Spin Transmission in Metallic Trilayers

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We investigate spin transmission in ferromagnetic-nonmagnetic-ferromagnetic junctions with particular focus on the recent experiment by Johnson. We argue that there is coupling of the charge and spin fluctuations for the electrons on different sides of the junction. In particular, the magnetic susceptibility should be calculated regarding the system as a whole, providing a possible explanation for the discrepancy between theory and experiment.

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In a series of papers, Johnson [1] and Johnson and Silsbee [2] presented a thermodynamic theory of ferromagnetic-paramagnetic interfaces under steady state conditions. Johnson [1] studied transport across a ferromagnetic (F_1)-paramagnetic (P) metallic interface whose thickness d is less than the spin diffusion length δ_s. He proposed that under steady state conditions, the electric current I_e corresponds to a magnetization current $I_M = \eta\beta I_e/e$, where β is the Bohr magneton, e is the electric charge, and η is the phenomenological factor describing the efficiency of transport of the dipoles. I_M induces in the paramagnetic film a magnetization $\delta M = I_M T_2/\Omega$, where Ω is the volume of the nonmagnetic film, and T_2 is the spin relaxation time. Corresponding to this magnetization is an energy change h_R between the spin up and spin down bands of the order of $\beta M/\chi$, where χ is the magnetic susceptibility. This voltage is measured in another ferromagnet F_2 on the other side of P. Experimental estimates of the left and right hand side of the above equation differs by 1 order of magnitude if one uses the magnetic susceptibility χ_0 of the paramagnetic metal. This motivates us to look more carefully into the above problem.

Some reflection makes it appear unlikely that the requisite enhancement of the h_R can come from a decrease in the susceptibility of the paramagnet alone. The free-electron model gives a value of the total susceptibility of Au (the paramagnet used in the experiment described above) which differs from the observed value by less than a factor of 2 [3]. The presence of magnetization itself could decrease χ, but this decrease goes as δM^2 and thus represents a small correction. If, however, we consider the entire trilayer as responding to the sudden appearance of a magnetization in the paramagnet, then there can indeed be a large enhancement of h_R due to the proximity of the ferromagnets, and, further, dramatic changes in h_R as a function of the angle between magnetizations of the two ferromagnets.

There are two ways to look at the problem. One can treat the coupling between the ferromagnet and the paramagnet by a "tunneling Hamiltonian" and incorporate its effect via perturbation theory. An implicit assumption behind this is that the two sides are weakly coupled and their physical properties are essentially unrelated. Another approach is to treat the bilayer without using perturbation.

In this second scenario, there is coupling of the charge and spin fluctuation between the ferromagnet and the paramagnet. The energy difference h_R measured experimentally between the spin up and spin down band corresponds to an effective magnetic field. A magnetic field causes a change in the spin up and spin down electron densities in opposite directions and with magnitudes proportional to their respective density of states. It will induce a charge fluctuation when the density of states of the spin up and spin down band are different, as in a ferromagnet [4]. As a result of this in a uniform system, the longitudinal magnetic susceptibility is proportional to the *product* of the density of states of the spin up and spin down band and not to their sum. Thus if one of these density of states is much smaller, the net susceptibility can be much reduced. In the experiment of interest, there is a voltage difference between the spin up and spin down band in *both the ferromagnet and the paramagnet.* Thus the difference in density of states of the spin up and spin down band in the ferromagnet must come in. The charge fluctuation is governed by the *long range* Coulomb potential. Thus the charge fluctuations in the paramagnet and the ferromagnet interact with each other, requiring that the system be treated as a whole.

The magnetic susceptibility contains contributions from the movements of domain walls and a Pauli electronic contribution. It is the latter that is of interest here and we shall refer solely to it from now on. In a self-consistent picture, the spin up and spin down electrons, which occupy the entire trilayer, experience the same potential in the paramagnet (P) but different potential in the ferromagnet (F). The total wave function is obtained by matching boundary conditions at the interface. For each wave function occupying *the entire trilayer,* there is a corresponding energy. χ

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should then be calculated using the wave function of the whole system and can be quite different from that of the paramagnet alone. In this Letter, we examine the physics, implications, and requirements of this picture and illustrate our results by simple model calculations. Self-consistent band structure calculations can be carried out for a few monolayers but becomes prohibitively time consuming for hundreds of monolayers (the experimental situation). Thus simple model calculations can provide complementary insight in films of intermediate thickness. We found that the effective susceptibility can be decreased and this may account for the discrepancy between theory and experiment. We now describe this in detail.

