Gauge Invariant Electromagnetic Two-Point Function for Heavy-Light Quark Systems

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A gauge invariant electromagnetic two-point function, crucial to the investigations of the violation of isospin symmetry, is derived for heavy-light quark systems. Thus QED can be consistently introduced into the QCD sum rule method, which was not previously possible.

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The origin of mass differences in isospin multiplets has long been of great interest in nuclear and particle physics as a source of information about symmetry violations. Hadronic isospin violations are particularly important in that they arise from nonperturbative quantum chromodynamics (QCD) as well as quark mass differences (see Ref. [1] for a review of the early work in this area), and, of course, electromagnetic effects. Among the first applications of the method of QCD sum rules was the study of isospin violations in the ρ-ω system [2], where it was recognized that the isospin splitting of the light-quark condensates can produce effects as large as the current-quark mass splittings and electromagnetic effects. Recently the QCD sum rule method has been used to study the neutron-proton mass difference [3,4], the octet baryon mass splittings [5], and the mass differences in the charmed meson systems (the D and D^* scalar and vector mesons) [6].

In comparison with hadronic quark models, the QCD sum rule method for calculating isospin splittings has the advantage that one can directly use QED field theory rather than rely on models to estimate Coulomb corrections. This, however, has not been done. In our earlier attempt to calculate the standard two-loop QED contributions to the isospin mass splittings in heavy-light quark systems (Fig. 1) using the sum rule method, we found [7] that for the charged mesons, involving charged currents, the calculation is not gauge invariant. The objective of the present Letter is to develop a gauge invariant theory for QED phenomena within the QCD sum rule method. Although our derivation is for heavy-light quark mesons, the method is quite general and can be used for baryons as well as mesons.

The basic approach of the QCD sum rule in heavy light quark systems is to study the two-point function in the Wilson operator product expansion (OPE), defined by

$$\bar{\Pi}^0_{\mu\nu}(q^2) = i \int d^4x\, e^{iqx} \langle T[J_\mu(x)\bar{J}_\nu(0)]\rangle = \sum_n I^n_{\mu\nu}(q^2) O_n \tag{1}$$

for the heavy-light quark current

$$J_\mu(x) = \bar{q}(x)\gamma_\mu Q(x). \tag{2}$$

The local operators $\{O_n\}$ consisting of quark and gluon fields and the Wilson coefficients functions $\{I^n_{\mu\nu}(q^2)\}$ have been extensively discussed in the literature in the studies of the masses and their decay constants for heavy-light quark systems.

In order to study the violations of the isospin symmetry, the electromagnetic effects should also be written in the framework of the operator product expansion. The leading electromagnetic effects in this approach are the two-point functions from a two-loop perturbative contribution, whose Feynman diagrams are shown in Fig. 1. For the charge neutral current, one could simply obtain the two-point functions by changing the gluons in QCD to the photons in QED in Fig. 1, since the two-point functions have been calculated in QCD [8]. Their imaginary parts are

$$\mathrm{Im}[\bar{\Pi}^{ps}(q^2)] = \frac{3e_q^2 M^2}{8\pi^2}(1-x)^2 f(x), \tag{3}$$

and

$$\mathrm{Im}[\bar{\Pi}^{v}(q^2)] = \frac{e_q^2 q^2}{8\pi^2}(1-x)^2\left[(2+x)[1+f(x)] - (3+x)(1-x)\ln\left(\frac{x}{1-x}\right) - \frac{2x}{(1-x)^2}\ln(x) - 5 - 2x - \frac{2x}{1-x}\right] \tag{4}$$

for pseudoscalar and vector currents, where $x = M^2/q^2$

$$f(x) = \frac{9}{4} + 2l(x) + \ln(x)\ln(1-x) + \left(\frac{5}{2} - x - \frac{1}{1-x}\right)\ln(x) - \left(\frac{5}{2} - x\right)\ln(1-x), \tag{5}$$

and $l(x) = -\int_0^x \ln(1-y)dy/y$ is the Spencer function. Obviously, this result is gauge invariant.

