

πNN and Pseudoscalar Form Factors from Lattice QCD

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The πNN form factor $g_{\pi NN}(q^2)$ is obtained from a quenched lattice QCD calculation of the pseudoscalar form factor $g_P(q^2)$ of the proton with pion pole dominance. We find that $g_{\pi NN}(q^2)$ is well fitted with a monopole form which agrees with the Goldberger-Treiman relation. The monopole mass is determined to be 0.75 ± 0.14 GeV, which shows that $g_{\pi NN}(q^2)$ is rather soft. The extrapolated πN coupling constant $g_{\pi NN} = 12.7 \pm 2.4$ is quite consistent with the phenomenological values. We also compare $g_{\pi NN}(q^2)$ with the axial form factor $g_A(q^2)$ to check pion dominance in the induced pseudoscalar form factor $h_A(q^2)$ vis-à-vis the chiral Ward identity.

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The πNN form factor $g_{\pi NN}(q^2)$ is a fundamental quantity in low-energy pion-nucleon and nucleon-nucleon dynamics. Many dynamical issues, such as πN elastic and inelastic scattering, NN potential, three-body force (triton and ^3He binding energies), pion photoproduction, and electroproduction, depend on it. Similarly, the pseudoscalar form factor is important in testing low-energy theorems, the chiral Ward identity, and the understanding of the explicit breaking of chiral symmetry. Yet, compared with the electromagnetic form factors and the isovector axial form factor of the nucleon, the pseudoscalar form factor $g_P(q^2)$ and the πNN form factor $g_{\pi NN}(q^2)$ are poorly known experimentally and theoretically.

Notwithstanding decades of interest and numerous works, the shape and slope of $g_{\pi NN}(q^2)$ remain illusive and unsettled. Upon parametrizing $g_{\pi NN}(q^2)$ in the monopole form

$$g_{\pi NN}(q^2) = g_{\pi NN} \frac{\Lambda_{\pi NN}^2 - m_\pi^2}{\Lambda_{\pi NN}^2 - q^2}, \quad (1)$$

with $g_{\pi NN} \equiv g_{\pi NN}(m_\pi^2)$, the uncertainty in the parametrized monopole mass $\Lambda_{\pi NN}$ can be as large as a factor of 2 or 3. For the sake of having a sufficiently strong tensor force to reproduce the asymptotic D -to- S -wave ratio and the quadrupole moment in the deuteron, $\Lambda_{\pi NN}$ is shown to be greater than 1 GeV [1]. Consequently, $\Lambda_{\pi NN}$ in the realistic NN potentials are typically fitted with large $\Lambda_{\pi NN}$ (e.g., $\Lambda_{\pi NN}$ ranges from 1.3 GeV [2] to 2.3–2.5 GeV [3]). On the other hand, arguments based on resolving the discrepancy of the Goldberger-Treiman relation [4] and the discrepancy between the $pp\pi^0$ and $pn\pi^+$ couplings [5] suggest a much softer $g_{\pi NN}(q^2)$ with $\Lambda_{\pi NN}$ around 0.8 GeV. Furthermore, hadronic models of baryons with meson clouds, such as the Skyrmin model, typically have a rather soft form factor (i.e., $\Lambda_{\pi NN} \sim 0.6$ GeV) [6] due to the large pion cloud, and such a small $\Lambda_{\pi NN}$ is needed for high-energy elastic pp scattering [7].

In view of the large uncertainty in $g_{\pi NN}(q^2)$, it is high time to study it in a lattice QCD calculation.

Since our recent calculations of the nucleon axial and electromagnetic form factors are within 10% of the experimental results [8,9], a prediction of $g_{\pi NN}(q^2)$ with a similar accuracy should be enough to adjudicate on the controversy over the πNN form factor. In this Letter, we extend our lattice calculation to the proton pseudoscalar form factor for a range of relatively light quark masses (around the strange quark mass). $g_{\pi NN}(q^2)$ is obtained by considering the pion pole dominance in $g_P(q^2)$ when the latter is extrapolated to the quark mass which corresponds to the physical pion mass.