We model the electronic properties of the ferromagnet and the paramagnet with different effective masses and different zeros in energy. The ferromagnets F_1 and F_2 are on the two sides, $0 < z < L/2$ and $L < z < 3L/2$, while the paramagnet P is in the middle, $L/2 < z < L$. The wave function of the spin up and spin down bands of the trilayer system are obtained by matching boundary conditions at the interfaces. The density of states of the combined system of these two bands are not the same.

In the spin transmission experiment, there is a magnetization current I_M as a result of the current from the ferromagnet to the paramagnet of volume Ω. Because of the finite relaxation time T_2 of the magnetization in the paramagnet, there is a change in the magnetization of $\delta M = I_M T_2/\Omega$. δM is confined mostly in P because the experimental arrangement is such that the current comes out of P. We thus assume δM to be constant in this region and zero outside. For a combined trilayer of thickness $3L/2$, δM can be represented by a Fourier series as $\delta M(z) = \frac{1}{3} + \sum_{j>0} \cos(q_j z) \sin(4\pi j/3)/\pi j$, where $q_j = 4\pi j/3L$.

δM is produced if electrons are moved from the spin down band to the spin up band so that they are separated by a voltage h_R which was measured experimentally. δM and h_R are related by an effective magnetic susceptibility, $\delta M(q) = \sum_{q'} \chi_R(q, q') h_R(q')$. We first derive the susceptibility χ_R, treating the charge and spin fluctuations in a self-consistent way. This calculation follows the earlier results of Ehrenreich and Cohen [5] for charge fluctuation alone that reproduces the results of the RPA and the earlier results of one of us [4] for the uniform system. Similar results can also be produced by summing series of ladder and bubble diagrams [6].

For a magnetic field h and a potential ϕ, the Fourier transform of the charge fluctuation of the spin up and spin down bands, $\rho_\pm(q)$, is given by $\rho_\pm(q) = \sum_p F_\pm(q, p)[\pm h(p) + \phi(p)]$, where $F(q, p) = \sum_{i,j}[f(i) - f(j)]\langle i|\exp(i\mathbf{q}\cdot\mathbf{r})|j\rangle\langle j|\exp(-i\mathbf{p}\cdot\mathbf{r})|i\rangle/(E_i - E_j)f(i)$, and $f(j)$ are Fermi occupation factors. For the trilayer there is no translation invariance, thus F is not diagonal. From this we get $\sigma = \rho_+ - \rho_- = F_s h + F_d \phi$, $\rho = \rho_+ + \rho_- = F_d h + F_s \phi$, where $F_s = F_+ + F_-, F_d = F_+ - F_-$. When the spin up and spin down density of states are not the same, $F_d \neq 0$, h is coupled to the charge fluctuation ρ, as we see from the second equation. Now in the absence of an external potential from Poisson's equation $\phi = V\rho$, where $V = 4\pi e^2/\Omega q^2$. Substituting back into the second equation we get $\rho = (1 - F_s V)^{-1} F_d h$. Substituting this back into the first equation, we get in matrix notation $\sigma = [F_s + F_d V(1 - F_s V)^{-1} F_d]h$. In the large V (small q) limit, this reduces to $\sigma = [F_s - F_d F_s^{-1} F_d]h$. In the presence of an exchange interaction v (which can be a matrix), $h = h_{\text{ext}} + v\sigma$, we get

$$h_{\text{ext}} = [F_s - F_d F_s^{-1} F_d]^{-1}[1 - v(F_s - F_d F_s^{-1} F_d)]\sigma . \tag{1}$$

In this Letter, for illustrative purposes, we shall perform our estimate assuming v to be zero and focus on

$$h_R = [F_s - F_d F_s^{-1} F_d]^{-1} \sigma_M , \tag{2}$$

where σ_M is the Fourier transform of the magnetization δM. We have evaluated h_R numerically by calculating the eigenfunctions $|i\rangle$, the form factor matrix elements, and the bare susceptibility F and eventually h_R. When $F_+ \gg F_-, F_s - F_d F_s^{-1} F_d \approx 4F_-$. We find this approximation works quite well when the magnetization of F_1 and F_1 are parallel to each other. This limit is consistent with that in the uniform case where the susceptibility is controlled by the smaller one when the spin up and spin down contribution are very different. Of course, in the present case F_- is a matrix. We describe the calculation of h_R next.