The calculation of the two-point functions for a charged current is much more complicated. Since the Feynman diagrams in Fig. 1 are weighted by the products of different charges, the corresponding two point functions are not

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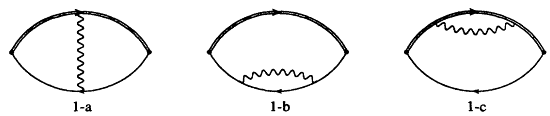

FIG. 1. The two-loop perturbative corrections for charge neutral currents with charged constituents.

gauge invariant. The physical origin of this problem is that under the gauge transformation

$$\begin{aligned} \bar{q}(x) &\longrightarrow e^{ie_q\Lambda(x)}\bar{q}(x)\,, \\ Q(x) &\longrightarrow e^{ie_Q\Lambda(x)}Q(x)\,, \\ A_\mu(x) &\longrightarrow A_\mu(x) - \partial_\mu\Lambda(x)\,, \end{aligned} \tag{6}$$

we have

$$\begin{aligned} J_\mu(x) \longrightarrow J'_\mu(x) &= e^{i\Lambda(x)(e_Q+e_q)}\bar{q}(x)\gamma_\mu Q(x) \\ &= e^{i\Lambda(x)(e_Q+e_q)}J_\mu(x) \end{aligned} \tag{7}$$

(notice $e_q = -e_{\bar{q}}$), therefore, the current J_μ becomes gauge dependent with $e_q + e_Q = e_T$. Of course, gauge invariance is a fundamental property of electromagnetic interactions, and any observable obtained using the current must be gauge invariant.

In order to obtain a solution to the problem, let us start with a gauge invariant form of the two-point function, which is obtained by inserting the link operator into the form given in Eq. (1), with the definition

$$\begin{aligned} \Pi_{\mu\nu}(q^2) &= i\int d^4x\, e^{iq(x-y)}\Big\langle T\Big[J_\mu(x)e^{iQ_{\rm op}\int_y^x A_\alpha(y)\,dy^\alpha}\bar{J}_\nu(y)\Big]\Big\rangle \\ &= i\int d^4x\, e^{iq(x-y)}\langle T[J^t_\mu(x)\bar{J}^t_\nu(y)]\rangle\,, \end{aligned} \tag{8}$$

where $Q_{\rm op}$ is the charge operator and

$$J^t_\mu(x) = \bar{q}(x)e^{iQ_{\rm op}\int_0^x A_\alpha(y)\,dy^\alpha}\gamma_\mu Q(x)\,. \tag{9}$$

The gauge transformation of the electromagnetic field $A_\alpha(x)$ cancels those of quark fields $\bar{q}(x)$ and $Q(x)$ so that two-point function is gauge invariant. This is the QED analog to the QCD treatment introduced [9] for the pion wave function.

Expanding to order α_e one finds that there are two currents at the two-loop level

$$J^t_\mu(x) = J^0_\mu(x) + J^e_\mu(x)\,. \tag{10}$$

The current $J^0_\mu(x)$ is given by insertion of photon vertices in the quark lines of the current given by Eq. (2), and the evaluation of the resulting two-point functions is carried out by the standard two-loop diagrams shown in Fig. 1. The charges involved are e_Q and e_q for these processes. The second current of Eq. (10),

$$J^e_\mu(x) = ie_T\bar{q}(x)\gamma_\mu\int_0^x A_\alpha(y)\,dy^\alpha Q(x)\,, \tag{11}$$

corresponds to an additional vertex function with the charge e_T, and in the evaluation of the two-point function gives rise to the additional diagrams shown in Fig. 2, which we now discuss in detail. The resulting gauge invariant two-point function is

$$\Pi_{\mu\nu}(q^2) = \Pi^0_{\mu\nu}(q^2) + \Pi^e_{\mu\nu}(q^2)\,, \tag{12}$$

where

$$\Pi^0_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx}\langle T[J^0_\mu(x)\bar{J}^0_\nu(0)]\rangle \tag{13}$$