In analogy to the study of the electromagnetic and axial form factors [8,9] of the nucleon, we calculate the following two- and three-point functions for the proton:

$$G_{pp}^{\alpha\alpha}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 | T(\chi^\alpha(x) \bar{\chi}^\alpha(0)) | 0 \rangle, \quad (2)$$

$$G_{pp}^{\alpha\beta}(t_f, \vec{p}, t, \vec{q}) = \sum_{\vec{x}_f, \vec{x}} e^{-i\vec{p}\cdot\vec{x}_f + i\vec{q}\cdot\vec{x}} \times \langle 0 | T(\chi^\alpha(x_f) P(x) \bar{\chi}^\beta(0)) | 0 \rangle, \quad (3)$$

where χ^α is the proton interpolating field and $P(x)$ is the mean-field improved isovector pseudoscalar current for the Wilson fermion

$$P(x) = \frac{2\kappa}{8\kappa_c} e^{m_q a} \bar{\psi}(x) i\gamma_5 \frac{\tau_3}{2} \psi(x). \quad (4)$$

Here, we have included the $2\kappa/8\kappa_c$ ($\kappa_c = 0.1568$ is the critical κ value for the chiral limit for our lattice at $\beta = 6.0$) and the $e^{m_q a}$ [$m_q a = \ln(4\kappa_c/\kappa - 3)$ is the quark mass] factors in the definition of the lattice current operator. These factors take into account the mean-field improvement and finite quark mass correction for the Wilson action [10], and have been shown to be an important improvement in the evaluation of the axial form factor in order to allow the perturbative lattice renormalization to work [8].

Phenomenologically, the pseudoscalar current matrix element is written as

$$\langle \vec{p}s | P(0) | \vec{p}'s' \rangle = g_P(q^2) \bar{u}(\vec{p}, s) i \gamma_5 u(\vec{p}', s'), \quad (5)$$

where $g_P(q^2)$ is the pseudoscalar form factor. It has been shown [8,9] that when $t_f - t$ and $t \gg a$, the lattice spacing, the combined ratios of three-point and two-point functions with different momentum transfers, lead to the desired form factors related to the probing currents. In the case of the pseudoscalar current in Eq. (4), the lattice pseudoscalar form factor $g_P^L(q^2)$ is given by the following ratio:

$$\frac{\Gamma^{\beta\alpha} G_{pp}^{\alpha\beta}(t_f, \vec{0}, t, \vec{q})}{G_{pp}^{\alpha\alpha}(t_f, \vec{0})} \frac{G_{pp}^{\alpha\alpha}(t, \vec{0})}{G_{pp}^{\alpha\alpha}(t, \vec{q})} \rightarrow \frac{q_3}{E_q + m} g_P^L(q^2), \quad (6)$$

where $\Gamma = \gamma_3 \gamma_5 (1 + \gamma_4)/2$ and m and E_q are the proton mass and energy with momentum \vec{q} , respectively.

Quark propagators have been generated on 24 quenched gauge configurations on a $16^3 \times 24$ lattice at $\beta = 6.0$ to study the nucleon electromagnetic and axial form factors [8,9]. We shall use the same propagators for the present calculation. Results are obtained for three relatively light quarks with $\kappa = 0.154, 0.152$, and 0.148 . They correspond to quark masses m_q of about 120, 200, and 370 MeV, respectively. [The scale $a^{-1} = 1.74(10)$ GeV is set by fixing the nucleon mass to its physical value.] Results of $g_P^L(q^2)$ for the momentum transfers $\vec{q}^2 a^2 = n(2\pi/L)^2$ (where $n = 1-4$, and L is the spatial extent of the lattice) are obtained from the plateaus of the ratio in Eq. (6) as a function of t , the time slice of the current insertion, away from the sink and source of the nucleon interpolation fields [8]. Since the ratio in Eq. (6) is proportional to q_3 , $g_P^L(q^2)$ at $q^2 = 0$ cannot be obtained directly. Rather, it will be obtained from extrapolation from the finite q^2 data as explained later.