To simplify matters, we shall assume the ferromagnets on the two sides to be the same and the paramagnet to have the same effective mass and the same zero in energy as the majority spin band. For the minority spin component the energy is the same in the ferromagnet, and in the paramagnet we get

$$\begin{aligned} E &= \hbar^2 k_1^2/2m_{1z} + \hbar^2 k_\perp^2/2m_{1\perp} + U \\ &= \hbar^2 k_2^2/2m_{2z} + \hbar^2 k_\perp^2/2m_{2\perp} . \end{aligned} \tag{3}$$

U is the splitting between the spin up and spin down band in the ferromagnet. We expect the ferromagnet to have a positive U and/or smaller effective mass for the minority spin band. If U is larger than the Fermi energy then all spins are lined up in the ferromagnet. In our calculation, we set $m_{iz} = m_{i\perp}$ for $i = 1, 2$. For a real k_{2z}, k_{1z} becomes imaginary when $\hbar^2 k_2^2/2m_2 + \hbar^2 k_\perp^2(1/2m_2 - 1/2m_1) - U_s < 0$.

If the magnetization of the ferromagnet on the two sides is parallel, the problem possesses parity symmetry. We thus need only to focus on matching the boundary condition at $z = L/2$. For both parities for $z < L/2$, the wave function is either $\psi = a\sin(k_1 z)$ or $a\sinh(|k_1|z)$, depending on whether k_1 is imaginary or not. For $L > z > L/2$ the wave function is $b\sin[k_2(z - 3L/4)]$ for odd

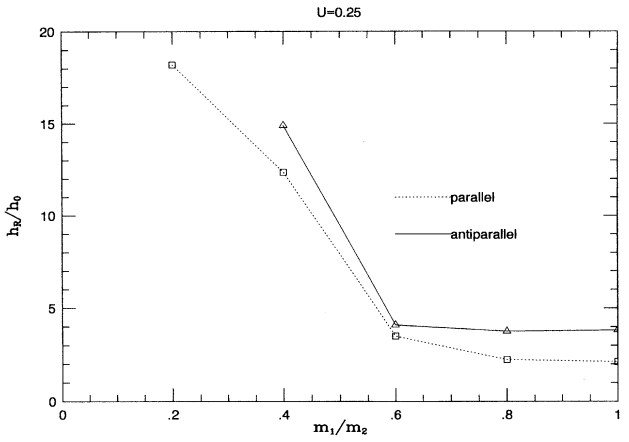

FIG. 1. The magnitude of the normalized effective voltage for parallel and antiparallel ferromagnets as a function of the effective mass ratio for $U = 0.25$. The lines are drawn through the points as a guide to the eye.

parity and $b\cos[k_2(z - 3L/4)]$ for even symmetry. ψ'/ψ needs to be continuous across the interface. For example, for odd parity and real k_2, we get

$$k_1 \cot(k_1 L/2) = -k_2 \cot(k_2 L/4). \quad (4)$$

Equations (3) and (4) were solved numerically for each spin component. The numerical result was checked for the case with $U = 0$ and $m_1 = m_2$, where $\psi(z) = \sqrt{4/3L}\sin(kz)$ and $k_1 = k_2 = 2\pi i/3L$ for integer i. With these wave functions, only the cosine form factors $T_{cij} = \langle i|\cos(\mathbf{q}\cdot\mathbf{r})|j\rangle$ come in. They involve integrals of elementary functions and can be evaluated analytically. In the limit when all the masses are equal and $U = 0$, $T_{cij} = 0.5[\cos(k_i - k_j - q) + \cos(k_i - k_j + q) - \cos(k_i + k_j + q) - \cos(k_i + k_j + q)]$. Again we verify that our result produces this limit correctly.