at the two-loop level is given by the Feynman diagrams in Fig. 1, and

$$\begin{aligned} \Pi^e_{\mu\nu} = i\int d^4x\, e^{iqx}T[J^0_\mu(x)\bar{J}^e_\nu(0) &+ J^e_\mu(x)\bar{J}^0_\nu(0) \\ &+ J^e_\mu(x)\bar{J}^e_\nu(0)] \end{aligned} \tag{14}$$

generates the additional Feynman diagrams shown in Fig. 2. These additional terms introduced by $\Pi^e_{\mu\nu}$ arise from the additional vertex function corresponding to J^e_ν. The fact that such additional vertex functions must be present for charged currents for gauge invariance has been observed previously. See, e.g., Ref. [10].

Since the masses for the heavy and light quarks are not the same, it is more convenient to calculate the two-point function in momentum space. The Fourier transformation of the operator $\int_0^x A_\mu(y)\,dy^\mu$ gives

$$\begin{aligned} F(q^2) &= \int d^4x\, e^{iqx}\int_0^x A_\mu(y)\,dy^\mu \\ &= \frac{q_\nu}{iq^2}\int d^4x\,(\partial^\nu e^{iqx})\int_0^x A_\mu(y)\,dy^\mu \\ &= -\frac{q_\nu}{iq^2}A^\nu(q)\,, \end{aligned} \tag{15}$$

where q_μ is the momentum carried by photon field $A_\mu(q)$, therefore the current $J^e_\mu(q)$ in the momentum space is given by

$$J^e_\mu(q) = -e_T\bar{q}(q-k_1)\gamma_\mu\frac{(k_1-k_2)_\nu}{(k_1-k_2)^2}A^\nu(k_1-k_2)Q(k_2)\,. \tag{16}$$

The sum of the Feynman diagrams in Fig. 2 has a simple form

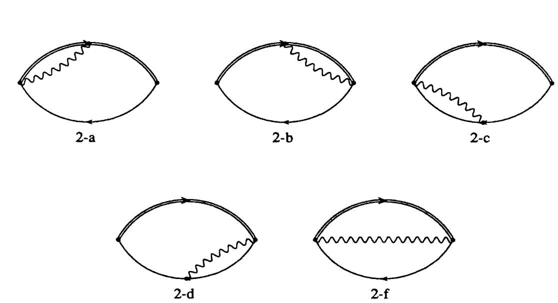

FIG. 2. Additional diagrams generated by $J^e_\mu(x)$ for charged heavy-light quark systems. See text.

$$\Pi^{e}_{\mu\nu}(q^2) = -\frac{e_T(e_q + e_Q)}{2^8\pi^4}\int \frac{d^Dk_1 d^Dk_2}{\pi^D} \times \frac{\mathrm{Tr}[\gamma_\mu \not{k}_2 \gamma_\nu (\not{k}_1 - \not{q})]}{(k_1 - k_2)^4 (k_2^2 - M^2)(k_1 - q)^2} = e_T(e_q + e_Q) I_{\mu\nu}(q^2). \quad (17)$$

To show that the addition of $\Pi^{e}_{\mu\nu}(q^2)$ indeed makes the total two-point function $\Pi_{\mu\nu}(q^2)$ gauge invariant, we separate $\Pi^{0}_{\mu\nu}(q^2)$ into the gauge dependent and independent parts

$$\Pi^{0}_{\mu\nu}(q^2) = e_q^2 I^{q}_{\mu\nu}(q^2) + e_Q^2 I^{Q}_{\mu\nu}(q^2) - e_q e_Q I^{qQ}_{\mu\nu}(q^2) = -e_q e_Q[I^{q}_{\mu\nu}(q^2) + I^{Q}_{\mu\nu}(q^2) + I^{qQ}_{\mu\nu}(q^2)] + e_T e_q I^{q}_{\mu\nu}(q^2) + e_T e_Q I^{Q}_{\mu\nu}(q^2), \quad (18)$$