Plotted in Fig. 1 are the lattice isovector pseudoscalar form factors $g_P^L(q^2)$ of the proton as a function of $m_q a$, the quark mass in dimensionless unit, which takes into ac-

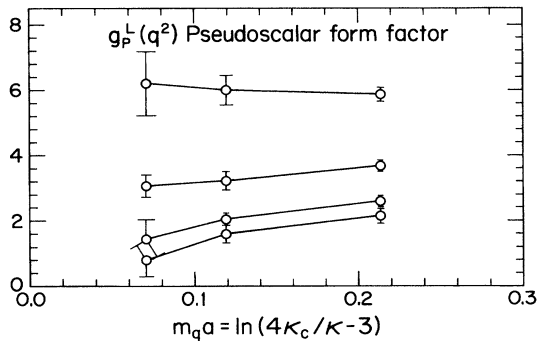


FIG. 1. The lattice isovector pseudoscalar form factors at various $-q^2$ as obtained from Eqs. (6) are plotted as a function of $m_q a$, the quark mass in lattice unit, for the three relatively light quark cases (Wilson $\kappa = 0.148, 0.152$, and 0.154). The top curve is for $\vec{q}^2 = (2\pi/La)^2$, the rest are for \vec{q}^2 from 2 to 4 times of $(2\pi/La)^2$ in descending order.

count the tadpole-improved definition for the quark mass [10]. They include different momentum transfers with \vec{q}^2 from 1 to 4 times $(2\pi/La)^2$ [$q^2 = (E_q - m_N)^2 - \vec{q}^2$ for the four-momentum transfer squared]. The errors are obtained through the jackknife in this case. The extrapolation of g_P^L to the quark mass $m_q a$ which corresponds to the physical pion mass is carried out with the correlated fit to a linear dependence on the quark mass $m_q a$ for $\kappa = 0.154, 0.152$, and 0.148 . The data covariance matrix is calculated with the single elimination jackknife error for g_P^L [8,11]. This fitting gives $\chi^2/N_{DF} = 0.005, 0.008, 0.65$, and 1.7 for $\vec{q}^2 a^2$ from 1 to 4 $(2\pi/L)^2$.

To extract the πNN form factor $g_{\pi NN}(q^2)$, we take the pion pole dominance in the dispersion relation for $g_P^L(q^2)$ so that

$$g_P^L(q^2) = \frac{G_\pi g_{\pi NN}(q^2)}{m_\pi^2 - q^2}, \quad (7)$$

where $G_\pi = \langle 0 | P(0) | \pi \rangle$ can be obtained from the two-point function

$$\left\langle \sum_{\vec{x}} P(\vec{x}, t) P(0, 0) \right\rangle \xrightarrow{t \gg a} \frac{G_\pi^2}{2m_\pi} e^{-m_\pi t}. \quad (8)$$

The lattice pseudoscalar form factor $g_P^L(q^2)$ is related to its continuum counterpart via $g_P(q^2) = Z_P g_P^L(q^2)$, where Z_P is the lattice renormalization (calculated to be 0.839 in the tadpole-improved perturbation theory at $\beta = 6.0$ [10]) for the pseudoscalar current. Since Z_P is associated with G_π , not $g_{\pi NN}(q^2)$, we will not discuss it any further.

Plotted in Fig. 2 is $g_{\pi NN}(q^2)$ defined via Eqs. (7) and (8). There is a caveat to extracting $g_{\pi NN}(q^2)$ this way which we wish to point out. Strictly speaking Eq. (7) is equivalent to PCAC (partial conservation of axial-vector current) where the physical pion field dominates and is thus valid for small q^2 . For q^2 as large as m_π^2 , with π' being the radially excited pion at 1.3 GeV, higher mass contribution to $g_P^L(q^2)$ may not be negligible. However,