At large $k_\perp$ when k_1 is imaginary, we expect the wave functions not to make a significant contribution to χ_R. This comes about in the following way. When k_1 is imaginary the wave function is mostly confined in the paramagnet and $k_2 \approx 2\pi i/L$ for integer i. This k_2 is less dense than the wave vector $q_l = 4l\pi/3L$ for integer l. There does not exist any k such that $k_i - k_j = \pm q_l$ unless l is a nonzero multiple of 3. Thus most of the matrix elements T_{ij} are very small. This effect greatly reduces the response function and eventually increases the voltage h_R.

The voltage shift measured in the spin transmission experiment depends on an effective susceptibility which in turn depends on the density of states and the wave function at the Fermi surface. The total density of states of the system will be less when $m_1 < m_2$ and $U > 0$. This provides another reason for the reduction of h_R.

Our result for h_R normalized by the corresponding value of the pure paramagnet h_0 at the lowest nonzero wave vector is shown in Fig. 1 for two values of U as a function of the mass ratio between the paramagnet and that of the minority spin band. We used a unit so that $\hbar = 1$, $m_2 = 1$. An arbitrary Fermi energy of 1.3 was used. To smooth out possible fluctuations due to discreteness of finite size systems, we have taken an average over a window of 0.1 about the Fermi energy. $L = 400a_B$. (a_B is the Bohr radius.) Similar ratios are also shown in Fig. 2 for equal masses but as a function of U. As we can see, h_R/h_0 can indeed be increased. Examination of the numerical results indicate that both the density of states and the spectral densities (wave function squared in the above) contribute to the final result.

A similar calculation can be carried out when the magnetization of F_1 is up and that for F_2 is down. For the spin up band for $z < L/2$, the wave function is either $\psi = a\sin(k_1 z)$ or $a\sinh(|k_1|z)$, depending on whether k_1 is imaginary or not. For $z > L/2$ the wave function is $b\sin[k_2(z - 3L/2)]$. ψ'/ψ needs to be continuous across the interface. For example, for real k_2, we get

$$k_1 \cot(k_1 L/2) = -k_2 \cot(k_2 L). \quad (5)$$

The spin down wave functions are parity conjugates of the spin up wave functions. For this case both the cosine (T_{cij}) and the sine form factors ($T_{sij} = \langle i|\sin(\mathbf{q}\cdot\mathbf{r})|j\rangle$) come in [7]. The magnitude of the voltage shifts h_R is also shown in Figs. 1 and 2. h_R for the spin up and spin down case are not equal to each other.

To present the essential physical picture, we have glossed over several issues in the above discussion. The susceptibility discussed has not taken into consideration the effect of electron-electron interactions. In the RPA, the result of Eq. (1) is obtained but now v is nonzero. More sophisticated treatment of the electron bands and/or improvements of the RPA will undoubtably provide for a more realistic result. But we expect the dependence on the product of the density of states to remain correct. This is the result we exploited.

Because the magnetization is decaying on the side of the paramagnetic metal, the magnetic susceptibility

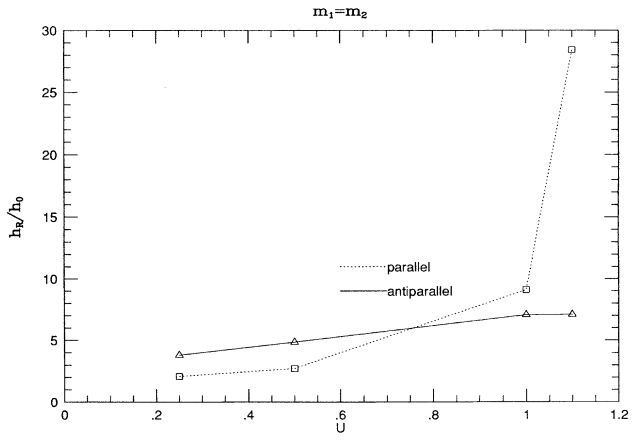

FIG. 2. The magnitude of the normalized effective voltage for parallel and antiparallel ferromagnets as a function of U for $m_1 = m_2$. The lines are drawn through the points as a guide to the eye.

that enters into consideration is not exactly that at momentum transfer $q = 4\pi l/3L$. Rather, it is expected to be integrated around $q = 0$ with a width of the order of the inverse spin diffusion length l_s^{-1}. Because $k_F l_s \ll 1$, we expect the effective susceptibility to be well approximated by that at $q = 0$, however.