where $I^{q}_{\mu\nu}(q^2)$ and $I^{Q}_{\mu\nu}(q^2)$ represent the self-energy diagram for the light quark $\bar{q}(x)$ and heavy quark $Q(x)$, respectively, and $I^{qQ}_{\mu\nu}(q^2)$ corresponds to the photon exchange between the light and heavy quarks in Fig. 1. The term with $I^{q}_{\mu\nu}(q^2) + I^{Q}_{\mu\nu}(q^2) + I^{qQ}_{\mu\nu}(q^2)$ in Eq. (18) is, of course, gauge invariant, and the analytical expressions of its imaginary part can be obtained from Eqs. (3) and (4). The last two terms in Eq. (18) are gauge dependent; if we substitute the gauge dependent part of the photon propagator $(D_{\mu\nu})_{\mathrm{gd}} = (k_1 - k_2)_\mu (k_1 - k_2)_\nu/(k_1 - k_2)^4$ into the two-loop integrals $I^{q}_{\mu\nu}(q^2)$ and $I^{Q}_{\mu\nu}(q^2)$, the gauge dependent part of the integrals is

$$[I^{q}_{\mu\nu}(q^2)]_{\mathrm{gd}} = [I^{Q}_{\mu\nu}(q^2)]_{\mathrm{gd}} = -I_{\mu\nu}(q^2), \quad (19)$$

where $I_{\mu\nu}(q^2)$ is given in Eq. (17). By substituting the gauge dependent photon propagator into $\Pi^{e}_{\mu\nu}(q^2)$, one can show that the gauge dependent part of the two-loop integral for $\Pi^{e}_{\mu\nu}(q^2)$ is identical to the expression in Eq. (17). Therefore, the gauge dependent two-loop integrals in $\Pi^{0}_{\mu\nu}(q^2)$ and $\Pi^{e}_{\mu\nu}(q^2)$ exactly cancel each other, and we have total gauge invariant results. This provides an important check at the two-loop level that the two-point function $\Pi_{\mu\nu}(q^2)$ for the current $J^{t}_{\mu}(x)$ in Eq. (10) is indeed gauge invariant.

To evaluate the two-point function $\Pi_{\mu\nu}(q^2)$, we rewrite Eq. (12) as

$$\Pi_{\mu\nu}(q^2) = \Pi^{qQ}_{\mu\nu}(q^2) + \Pi^{q}_{\mu\nu}(q^2) + \Pi^{Q}_{\mu\nu}(q^2), \quad (20)$$

where

$$\Pi^{qQ}_{\mu\nu}(q^2) = -e_q e_Q[I^{q}_{\mu\nu}(q^2) + I^{Q}_{\mu\nu}(q^2) + I^{qQ}_{\mu\nu}(q^2)], \quad (21)$$

whose analytical expressions can be obtained from Eqs. (3) and (4),

$$\Pi^{q}_{\mu\nu}(q^2) = e_T e_q[I^{q}_{\mu\nu}(q^2) + I_{\mu\nu}(q^2)], \quad (22)$$

and

$$\Pi^{Q}_{\mu\nu}(q^2) = e_T e_Q[I^{Q}_{\mu\nu}(q^2) + I_{\mu\nu}(q^2)]. \quad (23)$$

For $\Pi^{q}_{\mu\nu}(q^2)$, we have

$$I^{q}_{\mu\nu}(q^2) + I_{\mu\nu}(q^2) = \frac{1}{2^8\pi^4}\int \frac{d^Dk_2}{\pi^{D/2}} \frac{\mathrm{Tr}[\gamma_\mu(\not{k}_2 + M)\gamma_\nu(\not{k}_2 - \not{q})F(k_2)(\not{k}_2 - \not{q})]}{(k_2^2 - M^2)(k_2 - q)^4}, \quad (24)$$

where

$$F(k_2) = \int \frac{d^Dk_1}{\pi^{D/2}} \frac{\gamma_\mu(\not{k}_1 - \not{q})\gamma_\nu}{(k_1 - k_2)^2(k_1 - q)^2}\left(g^{\mu\nu} - \frac{(k_1 - k_2)^\mu (k_1 - k_2)^\nu}{(k_1 - k_2)^2}\right) \quad (25)$$

is a one-loop wave-function renormalization equivalently in Landau gauge. It has been shown that $F(k_2)$ vanishes [11] for a zero mass particle in dimensional regularization. This leads to

$$\Pi^{Q}_{\mu\nu}(q^2) = e_T e_Q[I^{Q}_{\mu\nu}(q^2) - I^{q}_{\mu\nu}(q^2)], \quad (26)$$

where $\Pi^{Q}_{\mu\nu}(q^2)$ is proportional to the mass of the heavy quarks. This shows that the divergence induced by the wave-function renormalization does not exist in Eq. (26), which implies that the Ward identity is restored in this approach.

