the model gives not only the ground band of ^{94}Mo and ^{212}Po as compact α-cluster states but also predicts other developed genuine α-cluster states below and near the Coulomb barrier.

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Alpha clustering is very important in light nuclei [1]. As the nucleus becomes heavier, the spin-orbit force becomes stronger; therefore it has been considered that α-cluster correlation may be suppressed in heavy nuclei. However, recent observations of a theoretically predicted $K = 0^-$ band of a parity doublet both in ^{44}Ti [2–4] and ^{40}Ca [5,6] have given firm evidence that the α-cluster model persists in the beginning of the fp-shell region. This encourages us to study further the α-clustering aspects in much heavier nuclei. ^{94}Mo and ^{212}Po are typical nuclei which have two protons and two neutrons outside the double closed shell; however, their α-clustering aspects are scarcely understood. It is the purpose of this Letter to show that the structure of ^{94}Mo and ^{212}Po is understood from the viewpoint of the α-cluster model.

As for the ^{212}Po nucleus, most of the studies done up to now have focused on how the observed large magnitude of the α width of the *ground state* is explained. Although a shell model calculation describes the level structure of the ground band well [7], it fails completely in reproducing the observed large α width of the ground state. A large-space configuration mixing shell model calculation [8] improves this; however, still the result is more than 1 order of magnitude smaller than the experiment. This suggests that other important correlations which are not involved in the shell model are necessary. To explain experimental results, Okabe [9] performed a hybrid model calculation with shell and cluster configurations assuming an existence of a genuine α-cluster state at $E_x = 5$ MeV and showed that the resultant α width of the ground state is greatly enhanced by a coupling. Varga *et al.* [10] reconfirmed Okabe's conclusion in a more rigorous hybrid model calculation. The above-mentioned approaches are based on the philosophy that the ground state which originates from the *shell model configuration* has a component of α-cluster correlation by a *coupling* with a genuine α-cluster state which lies at higher excitation energies. It is interesting to know whether the structure of ^{212}Po (not only the ground state but also the excited states) can be understood *starting from the α-cluster model.*

We investigate whether the approach of unifying bound and scattering states, which was very successful in solving a long-standing controversy about the α-cluster structure of ^{44}Ti [2,3], works in heavy nuclei. In the case of the $\alpha + ^{40}$Ca system, a backward angle anomaly (BAA) as well as rainbow scattering made it possible to determine unambiguously an interaction potential between α and ^{40}Ca up to the very inner region [11]. However, the BAA does not appear in α scattering from nuclei heavier than $A \approx 48$. Even in $\alpha + ^{90}$Zr scattering strong absorption masks the internal region. Furthermore, Put and Paans [12] reported through a systematic analysis of $\alpha + ^{90}$Zr scattering in a wide energy range that a shape of the real part of optical potentials in the low energy region should change from the one in the high energy region. This makes it difficult to extend an optical potential determined uniquely in a phenomenological analysis at higher energies to the very lower energy region while keeping its shape.

First we study the ^{94}Mo nucleus before we proceed to the ^{212}Po nucleus and confirm that a double folding model works well for the $\alpha + ^{90}$Zr system. We notice that the so-called unique potentials for $\alpha + ^{40}$Ca scattering [11] and $\alpha + ^{16}$O scattering [13] are very similar to the folding model potentials [3,14–16], and we take the double folding model with a two-body interaction of DDM3Y [17], which has not only density dependence but also energy dependence. The folding potential is given by

$$U(\vec{R}) = \lambda \int_0^\infty d\vec{r}_1 \int_0^\infty d\vec{r}_2 \, \rho_T(\vec{r}_1)\rho_\alpha(\vec{r}_2) \times t(E_\alpha, \rho_T, \rho_\alpha, \vec{s} = \vec{R} + \vec{r}_2 - \vec{r}_1), \quad (1)$$

where ρ_α and ρ_T are the matter density distributions of α and target nucleus, respectively, and t is the effective nucleon-nucleon interaction which depends on the local densities and bombarding energy E_α. The departure of the normalization factor λ from unity indicates a necessity to take into account the other effects not included in the bare folding model. To take account of absorption phenomenologically, an imaginary potential consisting of two

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terms of volume type and surface derivative type based on the square of the Woods-Saxon is introduced. The Coulomb potential due to a uniformly charged sphere of radius $R_c = 6.27$ fm is assumed. The matter density distributions for α and ^{90}Zr are taken from Refs. [16] and [18], respectively. By taking λ values which decrease gradually with decreasing incident energy ($\lambda = 1.33$ at $E_\alpha = 79.5$ MeV and 1.21 at $E_\alpha = 23.4$ MeV), α scattering from ^{90}Zr [17,19] can be reproduced well from high energy to low energy (Fig. 1). This suggests that not only the surface region but also the interior region of the real part of the potential is well reproduced. The lowest incident energy in Fig. 1 is a little bit above the Coulomb barrier. Therefore it is meaningful to study the quasibound and bound states of the composite system ^{94}Mo using this potential.