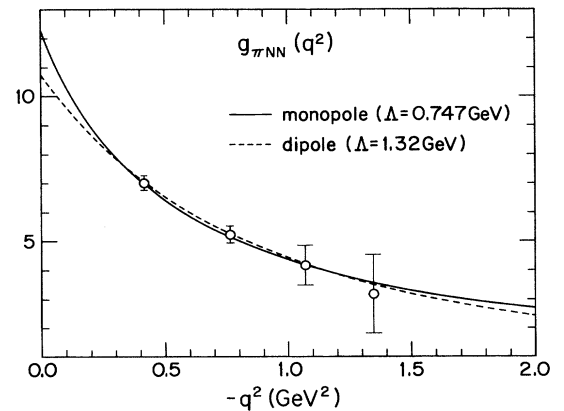


FIG. 2. $g_{\pi NN}(q^2)$ at the quark mass which corresponds to the physical pion mass. The solid and the dashed curves represent the monopole and dipole fits with the respective monopole and dipole mass Λ . They give somewhat different extrapolations at $q^2 = 0$.

$g_{\pi'NN}$ is expected to be an order of magnitude smaller than $g_{\pi NN}$ due to the fact that a node in the internal $q\bar{q}$ wave function of π' will lead to cancellation in the vertex function. Therefore, we estimate that the pion pole dominance [Eq. (7)] may have an error as large as 5%–10% at the highest q^2 we calculated. This is much smaller than the statistical error we have at the highest q^2 . This is also consistent with the estimate that PCAC and chiral perturbation are good to a scale of $4\pi f_\pi$. Keeping this in mind, we discuss the behavior of $g_{\pi NN}(q^2)$. We fitted it with both a monopole form [i.e., Eq. (1)] and a dipole form. We found that the monopole form with $\Lambda_{\pi NN} = 0.75 \pm 0.14$ GeV and a $\chi^2/N_{DF} = 0.13/2$ is only slightly better than the dipole form with a dipole mass of 1.32 ± 0.17 GeV and a $\chi^2/N_{DF} = 0.57/2$, in so far as the χ^2 is concerned. However, at this point we can inject our knowledge at $q^2 = 0$, where the Goldberger-Treiman (GT) relation relates coupling constants

$$m_N g_A(0) = f_\pi g_{\pi NN}(0). \quad (9)$$

Using $g_A(0) = 1.20 \pm 0.11$ [8] and $f_\pi = 89.8 \pm 4.5$ MeV calculated from the two-point functions $\langle \sum_{\vec{x}} A_4(t, \vec{x}) \times P(0, 0) \rangle$ and Eq. (8) using the point-split axial current, $g_{\pi NN}(0)$ is predicted to be 12.7 ± 1.3 from the above GT relation. This agrees well with $g_{\pi NN}(0) = 12.66 \pm 0.04$ from the experimentally known $g_A(0) = 1.2573 \pm 0.0028$ and $f_\pi = 93.15 \pm 0.11$ MeV [12]. On the other hand, the extrapolation of the monopole (dipole) fit of $g_{\pi NN}(q^2)$ yields $g_{\pi NN}(0) = 12.2 \pm 2.3$ (10.8 ± 1.3). Comparing with 12.7 ± 1.3 from the GT relation, we see that the dipole form is less favored than the monopole form; however, it cannot be ruled out in our present study with limited statistics. It is interesting to note that the monopole mass $\Lambda_{\pi NN} = 0.75 \pm 0.14$ GeV thus obtained is quite a bit smaller than those typically used in the NN potential, but agrees well with those based on the consideration of the GT relation [4], the apparent discrepancy between $g_{\pi^0 pp}$ and $g_{\pi^+ np}$ [5], and nucleon models such as the Skyrmin model [6]. To salvage the nice fit of the NN scattering data and the deuteron properties based on a hard πNN form factor, attempts have been made to incorporate a soft $g_{\pi NN}(q^2)$ either by appending a heavy pion at ~ 1.2 GeV [13] or by including multimeson exchanges [14] (e.g., $\pi\rho$ and $\pi\sigma$). Extrapolating $g_{\pi NN}(q^2)$ to $q^2 = m_\pi^2$, we obtain $g_{\pi NN}$, the πN coupling constant, to be 12.7 ± 2.4 . This compares favorably with the empirical value of 13.40 ± 0.17 [15] and 13.13 ± 0.07 [16]. The 4% change in $g_{\pi NN}(q^2)$ from $q^2 = 0$ to m_π^2 indeed can account for the 4% discrepancy in the GT relation when the physical $g_{\pi NN}$ is used in Eq. (9) instead of the $g_{\pi NN}(0)$ [4].