In the above, we presented a simplified picture of the electronic structure of the ferromagnetic-paramagnetic interface which we think illustrates our point. In real systems there are other issues which come in and enrich the physics. First of all, there will be electron transfer and a dipole layer at the interface. This in principle can induce a polarity effect but it is not clear how big the effect will be. Interfaces can also be constructed between a ferromagnetic metal and a semiconductor. The dipole layer is wider because there is no metallic screening in the semiconductor. This polarity effect may be much larger under those situations. Second, the paramagnetic atoms close to the interface may also develop magnetic moments. This will change the weighting of the wave function on the left and on the right.

A requirement for treating the wave function of the combined system as a whole is that there is phase coherence throughout the system. This requires the inelastic mean free path to be larger than the width of the bilayer. One can estimate the inelastic mean free path in the following way. Static impurities including the presence of the interface will affect the elastic scattering time but only inelastic processes such as scattering by phonons will affect the inelastic scattering time. Thus an estimate of the inelastic time τ_i can be obtained from the total scattering time of very pure materials. At 4.2 K, the mean free path of typical metals such as Cu is of the order of 0.3 cm. The experimental sample width can be of the order of microns. The interface may affect τ_i by changing the nature of the interface phonons. This operates when the electron is at the interface [8]. But since the interface width is much less than the inelastic mean free path, we expect the correction to be small. Thus we expect the inelastic mean free path to be much larger than the width of the trilayer. There is independent evidence that the inelastic mean free path is larger than the sample width. If it were smaller, the experimental results would be temperature dependent. Thus phase coherence seems to be a reasonable assumption. Experiments are currently being carried out where the trilayer widths are of the order of 100 monolayers [9]. For these cases, phase coherence is even less of an issue. Elastic scattering is not a problem in the calculation we performed because the L that we use is less than the mean free path of a noble metal. However, for some of the films of Ref. [1] the thickness of the paramagnet is greater than the elastic mean free path. For those cases, one might have to modify the present calculation to include elastic scattering.

We next discuss possible ways by which the above suggestion can be tested experimentally. If the effect of the ferromagnet is unimportant, the magnitude of h_R is the same whether the ferromagnets are parallel or antiparallel. We found that the magnitude of the voltage difference h_R when F_1 and F_2 are parallel is different from that when they are antiparallel to each other. This can be measured experimentally. The present calculation predicts dependences of the voltage on the energy splitting between the spin up and spin down band of the ferromagnet. One may be able to tune this near the phase transition of the ferromagnet. If a ferromagnet with a relatively low transition temperature is used, this may be experimentally accessible. As the temperature is increased, the inelastic scattering length will be decreased. If it becomes smaller than the width of the paramagnet, the value of χ may be changed. For pure Cu, the mean path changes from 0.3 cm at 4 K to 300 Å at 300 K. Another possibility will be to change the paramagnetic metal so that the work function difference on both sides is very different. As we discuss above, this would change U: the matching of the wave function at the boundary and hence the resulting magnetic susceptibility. A possibility would be to use an alkali metal for the paramagnet. Experimental measurement of the susceptibility of the trimetallic system will also provide insight into the validity of the above reasoning.

In this Letter, we assumed that the change in the magnetization, as a result of the current, is in the paramagnet. There may also be a change in magnetization in the ferromagnet that is not yet explicitly specified. A more sophisticated treatment would require careful treatment of this issue, which may entail an iteration.

To summarize, we calculated the magnetization potential of a model ferromagnetic/paramagnetic/ferromagnetic trilayer. We found that (1) this potential is enhanced by the proximity of the ferromagnetic layers, by as much as a factor of 30 for large spin-split ferromagnets; and (2) the magnitude of this potential is a strong function of the angle between the magnetization of the ferromagnets, especially for large spin splitting.

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