The energy levels of ^{94}Mo are calculated in the bound state approximation by using the potential at the lowest incident energy. The bound states with $N = 2n + l < N_0$ have to be discarded due to the Pauli principle. We note that the $K = 0^+$ band with $N = 16$ corresponds to the experimental ground band of ^{94}Mo. $N_0 = 16$ seems to be consistent with the microscopic calculations in the resonating group method [20]. By slightly modifying the normalization factor (~4%) to $\lambda = 1.161$ (volume integral per nucleon pair $J_V = 322$ MeV fm^3), the energy of the calculated 0^+ state corresponds exactly with the experimental ground state. The calculated ground band with this value shows a rotational spectrum. To reproduce the experimental spectrum very well which deviates from a perfect rotational spectrum, a small l dependence in λ is required, as was the case in ^{44}Ti [2] and ^{20}Ne [15]. The linear dependence of $\lambda = \lambda_0 - cl$ with $\lambda_0 = 1.160$ and $c = 0.0041$ reproduces very well the experimental spectrum. The decreasing trend and its rate (c/λ_0) is nearly the same as the one required in ^{44}Ti [2] ($c/\lambda_0 = 0.0033$ and 0.0035 for ^{44}Ti and ^{94}Mo, respectively). The necessity of l dependence of the potentials in the low energies is reasonable. In fact, microscopic studies of interaction between composite particles have revealed that the equivalent local potential should have an l dependence (although small) due to the Pauli principle and that the strength of the potential should decrease as l becomes higher [21]. It was also shown that this l dependence is indispensable when we investigate the low energy properties of the composite systems using a local potential [22]. Since the folding model does not take into account the Pauli principle explicitly, it is quite reasonable to supplement the model by introducing an l dependence in λ in the present study of the low energy and negative energy region. The calculated intercluster rms radius of the ground state (Table I) amounts to 89% of the sum of the experimental radii [23] for α (1.674 fm) and ^{90}Zr (4.244 fm), which is as large as that in ^{44}Ti [2]. This suggests that the ground state has a compact α-cluster structure. The calculated $B(E2)$ values (Table I) are very much enhanced and by introducing a small effective charge, $\delta e = 0.2e$, good agreement with experiment [24] is obtained. The values for high spin states for which shell-model-like character increases may be overestimated [25]. In the shell model [24] the large effective charges are needed to reproduce the experimental $B(E2)$ values. The present calculations locate the α-cluster states with $N = 17$ $K = 0^-$ band, which is a parity-doublet partner of the ground band, at about 6 MeV, and the higher nodal $N = 18$ $K = 0^+$ band just below the Coulomb barrier.

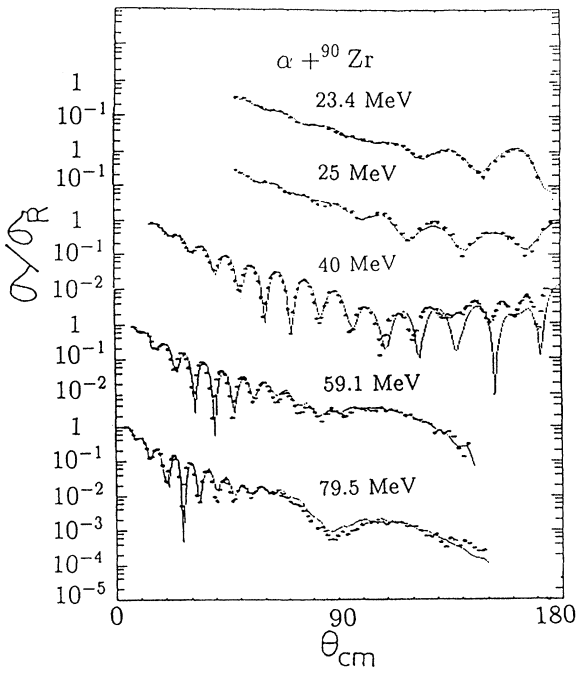

FIG. 1. Calculated angular distributions for $\alpha + {}^{90}$Zr scattering (solid lines) are compared with the experimental data.