Putting the chiral Ward identity $\partial_\mu A_\mu^a = 2m\bar{\Psi}i\gamma_5\tau_a/2\Psi$ with pion pole dominance or equivalently PCAC ($\partial_\mu A_\mu^a = f_\pi m_\pi^2 \phi^a$) between nucleon states, we find

$$2m_N g_A(q^2) + q^2 h_A(q^2) = \frac{2m_\pi^2 f_\pi g_{\pi NN}(q^2)}{m_\pi^2 - q^2}. \quad (10)$$

In addition to PCAC, if one further assumes that the induced pseudoscalar form factor $h_A(q^2)$ is dominated by the pion pole, i.e., $h_A(q^2) = 2f_\pi g_{\pi NN}(q^2)/(m_\pi^2 - q^2)$, then $g_{\pi NN}(q^2) = (m_N/f_\pi)g_A(q^2)$. In other words, $g_{\pi NN}(q^2)$ has the same q^2 dependence as $g_A(q^2)$ which has been frequently used in the literature [17,18]. As there is no *a priori* reason why $g_{\pi NN}(q^2)$ should have the same falloff as $g_A(q^2)$ at all q^2 , and, furthermore, chiral perturbation calculation [19] at one loop suggests that they acquire different contributions, we compare $g_{\pi NN}(q^2)$ from Eq. (7) and $g_A(q^2)$ obtained on the same set of gauge configurations [8] for the present quark cases. Both $g_{\pi NN}(q^2)$ and $g_A(q^2)$, normalized at $q^2 = 0$, are plotted in Fig. 3 for $\kappa = 0.148, 0.152, 0.154$, and 0.1567 . The last κ corresponds to the physical pion mass. We find that in all these relatively light quark cases, there is a tendency for the normalized $g_{\pi NN}(q^2)$ to lie lower (higher) than the normalized $g_A(q^2)$ at lower (higher) $-q^2$. This presumably reflects the preferred monopole vs dipole fit for the $g_{\pi NN}(q^2)$ and $g_A(q^2)$. Our data do not discern this well, though. If this behavior is verified, it would imply that the induced pseudoscalar form factor $h_A(q^2)$ [not the pseudoscalar form factor $g_P(q^2)$] is not entirely dominated by the pion for higher $-q^2$ as it is at very low $-q^2$ (< 0.1 GeV², say).

Lastly, from the chiral Ward identity [Eq. (10)], we can obtain the pseudoscalar form factor $h_A(q^2)$ from $g_A(q^2)$ [8] and $g_{\pi NN}(q^2)$. We plot $h_A(q^2)$ in Fig. 4. Also plotted in the inset are experimental data obtained from pion electroproduction [18]. It turns out that the momentum transfer ranges of our lattice calculation and the available experiment do not overlap. We cannot compare them directly. However, if we use the monopole fit of $g_{\pi NN}(q^2)$ and the dipole fit of $g_A(q^2)$ [8], we find the extrapolation of $h_A(q^2)$ (solid line in Fig. 4) does agree with the experimental data at small $-q^2$. We note the errors of the fit start to diverge as $-q^2 \rightarrow 0$ due to the q^2 singularity in Eq. (10). As a result we are not able to extrapolate to $-q^2 = 0.88m_\pi^2$ to compare with the muon capture experiment.