After including the mass renormalization, the two-loop integral $\Pi^{Q}_{\mu\nu}(q^2)$ in the dimensional regularization is

$$\left(\frac{\mu^2 e^\gamma}{4\pi}\right)^\epsilon \frac{\Pi^{Q}_{\mu\nu}(q^2)}{e_Q e_T} = 3\left(\frac{\mu^2}{-q^2}\right)^\epsilon \left[-\frac{3\alpha_e}{4\pi\epsilon}\bar{M}\frac{\partial}{\partial \bar{M}} I^{0}_{\mu\nu}(q^2) + \left(\frac{\mu^2}{-q^2}\right)^\epsilon [I^{Q}_{\mu\nu}(q^2) - I^{q}_{\mu\nu}(q^2)]\right], \quad (27)$$

where

$$I^{q}_{\mu\nu}(q^2) = -i\frac{(-q^2 e^\gamma)^\epsilon}{16\pi^2}\int \frac{d^Dk}{\pi^{D/2}} \frac{\mathrm{Tr}[\gamma_\mu(\not{k} + M)\gamma_\nu(\not{k} - \not{q})]}{(k^2 - M^2)(k - q)^2} \quad (28)$$

is the one-loop integral, $D = 4 - 2\epsilon$, $\epsilon \to 0^+$, is the number of spacetime dimensions, and the running mass $\bar{M}(\mu)$ is related to the pole mass M by [8]

$$\bar{M}(\mu) = M\left\{1 - \frac{\alpha_e}{4\pi}\left[3\ln\left(\frac{\mu^2}{M^2}\right) + 4\right]\right\}. \quad (29)$$

The evaluation of Eq. (27) is performed in the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$). Here we only present the imaginary parts of $\Pi^1_{\mu\nu}(q^2)$ for the pseudoscalar and the vector states,

$$\mathrm{Im}\Big(\Pi^Q_{\mathrm{ps}}(q^2)\Big) = \frac{3\alpha_e e_Q e_T M^2}{16\pi^2}\left[2x(1-x) + 2\ln(x) + (1-x)^2\ln\left(\frac{1-x}{x}\right) + \frac{3}{2}(1-x)^2\right], \tag{30}$$

and

$$\mathrm{Im}\Big(\Pi^Q_{v}(q^2)\Big) = \frac{9\alpha_e e_Q e_T}{16\pi^2}\left\{q^2(2+x)\left[x(1-x) + \ln(x) + (1-x)^2\ln\left(\frac{1-x}{x}\right)\right] + M^2\left(x(1-x) + \ln(x) + \frac{3}{2}(1-x)^2\right)\right\} \tag{31}$$

for pseudoscalar and vector currents, respectively, in which the running mass $\bar{M}(\mu)$ is replaced by the pole mass M. In the limit of $M \rightarrow 0$, $\Pi^Q(q^2)$ vanishes for both pseudoscalar and vector currents. Thus, the two-point functions are dominated by the term proportional to $e_q e_Q$ in the small M limit, this is exactly what one would expect from the phenomenological model. However, how large effects the contributions from $\Pi^1(q^2)$ are in the heavy quark limit remains to be studied.

In summary, we have derived a gauge-independent form for QED corrections to QCD sum rule by identifying processes with additional vertices which must be included for charged currents. The result here presents an important step toward a consistent treatment of the violations of isospin symmetry in the QCD sum rule framework. The applications of this result to isospin splittings of heavy-light quark systems as well as the kaon systems are in progress, and will be given elsewhere.

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