Now we come back to ^{212}Po and study its structure from the same viewpoint. The matter density distribution for ^{208}Pb in Ref. [26] is used in the folding model calculation. By taking λ values which decrease gradually with decreasing incident energy ($\lambda = 1.225$ at $E_\alpha = 42$ MeV and 1.20 at $E_\alpha = 19$ MeV) experimental angular distributions for $\alpha + {}^{208}$Pb scattering [27] are reproduced well (Fig. 2). The obtained $J_V = 328$ MeV fm^3 (for $\lambda = 1.225$) is consistent with the values in other α-particle scattering from heavy and light nuclei. The incident energy of 19 MeV is near the Coulomb barrier height. The calculated bandhead of the quasibound state with $N = 22$ falls above the α threshold in the range of the experimental ground band of ^{212}Po. By slightly decreasing the normalization factor (~3%) to $\lambda = 1.1805$, the energy of the calculated 0^+ state corresponds exactly with the experimental ground state. In cases where the absolute binding energy of the 0^+ state with $N = 20$ ($N = 24$) with respect to the α threshold would correspond with the ground

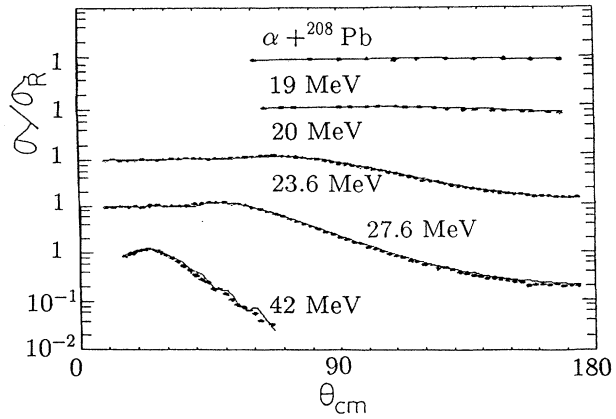

FIG. 2. Same as Fig. 1 but for α + ^{208}Pb scattering and R_c = 7.41 fm.

state of ^{212}Po, the value of λ needs to be decreased (increased) to $\lambda = 1.0422$ (1.3297) with $J_V = 279$ MeV fm^3 (356 MeV fm^3). $N_0 = 22$ seems physically favored and coincides with the result in the resonating group method [20]. As in the case of the α + ^{90}Zr system, introduction of a very weak l dependence in the strength of λ ($\lambda_0 = 1.1778$ and $c = 0.0025$) reproduces the spectrum of the experimental ground band. This dependence shows a similar trend to the α + ^{90}Zr system. In Table II the λ values which reproduce exactly the experimental energies with respect to the α threshold together with the calculated rms radii and $B(E2)$ values are shown. It is noted that the volume integral changes little from $J = 0^+$ (316 MeV fm^3) to 10^+ (309 MeV fm^3). The calculated intercluster distance of the ground state amounts to 90% of the sum of the experimental rms radii [23] for α and ^{208}Pb (5.502 fm), which suggests the ground band has a significant amount of α clustering. The calculated $B(E2)$ values are enhanced very much without effective charges: The agreement with experiment [28] is gratifying for the $6^+ \rightarrow 4^+$ transition if one takes into account that the shell model calculations give results which are 1 to 2 orders of magnitude smaller than the experiment even with effective charges. For high spin states, mixing of symmetry breaking components due to the spin-orbit force would decrease the calculated values [25]. The calculated wave function of the ground state is very similar to the harmonic oscillator wave function with $N = 22$ in the inner region; however, at the surface the wave function is slightly pushed outward with its outermost peak at 8.2 fm, which brings about the enhancement of $B(E2)$ values and α widths.

We calculated α-reduced widths using R-matrix theory. The R-matrix reduced width is given by $\gamma_\alpha^2 = (\hbar^2 a/2\mu)\Omega_l^2(a)$, where $\Omega_l(a)$ is the amplitude of the amount of clustering [10] at a channel radius a and μ the reduced mass. In the present model the $\Omega_l(a)$ is taken to be a wave function $\psi_l(a)$ at a beyond which only the Coulomb potential works. A channel radius $a = 10.5$ fm is used. For the 0^+ and 2^+ states reduced widths γ_α^2 experimentally deduced in the α-transfer reactions, $\gamma_\alpha^2(0^+) = 1.4$ keV and $\gamma_\alpha^2(2^+) = 0.48$ keV [29] are available. The value of $\gamma_\alpha^2(0^+)$ is consistent with the value 1.3 keV deduced from an analysis of α decay [29]. Our calculated results are $\gamma_\alpha^2(0^+) = 1.23$ keV and $\gamma_\alpha^2(2^+) = 1.36$ keV. The agreement with the experiment is good for the 0^+ state. For the 6^+ and 8^+ states, α-decay widths $\Gamma_\alpha = 4.26 \times 10^{-13}$ MeV and 1.12×10^{-14} MeV were experimentally reported [30]. Our calculation gives 1.23×10^{-12} MeV and 1.62×10^{-13} MeV for the 6^+ and 8^+ states, respectively. The overestimated result for the 8^+ state is consistent with the overestimated result of $B(E2)$ value in Table II. The discrepancy between theory and experiment which increases as the spin increases is qualitatively understood if we consider that the mixing of symmetry broken shell model components due to the spin-orbit force increases as the spin becomes higher [25] and will be remedied by taking account of shell model configurations which are orthogonal to the present cluster model configurations [10]. The (18^+) state at 2.922 MeV with a long half-life of 45.1 s [30] has been considered to be an isomer. Our calculated half-life, 25 ms, is 3 orders of magnitude smaller than the experiment. This means that the 18^+ state is strongly mixed by shell model configurations and the α + ^{208}Pb cluster structure is greatly broken. We see that the structure change from an α-cluster state to a shell model state as the spin becomes higher