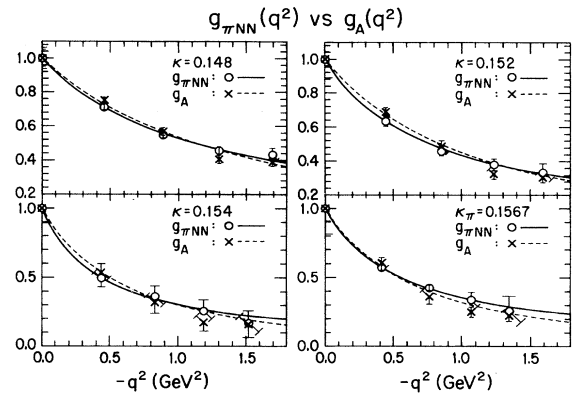


FIG. 3. Comparison of $g_A(q^2)$ and $g_{\pi NN}(q^2)$ (both normalized to 1 at $q^2 = 0$) as a function of $-q^2$ for the four quark cases ($\kappa = 0.148, 0.152, 0.154$, and 0.1567).

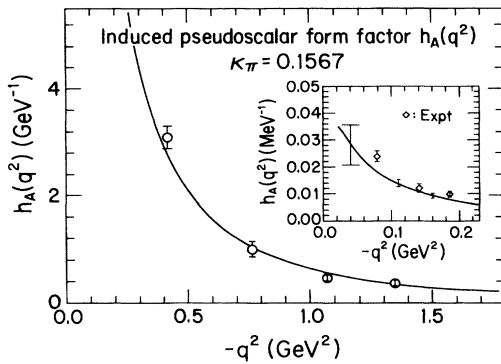


FIG. 4. The induced pseudoscalar form factor $h_A(q^2)$ from Eq. (10). The solid line is from the fits to $g_A(q^2)$ and $g_{\pi NN}(q^2)$. Also plotted in the inset are data from the electroproduction of pion [18]. The typical size of the error bars for the solid line is indicated in the inset.

Although we mentioned earlier that our calculations of the electromagnetic and axial form factors are within 10% of the experimental values, the work on weak matrix elements on similar lattice sizes and β has shown that the systematic errors are about 30% or less [20]. As an effort to estimate the size of the finite-lattice-spacing error, we used the lattice boson propagator for the pion pole [21] and the lattice version for the fermion kinetic factor in Eq. (6) to extract $g_{\pi NN}(q^2)$. We found that the monopole mass is increased by 7% and $g_{\pi NN}$ decreased by 3% in this case.

Keeping the caveat of the systematic errors in mind, the main results we gleaned are the following:

(1) $g_{\pi NN}(q^2)$ can be well described by a monopole form which agrees with the Goldberger-Treiman relation. The monopole mass $\Lambda_{\pi NN} = 0.75 \pm 0.14$ GeV is much smaller than commonly used in the NN potential.

(2) $g_{\pi NN} = 12.7 \pm 2.4$ agrees with the phenomenological values of 13.40 ± 0.17 [15] and 13.13 ± 0.07 [16]. It is also consistent with the lattice calculation of 14.8 ± 6.0 with staggered fermions [22].

(3) The falloff of $g_{\pi NN}(q^2)$ is about the same as $g_A(q^2)$ at very small $-q^2$ (< 0.3 GeV²), but is likely to fall slower at higher $-q^2$. This suggests that the induced pseudoscalar form factor $h_A(q^2)$ is not entirely dominated by the pion pole at higher $-q^2$. This point needs to be verified further with higher statistics study.

(4) From the chiral Ward identity and PCAC, we obtain $h_A(q^2)$, which can be checked experimentally in the future.

To conclude, we have calculated the isovector pseudoscalar form factor of the nucleon in a lattice QCD calculation for quark masses from about one to about two times that of the strange quark. From these we extracted $g_{\pi NN}(q^2)$ with the help of the pion pole dominance. The soft $g_{\pi NN}(q^2)$ form factor agrees with the predictions based on the discrepancy of the Goldberger-Treiman relation [4]

and between the $pp\pi^0$ and $pn\pi^+$ couplings [5]. This will have a large impact on the study of NN potential, the three-body force, and other processes which involve the πN coupling. For future studies, it is essential to improve the calculation by expanding the volume in order to access smaller $-q^2$ and to study the systematic errors related to the infinite volume as well as the continuum limits.

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