TABLE I. Theoretical and experimental values for the $J \rightarrow J - 2$ transitions and intercluster rms radii for the ground band of ^{94}Mo. The normalization factor λ given is that reproducing the experimental energies with respect to the α threshold. W.u. denotes Weisskopf units.

J^π	E_x (MeV)	λ	$\sqrt{\langle R^2\rangle}$ (fm)	$B(E2)$ (W.u.) Exp. [24]	Cal.
0^+	0.0	1.161	5.28		
2^+	0.871	1.150	5.30	17.2	17.4
4^+	1.574	1.143	5.26	26.6	24.1
6^+	2.423	1.136	5.20		24.3
8^+	2.955	1.135	5.09		21.6

TABLE II. Same as Table I but for ^{212}Po.

J^π	E_x (MeV)	λ	$\sqrt{\langle R^2\rangle}$ (fm)	$B(E2)$ (e^2 fm^4) Exp. [28]	Cal.
0^+	0.0	1.1805	6.60		
2^+	0.727	1.1711	6.62		579
4^+	1.132	1.1655	6.60		814
6^+	1.355	1.1620	6.54	1013	844
8^+	1.476	1.1590	6.46	297	801
10^+	1.834	1.1531	6.38	164	729

persists more purely in the ground band of ^{212}Po. That the 18^+ state is dominated by the shell model configurations is supported by the fact that the shell model wave function in Ref. [31] can reproduce the reduced α width of the isomer within a factor $\frac{1}{1.7}$ [8], although it gives, for the ground state, a reduced α width which is 3 orders of magnitude smaller than the experiment. This 18^+ isomer, which is in the line of the 8^+ state of ^{20}Ne and 12^+ state of ^{44}Ti, is understood as a state whose shell model character is extremely intensified.

Our calculation also gives excited states with α-cluster structure with higher N. The $N = 23$ $K = 0^-$ band starts from $E_x = 5.4$ MeV and the second $K = 0^+$ band with $N = 24$ appears from 9.7 MeV. The calculated dimensionless reduced width θ_α^2 for the $N = 24$ 0^+ state is 48%, which is large enough to be observed in experiment near the Coulomb barrier. The present calculation does not give a genuine α-cluster 0^+ state at such a low excitation energy as 2.5 [10] or 5 MeV [9]. Since the folding potentials do not differ very much for neighboring nuclei, persistency of α clustering in the ^{212}Po region is expected. Recently Buck *et al.* [32] studied the α-cluster structure of ^{212}Po using phenomenological potentials; however, it may be worth mentioning that the potentials whose volume integral ranges from $J_V = 287$ ($J = 0^+$) to 258 MeV fm^3 ($J = 10^+$) within the ground band are not consistent with the values in this region and cannot describe α scattering due to the short tail.

In conclusion, we have shown that the ground state of ^{212}Po is described in the $\alpha + ^{208}$Pb cluster model by the use of the intercluster potential calculated in the double folding model. It is found that although the ^{212}Po ground band does not have a typical cluster structure it is understood in the picture of unification of bound and scattering states. The structure change of the ground band from a cluster state to a pure shell model state of isomer (18^+) is understood from the viewpoint of the α-cluster model that the α-cluster structure is destroyed by the increase of the mixing of the symmetry broken shell model components due to the strong spin-orbit force as the spin gets higher. Developed α-cluster states are expected at about 10 MeV which are analogs of the higher nodal states observed in ^{20}Ne and ^{40}Ca. Similar α-cluster band structure is also obtained for ^{94}Mo. It seems that the α-cluster model persists throughout the periodic table.

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